# A New CMOS 4Q-Multiplier Using Linear and Saturation Regions Complementally

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#### Abstract

A New Four-Quadrant (4Q) Multiplier complementally using linear and saturation regions of MOSFET (Metal Oxide Semiconductor Field Effect Transistor) is proposed. This multiplier operates in the region except for the threshold voltage  $V_T$  to zero, using a novel non-linearity cancellation method. The validity of proposed circuit is confirmed through HSPICE simulation.

# 1. Introduction

A multiplication of two signals is one of the most important signal operations in analog signal processing. The multiplier is used not only as computational building blocks but also as a programming element in systems such as filters, neural networks, mixers and modulators in communication systems. Thus far, several 4Q-multipliers have been reported [1][2][3]. However these 4Q-multipliers utilize only either in the saturation region or the linear region of MOSFET. Therefore the input range of 4Q-multipliers has been restricted and narrow. On the other hand, rail-to-rail components have been proposed to expand the input range of OTA circuit. One of them is the novel rail-to-rail VCCS (Voltage Controlled Current Sources) [4].

In this paper, we propose a novel non-linearity cancellation method and 4Q-multiplier, which operates in the saturation and linear regions of MOSFET in which the input range of the multiplier is expanded [6]. We have simulated the proposed circuit by using HSPICE. Finally, we confirm that the proposed circuit is useful as the 4Q-multiplier.

### 2. Non-linearity cancellation

The drain current  $I_{DS}$  of the simple MOSFET model is expressed as

$$I_{DS} = K(V_{GS} - V_{\rm T} - \frac{V_{DS}}{2})V_{DS}$$
(1)

for the linear region( $V_{GS} - V_T > V_{DS}$ ), and

$$I_{DS} = \frac{K}{2} (V_{GS} - V_{\rm T})^2$$
 (2)



Fig.1 Operating conditions of the multipliers. (a) Using  $V_{GS}V_{DS}$  (b) Using  $V_{GS}^2$ 

for the saturation region( $0 < V_{GS} - V_T < V_{DS}$ ), respectively, where  $V_{GS}$ ,  $V_{DS}$  and  $V_T$  are the gate-source, drain-source and the threshold voltages, and K (=(W/L) $\mu_0 C_{OX}$ ) is the transconductance parameter. When  $V_{GS}$  is lower than  $V_T$ , the transistor in the cut-off region.

The conventional architecture of MOS 4Q-multiplier is categorized in a few types, namely, multipliers that utilize the term  $V_{\rm GS}V_{\rm DS}$  or  $V_{\rm DS}^2$  of the drain current model (1) of MOSFET operating in the linear region, multipliers which utilize the term  $V_{\rm GS}^2$  of the drain current model (2) of MOSFET operating in the saturation region, and the Gilbert cells [5] etc. In the following sections, we discuss two conventional multiplier topologies and their dynamic ranges. One is the multiplier operating in the linear region using the term  $V_{\rm GS}V_{\rm DS}$  in (1) and another is the multiplier operating in the saturation region using the term  $V_{\rm GS}^2$  in (2).

### 2.1. Multiplier Using $V_{GS}V_{DS}$

Fig.1 (a) shows the operating condition of 4Qmultiplier using the term  $V_{GS}V_{DS}$  in (1). In Fig. 1 (a), the transistor operates in the linear region when proper bias voltages X and Y are provided, where x and y indicate the input signals. In this configuration, the drain current  $I_d$  of the transistor is given by

$$I_{d} = K \left\{ X + x - V_{\rm T} - \frac{1}{2} (Y + y) \right\} (Y + y), \qquad (3)$$

according to the drain current model of MOSFET operating in the linear region (1). Then the desired term xy can be obtained in (3). All the higher order terms and common-mode components (X, Y and  $V_T$ ) can be



Fig.2 Basic elements of proposed circuits.

cancelled with the subtractions of the currents by using current mirrors circuits.

# **2.2.** Multiplier Using $V_{GS}^2$

Fig.1 (b) shows the operating condition of 4Qmultiplier using the term  $V_{GS}^2$  in (2). In Fig.1 (b) the transistor M<sub>B</sub> operates in the saturation region, where x and y are the input voltages, and X and Y indicate the bias voltages. The drain current is given by

$$I = K \{ (X + x) - (Y + y - V_{\rm T}) - V_{\rm T} \}^2, \qquad (4)$$

from the substitutions the voltages into the drain current model of MOSFET operating in the saturation region (2). Then the desired term xy can be obtained in (4). Undesired terms can be cancelled with differential configuration by using current mirrors circuits.

# 2.3. Input Range of Multipliers

The input voltages for the configurations of Fig.1 (a) and Fig. 1 (b) have the following restrictions.

In Fig.1 (a), the input voltages must be satisfied with (5) to make the transistor operate in the linear region.

$$V_{SS} < X + x - V_{\rm T} < Y + y < V_{DD} , \qquad (5)$$

where  $V_{DD}$  and  $V_{SS}$  represent the positive and negative power supply voltages.

In Fig.1 (b), on the other hand, the input voltages have to be satisfied with (6) to make the transistor operate in the saturation region, where  $V_d$  is the drain voltage of the transistor.

$$V_{SS} < Y + y < X + x - V_{\rm T} < V_d < V_{DD} .$$
(6)

Due to the restrictions mentioned above, the input ranges of the multipliers are restricted and narrowed.

# 3. VCCS circuit

Fig.2 shows the basic elements of proposed circuit, which consists of the transistors  $M_A$  and  $M_B$ . In this figure,  $V_{in}$ ,  $V_C$ ,  $I_{DA}$  and  $I_{DB}$  are the input voltage, the control voltage, the drain current flowing in the transistor  $M_A$  and the drain current flowing in the transistor  $M_B$  respectively. To make the transistors  $M_A$ and  $M_B$  operate complementally, the drain voltages  $V_{DA}$ and  $V_{DB}$  are determined as follows,

$$V_{DA} = V_C - V_T,$$
  

$$V_{DA} = V_C - V_T.$$
(7)





Fig.4 I/V Characteristics of VCCS.

For (7), let the point of transition from saturation region to linear one for  $M_A$  be equal to that from linear region to saturation one for  $M_B$ .

Now we name the element as VCCS, which consists of the transistors  $M_A$  and  $M_B$ , whose output current  $I_{out}$  is the sum of the drain currents  $I_{DA}$  and  $I_{DB}$  flowing in the transistors  $M_A$  and  $M_B$ .

When the input voltage  $V_{in}$  is less than the control voltage  $V_C$ , the transistor  $M_A$  operates in the saturation region and the transistor  $M_B$  operates in the linear region.

Therefore the output current  $I_{out}$  is given by

$$I_{out} = I_{DA} + I_{DB}$$
  
=  $\frac{K}{2} (V_{in} - V_{T})^{2}$   
+  $K (V_{C} - V_{T} - \frac{V_{in} - V_{T}}{2}) (V_{in} - V_{T})$   
 $I_{out} = K (V_{in} - V_{T}) (V_{C} - V_{T}).$  (8)

On the other hand, in the case of the input voltage  $V_{\rm in}$  is higher than the control voltage  $V_{\rm C}$ , the transistor  $M_{\rm A}$  operates in the linear region and the transistor  $M_{\rm B}$  operates in the saturation region. Therefore the output current  $I_{\rm out}$  is given by



Fig.5 proposed 4Q-multiplier circuits.

$$I_{out} = I_{DA} + I_{DB}$$
  
=  $K(V_{in} - V_{\rm T} - \frac{V_C - V_{\rm T}}{2})(V_C - V_{\rm T})$   
+  $\frac{K}{2}(V_C - V_{\rm T})^2$   
 $I_{out} = K(V_{in} - V_{\rm T})(V_C - V_{\rm T}).$  (9)

From (8) and (9), the undesired terms are cancelled so that the output current  $I_{out}$  yields a multiplication of  $V_{in}$ and  $V_C$ . Therefore this VCCS operates as the multiplier between the threshold voltage  $V_T$  and the supply voltage  $V_{DD}$ . The proposed VCCS circuit is shown in Fig.3. Here the transistors  $M_{AS}$  and  $M_{BS}$  are fixed to operate in the saturation region. Then the drain voltages of the transistors  $M_A$  and  $M_B$  are

$$V_{DA} = V_C - V_T - \sqrt{\frac{2I_{DA}}{K_s}} , \qquad (10)$$

$$V_{DB} = V_{in} - V_{\rm T} - \sqrt{\frac{2I_{DB}}{K_{\rm S}}},$$
 (11)

from (2), where  $K_S$  represents the aspect ratio of the transistors  $M_{AS}$  and  $M_{BS}$ .

Substituting (1) or (2) into (10), the drain voltage  $V_{DA}$  of the transistor  $M_A$  is given by

$$V_{DA} = V_C - V_T - \sqrt{\frac{K}{K_S}} (V_{in} - V_T), \qquad (12)$$

when the transistor  $M_{\text{A}}$  operates in the saturation region, and

$$V_{DA} = V_{C} - V_{T} - \sqrt{\frac{K}{K_{S}}} \sqrt{(V_{in} - V_{T} - \frac{V_{C} - V_{T}}{2})(V_{C} - V_{T})}$$
(13)

when the transistor M<sub>A</sub> operates in the linear region.

If  $K_S$  is sufficiently larger than K, the third term  $(K/K_S)^{1/2}$  of (12) and (13) can be assumed to be zero. Then the drain voltage of transistor  $M_A$  becomes  $V_C - V_T$ . Similarly the drain voltage of transistor  $M_B$  becomes  $V_{in} - V_T$ , thus these are satisfied with (7). When  $V_{in}$  is lower than the threshold voltage  $V_T$ , the transistors  $M_A$ and  $M_{SB}$  are in the cut-off region and the output currents become zeros.

The performances of the proposed VCCS circuits are analysed through the use of HSPICE simulator. Fig. 4 shows the *I*-V curves of the proposed VCCS circuit in Fig.3. The power supply voltages  $V_{DD}$  and  $V_{SS}$  for the circuit in Fig. 3 are 1.5V and -1.5V respectively. Here, the input voltage  $V_{in}$  and the control voltage  $V_C$  are varied from -1.0V to 1.5V. From Fig.4, it is clear that the proposed VCCS circuit operates as the multiplier between the threshold voltage  $V_T$  and the supply voltage  $V_{DD}$ .

# 4. 4Q-Multiplier

We propose the 4Q-multiplier, which consists of 4 similar VCCS circuits. Fig.5 shows the proposed 4Q-multiplier. The output currents  $I_1$ ,  $I_2$ ,  $I_3$ , and  $I_4$  of VCCS are shown as follows,

$$I_1 = K(V_{X1} - V_T)(V_{Y1} - V_T), \qquad (14)$$

$$I_2 = K(V_{X2} - V_T)(V_{Y1} - V_T), \qquad (15)$$



$$I_{3} = K(V_{X2} - V_{T})(V_{Y2} - V_{T}), \qquad (16)$$

$$I_4 = K(V_{X1} - V_T)(V_{Y2} - V_T).$$
(17)

Here, we define the current  $I_{o1}$  as sum of the output currents  $I_1$  and  $I_3$ , whereas  $I_{o2}$  as of  $I_2$  and  $I_4$ . Therefore the difference between output currents  $I_{o1}$  and  $I_{o2}$  yields a multiplication of  $(V_{X1}-V_{X2})$  and  $(V_{Y1}-V_{Y2})$ .

$$I_{out} = I_{o1} - I_{o2}$$
  
=  $(I_1 + I_3) - (I_2 + I_4)$   
=  $K(V_{X1} - V_{X2})(V_{Y1} - V_{Y2})$  (18)

From (18), the output current  $I_{out}$  can be realized precisely, because it is independent of the threshold voltage  $V_{\rm T}$ . Fig.6 shows the *I-V* curves of 4Q-multiplier in Fig.5. The power supply voltages  $V_{DD}$  and  $V_{SS}$  are 1.5V and -1.5V respectively. Here, the input voltages  $V_{X1}$  and  $V_{Y1}$  are varied from -1.0V to 1.5V and the input voltage  $V_{X2}$  and  $V_{Y2}$  are 0.5V. From Fig.6, we have confirmed that the proposed 4Q-multiplier operates as the 4Q-multiplier for wide input ranges compared to the power supply voltages. Fig.7 shows the multiplications of two sinusoidal waves, which are 25kHz(dashed) and 2.5kHz(solid). In this simulation, the input voltages  $V_{X1}$ ,  $V_{X2}$ ,  $V_{Y1}$  and  $V_{Y2}$  are set as below,

$$V_{x1} = V_{DC} + A\sin(2\pi f_1), \qquad (19)$$

$$V_{x2} = V_{DC} - A\sin(2\pi f_1), \qquad (20)$$

$$V_{y_1} = V_{DC} + A\sin(2\pi f_2), \qquad (21)$$

$$V_{\rm v2} = V_{\rm PC} - A\sin(2\pi f_2), \qquad (22)$$

where the bias voltage  $V_{DC}$ =0.5[V], the amplitude A=1.0[V], frequency  $f_1$ =25[kHz], frequency  $f_2$ =2.5[kHz] respectively. As a result we confirmed that the proposed circuit carries out the multiplication of two signals.



Fig.7 Multiplication of two sinusoidal waves.

# 6. Conclusion

In this paper, we have proposed a novel 4Q-multiplier, which uses complementally in the saturation and linear regions of MOSFET. Thus the proposed 4Q-multiplier operates between the threshold voltage  $V_{T}$  and the supply voltage  $V_{DD}$ . Furthermore we have simulated the proposed circuit by using HSPICE. As a result, we have confirmed that the proposed circuits are useful as the 4Q-multiplier.

- H. Song, and C. Kim, "An MOS Four-Quadrant Analog Multiplier Using Simple Two Squaring Circuit with Source Followers, "IEEE J. Solid-State Circuits, Vol.25 pp. 741-748 NO.3 June 1990.
- [2] S. Liu and Y. Hwang, "CMOS Four Quadrant Multiplier Using Bias Feedbacks, "IEEE J. Solid-State Circuits, Vol.29, pp. 750-752 NO.3 June 1994.
- [3] B. Gilbert, "A precision four-quadrant multiplier with subnanosecond response," IEEE J. Solid-State Circuits, vol. SC3, pp. 353-365, Dec. 1968.
- [4] T. Sato, S. Takagi, and N. Fujii, "Rail-to-Rail OTA Using One Kind MOSFET's as a VCCS," Technical meeting on Electronic Circuits, ECT-00-95, 2000.
- [5] Han, G. and Sánchez-Sinencio, "CMOS Transconductance Multipliers: A Tutorial," IEEE Trans. on Circuit and Systems II, vol. 45, No. 12, Dec 1998.
- [6] T.Suzuki, T.Oura, T.Yoneyama, H.Asai, "Design of 4Q-Multiplier Using Linear and Saturation Regions Complementally," IEICE Trans. on Fundamentals, vol. E85-A, No. 6, Jun 2002. (to appear)