

High Speed Pipelined ADCs: Fundamentals and Variants

Ahmed M. A. Ali
Analog Devices

2018

Outline

- ◆ Introduction
- ◆ Pipelined ADCs
- ◆ Design challenges
 - Quantizer linearity
 - Noise
- ◆ Digitally assisted converters

INTRODUCTION

ADC Performance Metrics

Some of the Most Commonly Used

- ◆ **Sampling Rate**
- ◆ **Resolution**
- ◆ **Signal-to-Noise Ratio (SNR)**
- ◆ **Signal-to-Noise-and-Distortion Ratio (SNDR or SINAD)**
- ◆ **Spurious-Free Dynamic Range (SFDR)**
- ◆ **Inter-modulation Distortion (IMD)**
- ◆ **Integral Non-linearity (INL)**
- ◆ **Differential Non-linearity (DNL)**
- ◆ **Offset and Gain Error**
- ◆ **Bit-Error-Rate (BER)**
- ◆ **Power**
- ◆ **Jitter**

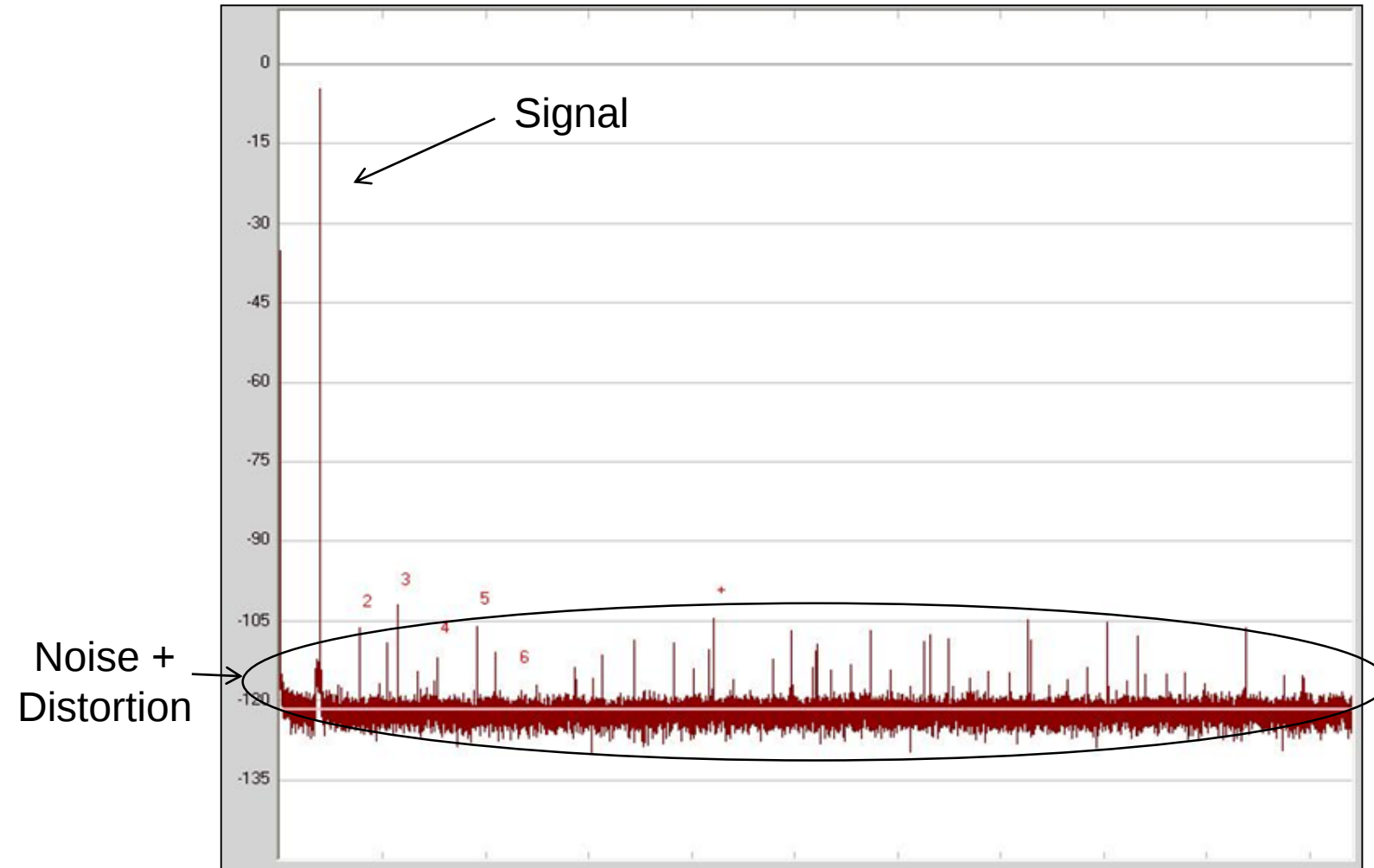
Signal-to-Noise Ratio (SNR)

Signal-to-Noise-and-Distortion Ratio (SNDR or SINAD)

- ◆ SNR/SINAD are defined as the ratio of the signal power to the noise power excluding DC:
 - $\text{SNR} = 10\log(\text{Signal Power} / \text{Noise Power excluding harmonics})$
 - $\text{SINAD} = 10\log(\text{Signal Power} / (\text{Noise} + \text{Harmonic Power}))$
- ◆ They can be specified in dBc or dBFS
- ◆ Usually specified using a single tone input signal
- ◆ Sources:
 - Quantization noise and distortion
 - Thermal noise in the signal path
 - Noise and distortion in the clock path (Jitter)
 - Non-linearity in the signal path
- ◆ Effective Number of Bits (ENOB) = $(\text{SINAD} - 1.76\text{dB}) / 6.02$
- ◆ Effective Resolution = Full scale / Input referred noise in LSBs

Example: AD9467 (AD80264) at 250MS/s

SNR = 74dB, SINAD = 74dB

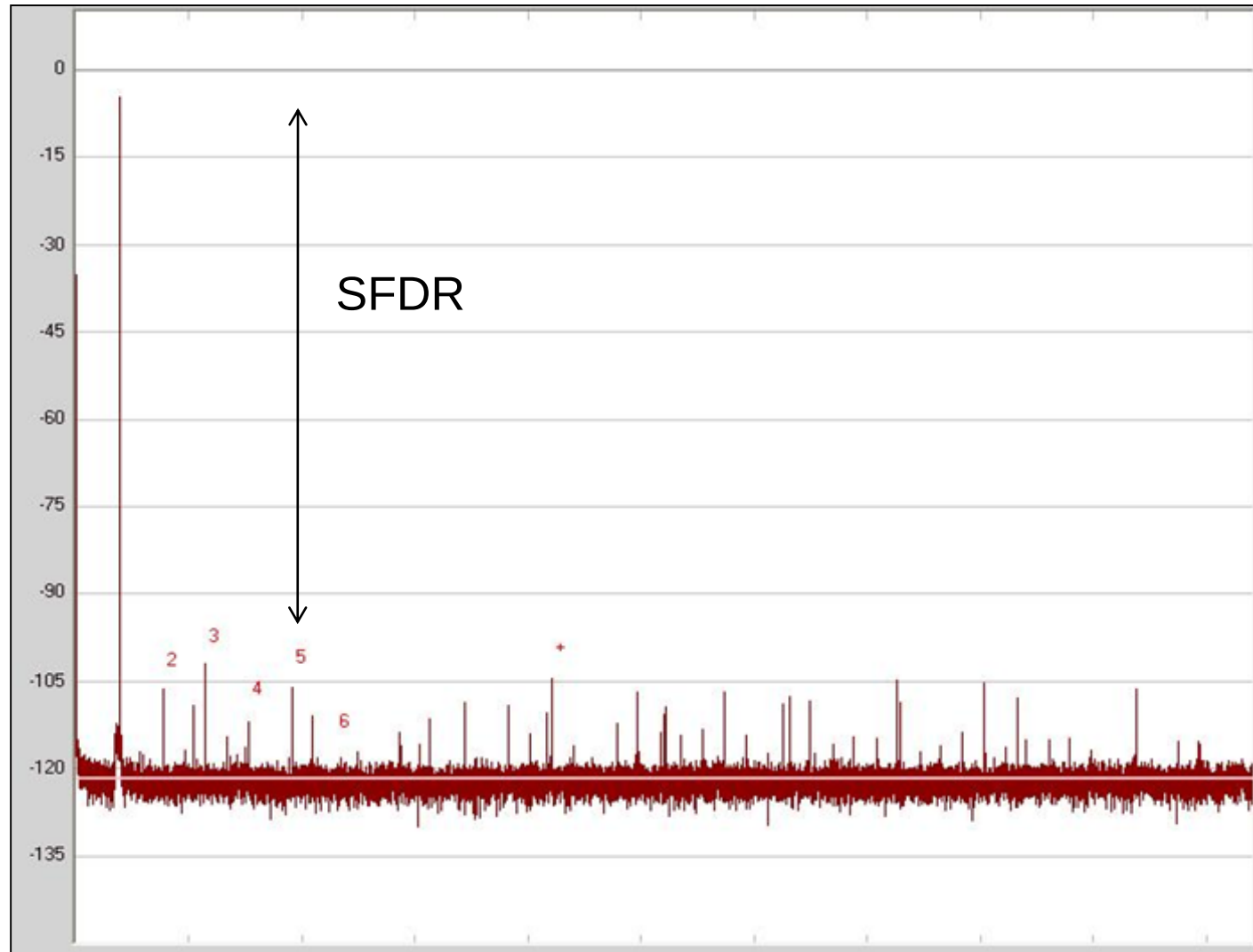


Spurious-Free Dynamic Range (SFDR)

- ◆ It is the ratio of the signal power to the power of the largest undesired harmonic or spur
- ◆ SFDR can be defined in dBc or dBFS
- ◆ Usually specified using a single tone input signal
- ◆ It is usually evaluated across the input amplitude
- ◆ THD is the ratio of the signal power to the total harmonic power of the significant harmonics

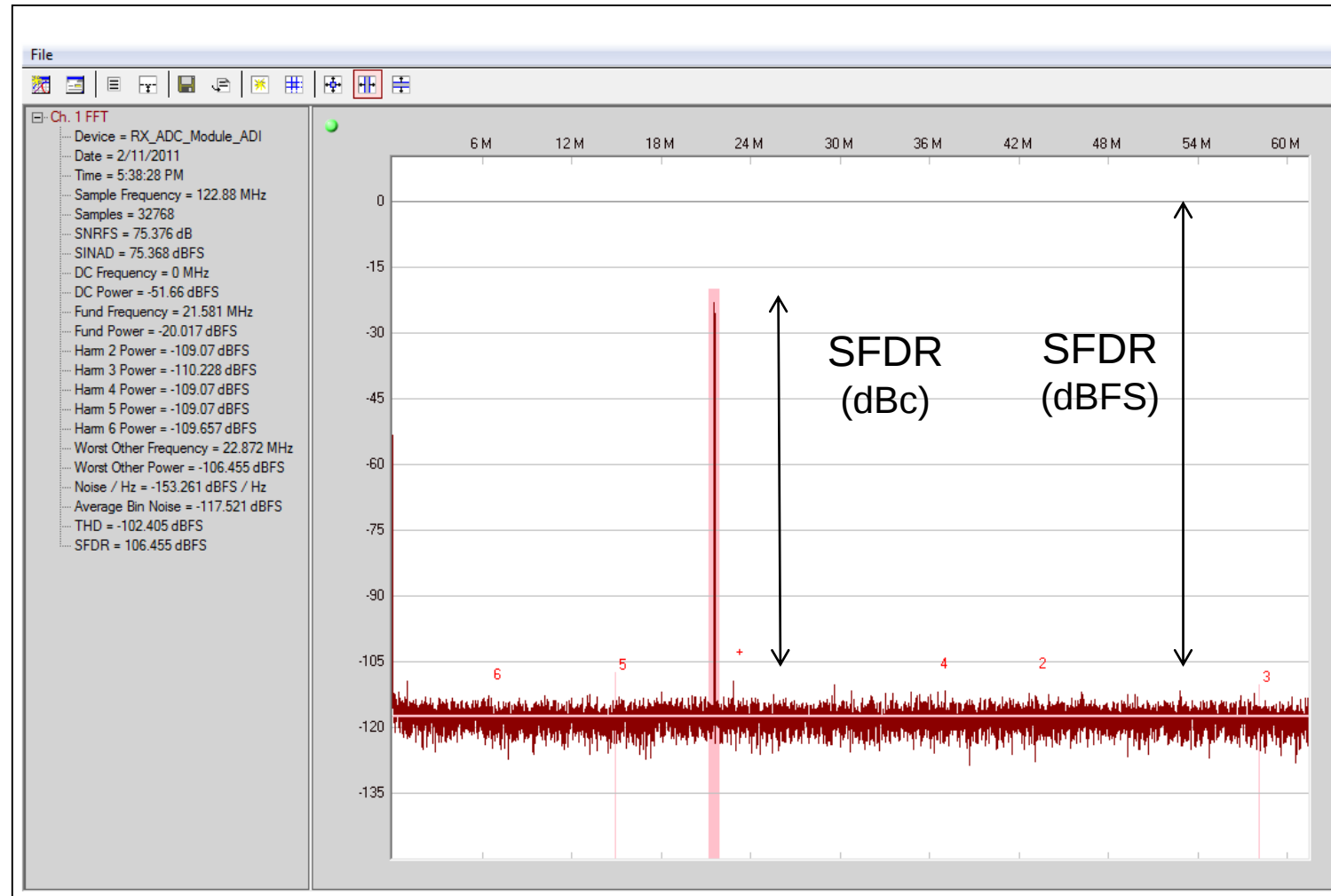
Example: AD9467 at 250MS/s

SFDR = 100 dBc

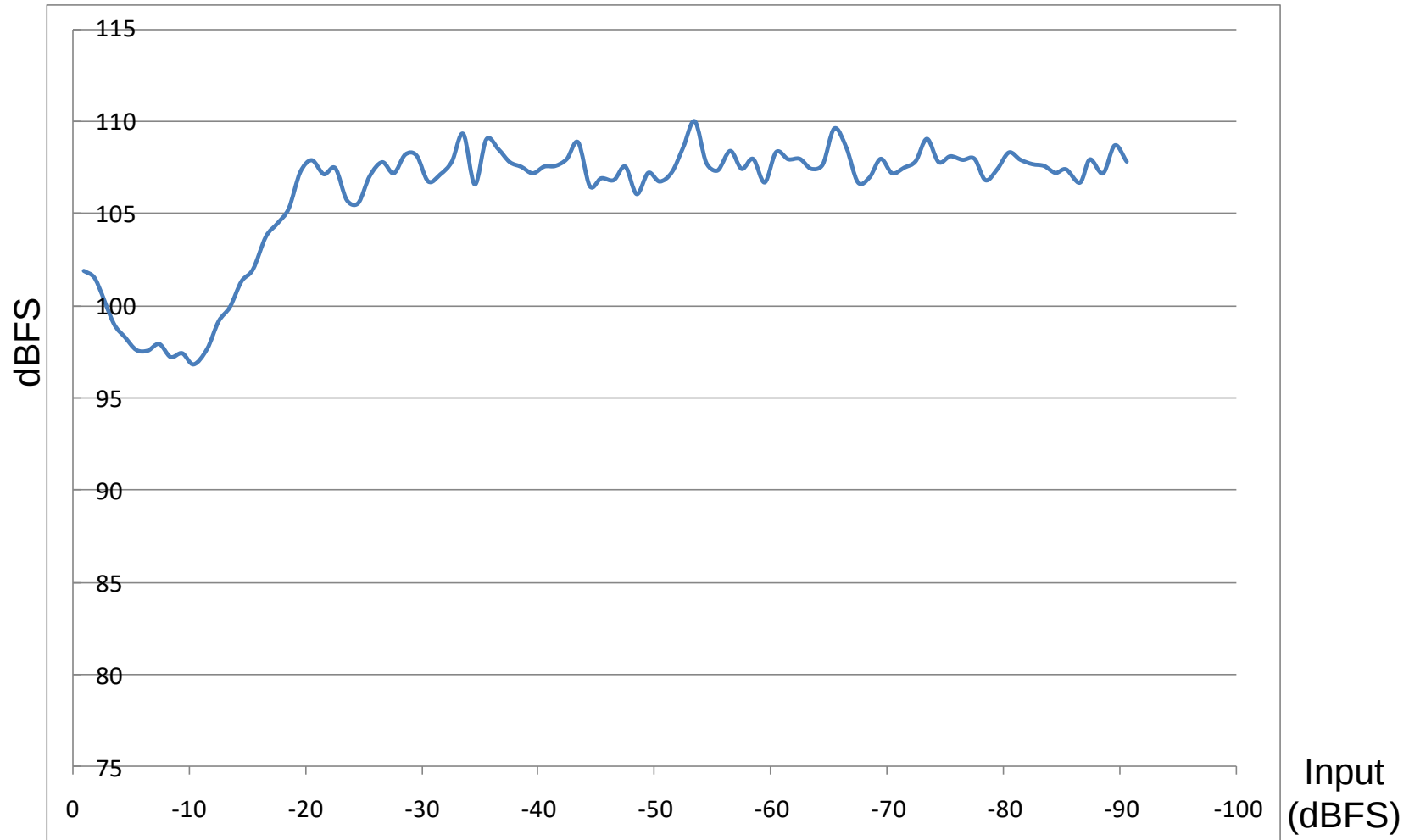


Example: AD9467 at 122.88 MS/s

At -20 dBFS, SFDR = 106 dBFS = 86dBc

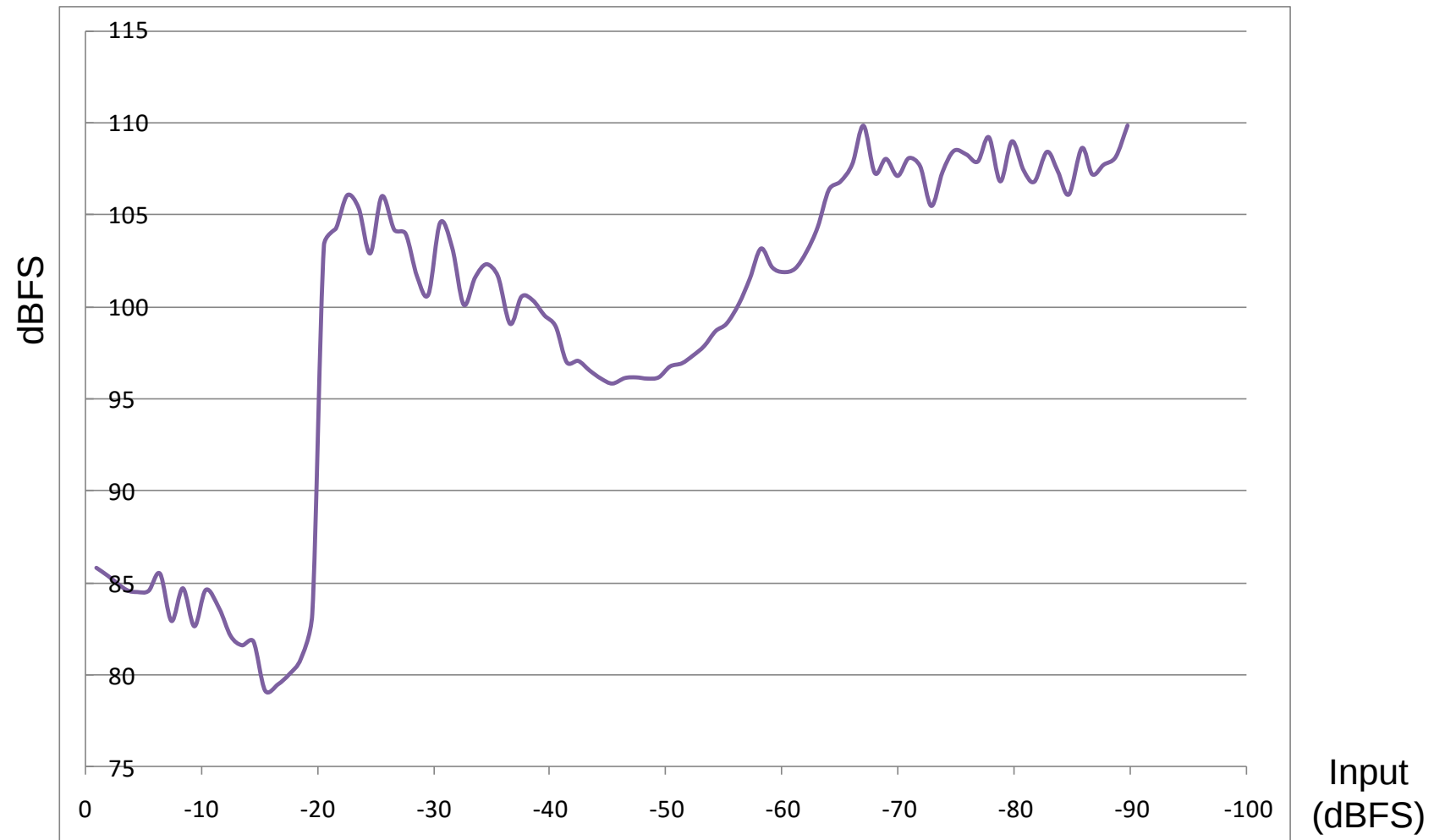


SFDR versus Input Amplitude

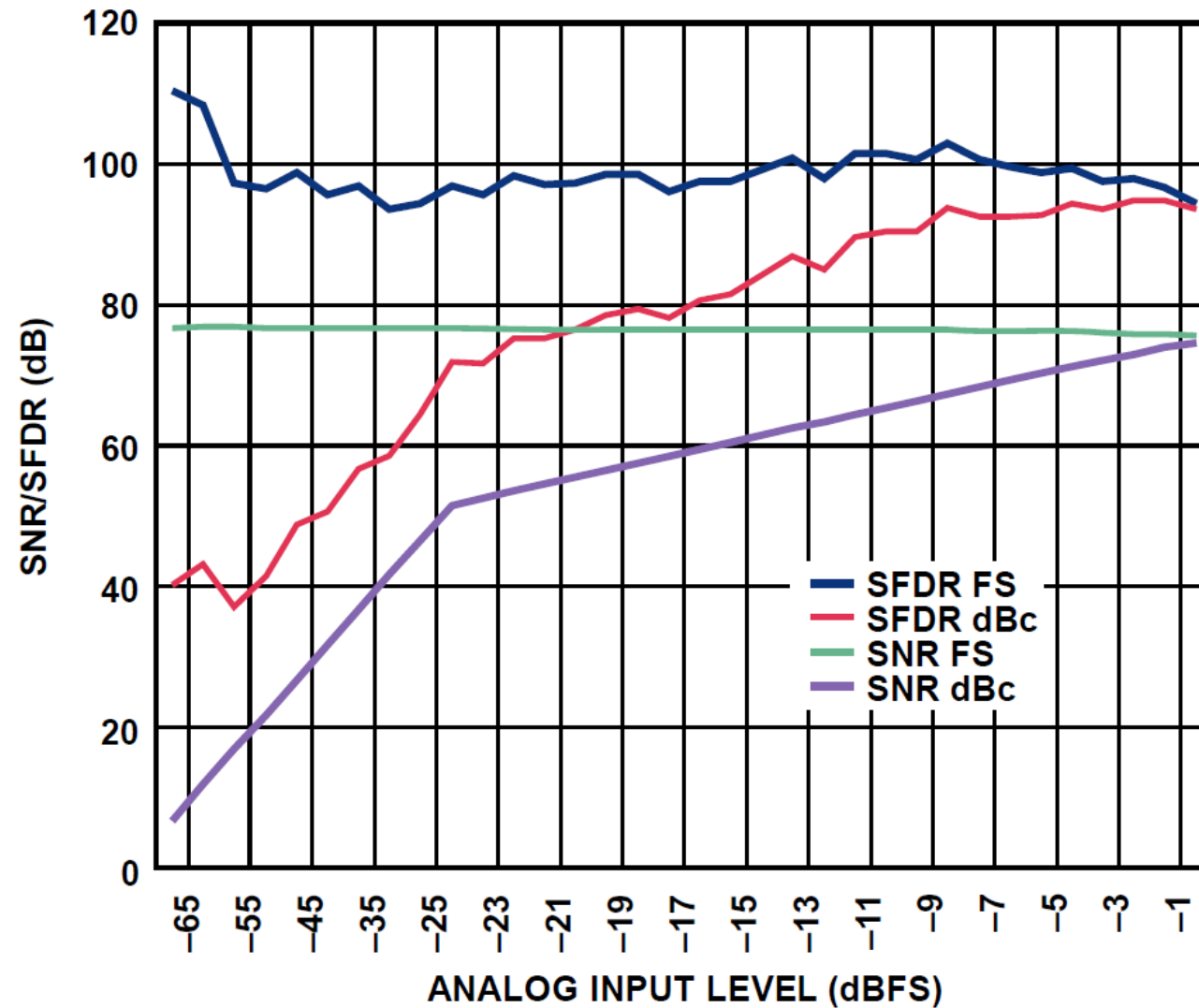


SFDR versus Input Amplitude

Example



SFDR versus Input Amplitude (Example)

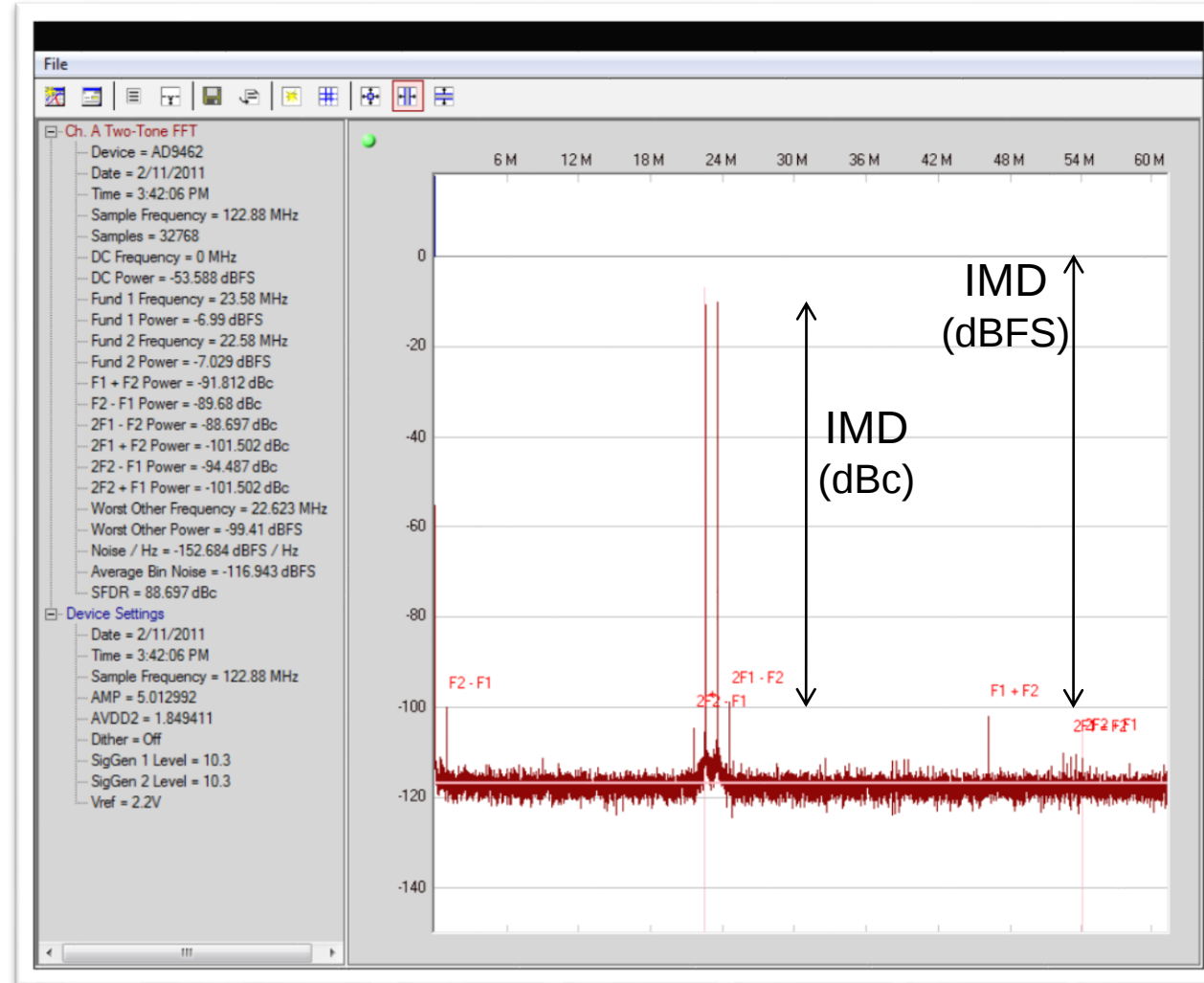


Inter-Modulation Distortion (IMD)

- ◆ It is tested using two input tones equal in amplitude and closely spaced in frequency (f_1 and f_2)
- ◆ It is the ratio of the power of one of the input tones to the power of the largest undesired spur
- ◆ It is useful in some systems to evaluate distortion without the effects of filtering or frequency planning
- ◆ The second order products (IM2) are at:
 - (f_1+f_2) and (f_1-f_2)
- ◆ The third order products (IM3) are at:
 - $(2f_1+f_2)$ and $(2f_2+f_1)$
 - $(2f_1-f_2)$ and $(2f_2-f_1)$

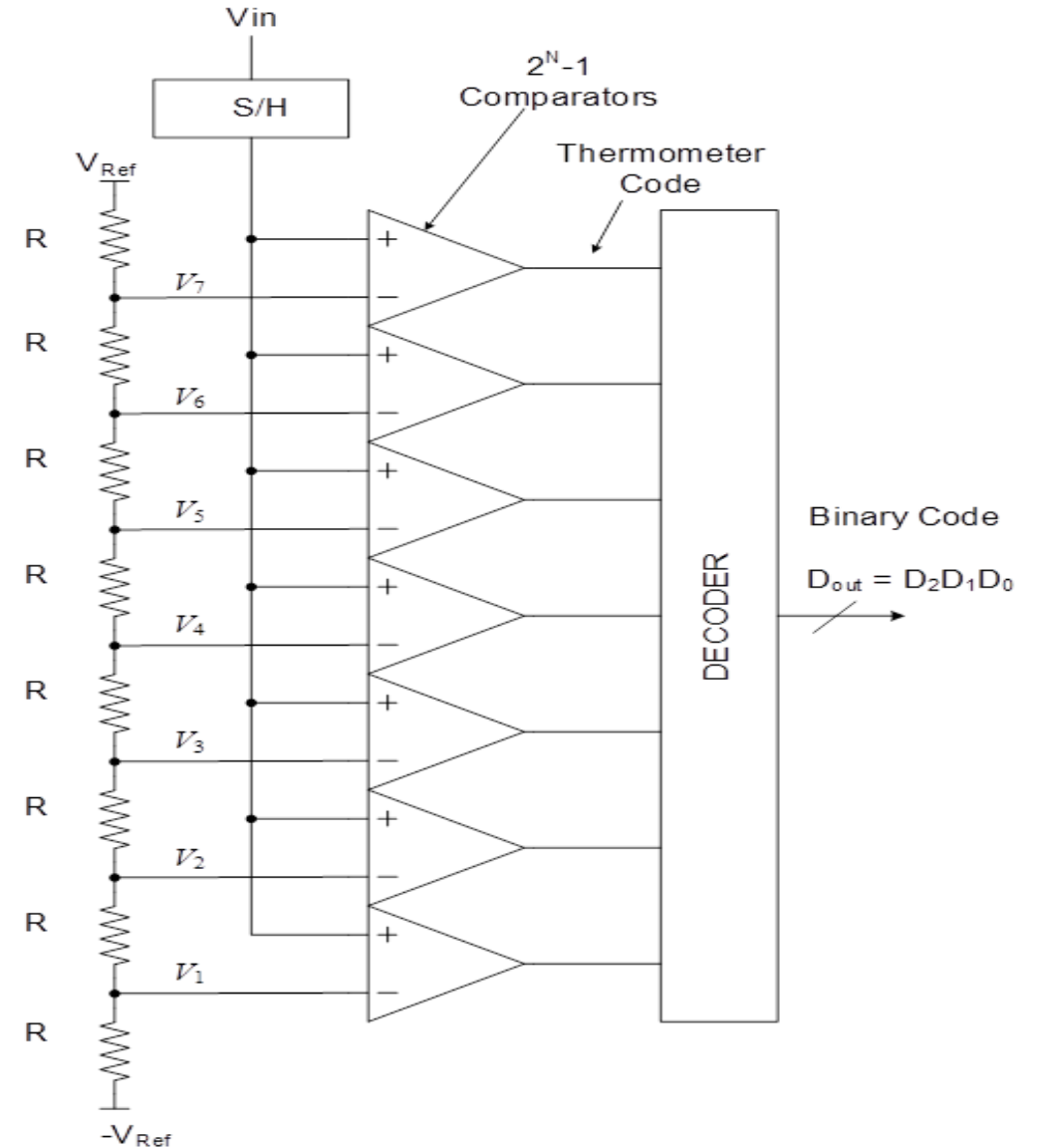
Example: AD9467 at 122.88MHz

IMD = -89dBc (-96dBFS)



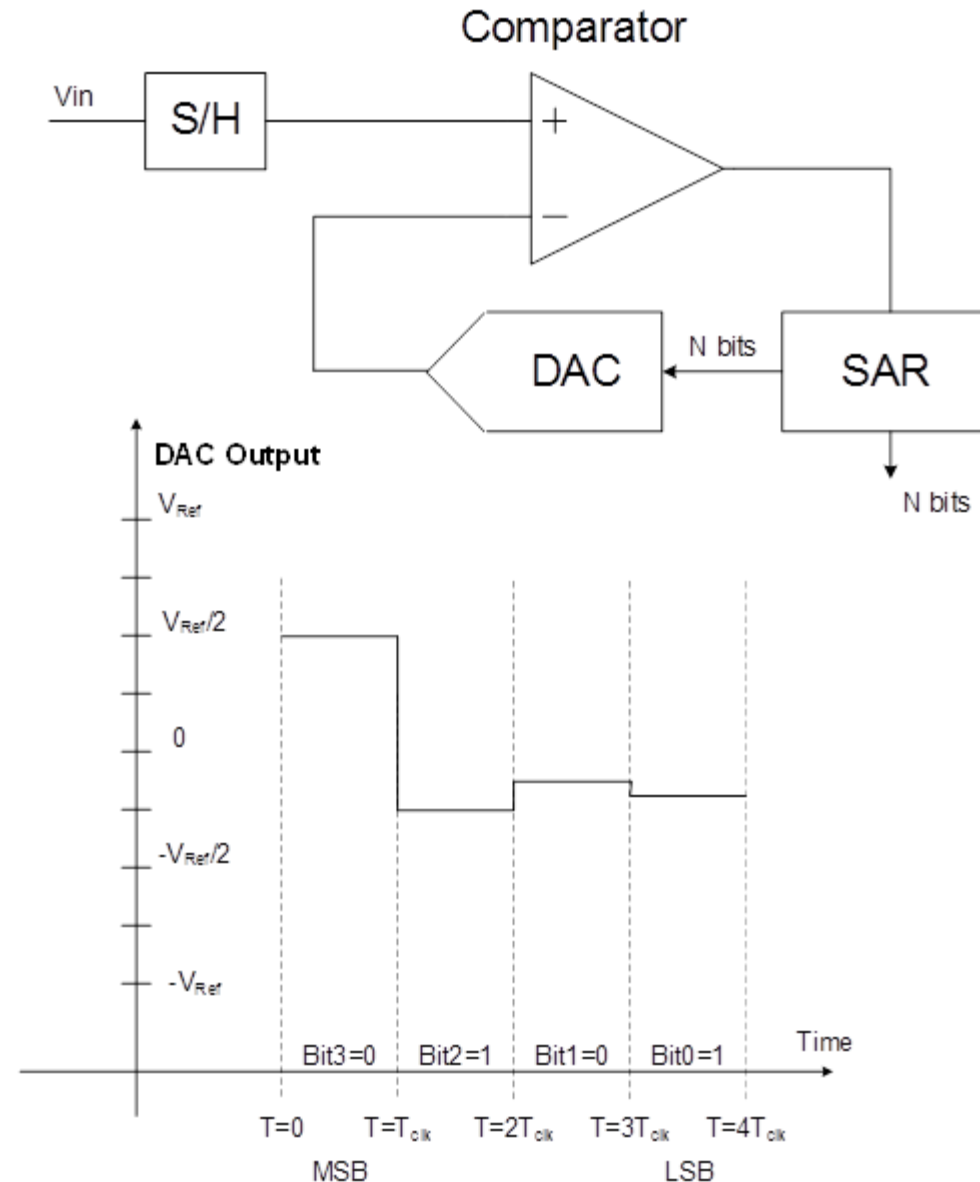
Flash A/D Converters

- ◆ Compares input to all levels simultaneously for maximum speed
- ◆ Very high speed (GS/s):
 - Sample rate $f_s = f_{clk}$
- ◆ Not efficient for high resolutions:
 - Area & Power $\propto 2^N$
- ◆ Limited accuracy: (6-bit)
- ◆ Variations include: Interpolation and Folding Architectures

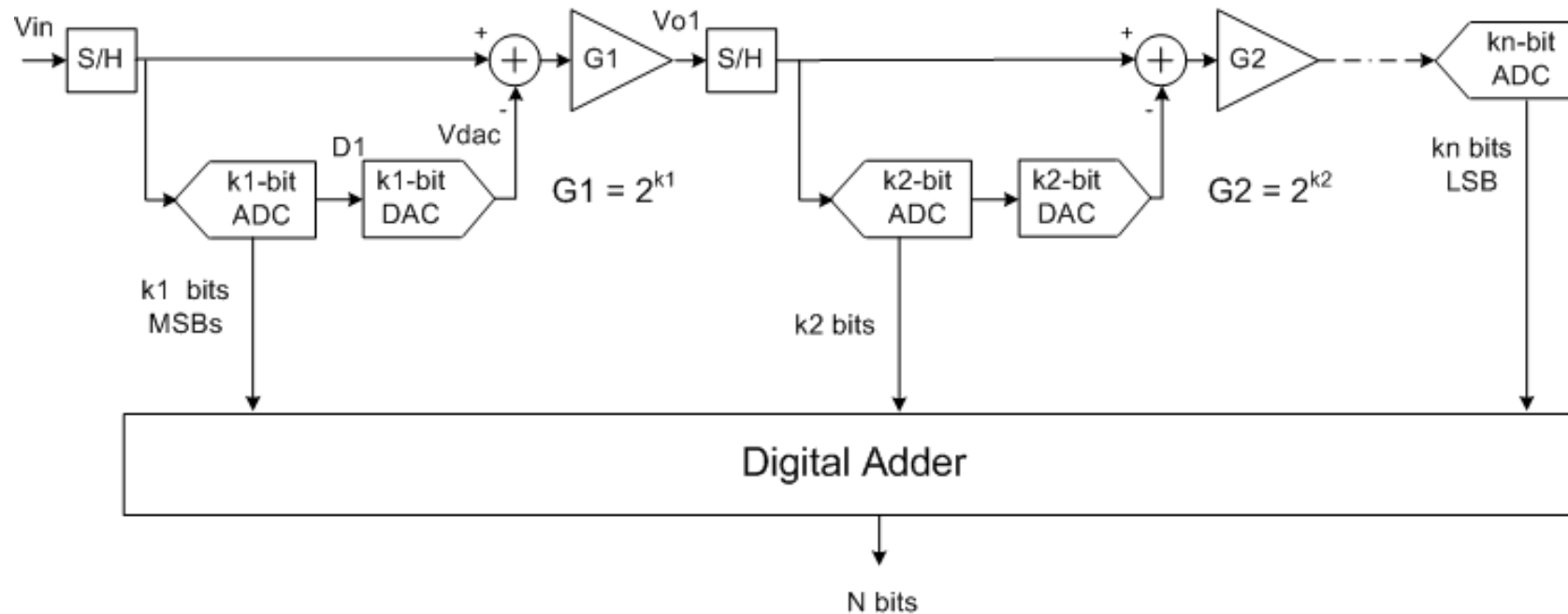


SAR A/D Converters

- ◆ Quantize the input by successively approaching the right answer using a binary search algorithm
- ◆ Very efficient in power and hardware
- ◆ Relatively low speed:
 - Conversion time proportional to N
- ◆ Small acquisition time
- ◆ High accuracy
- ◆ Currently: 18-20+ bit, 10-20+ MS/s



Pipelined A/D Converter



$$D_1 = 0, \pm 1, \pm 2, \pm 3 \dots, \pm 2^{k_1-1}$$

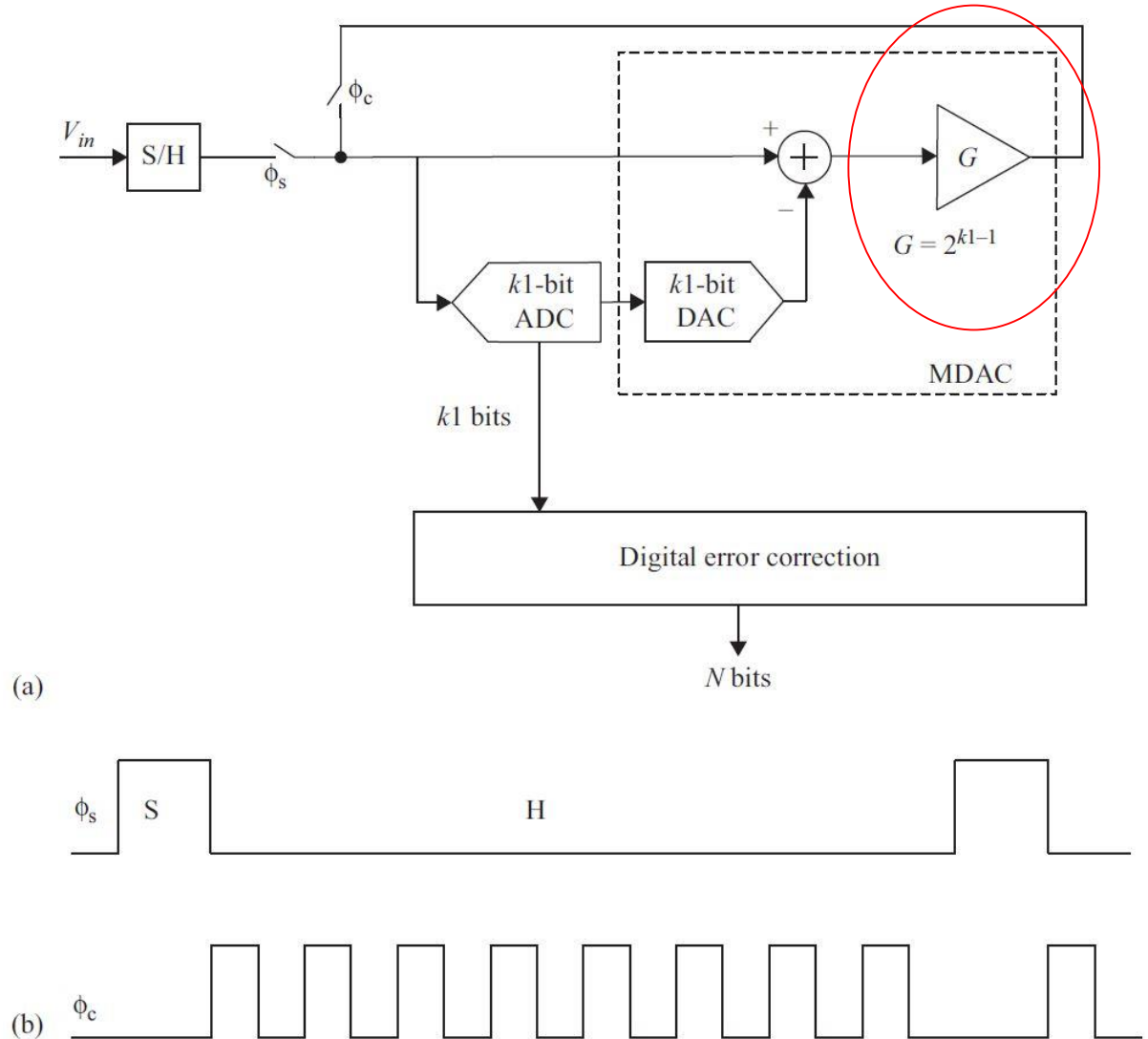
$$V_{o1} = G_1 (V_{in} - V_{dac})$$

$$V_{o1} = G_1 \left(V_{in} - \frac{D_1 \cdot V_{Ref}}{2^{k_1-1}} \right)$$

$$G_1 = 2^{k_1}$$

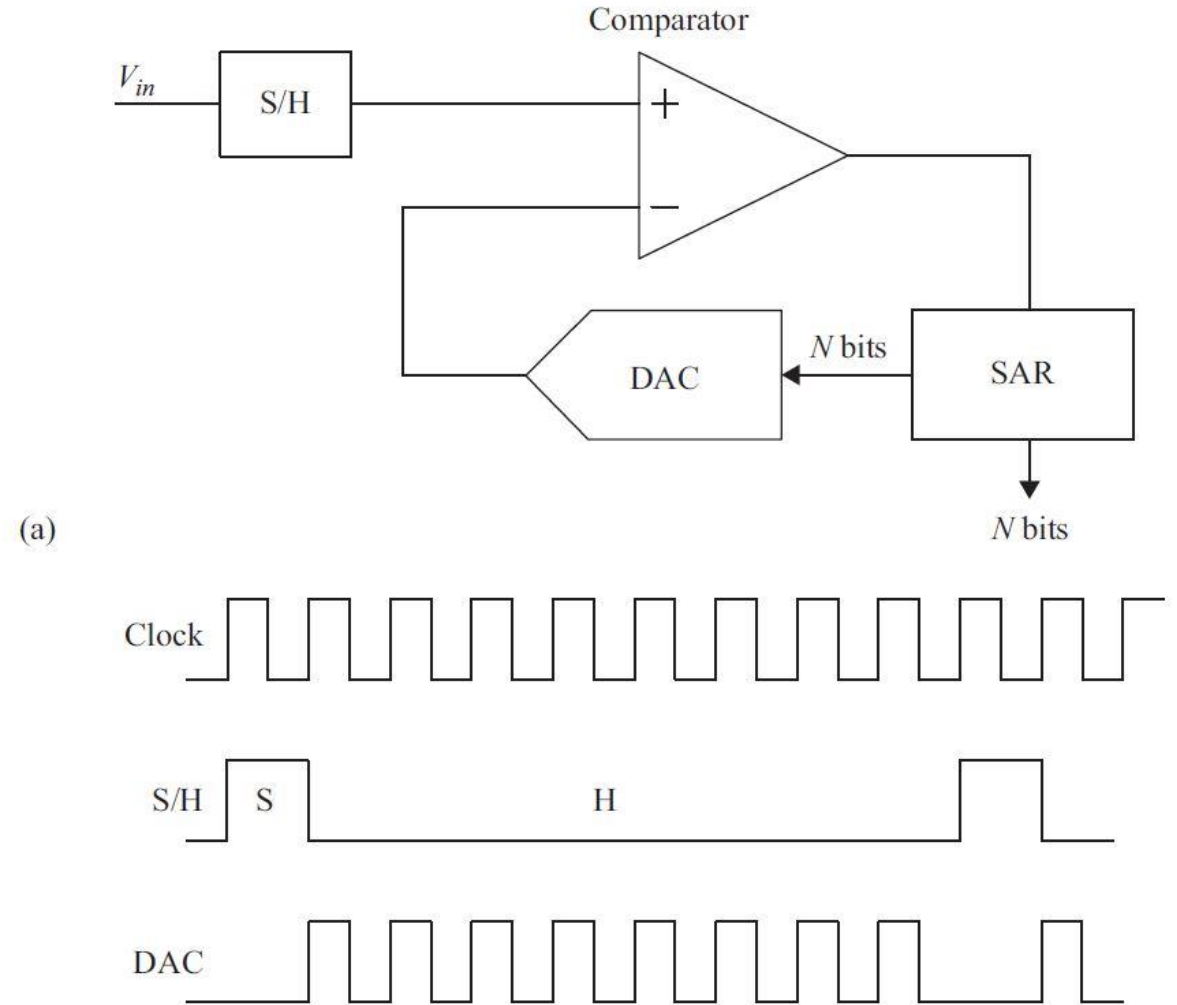
Algorithmic/Cyclic (pipelined) ADC

- ◆ Folding the n pipeline stages into one stage
- ◆ For n cycles $\Rightarrow$ n times slower
- ◆ We need n in parallel to achieve the same speed
- ◆ Inter-stage amplifier is needed



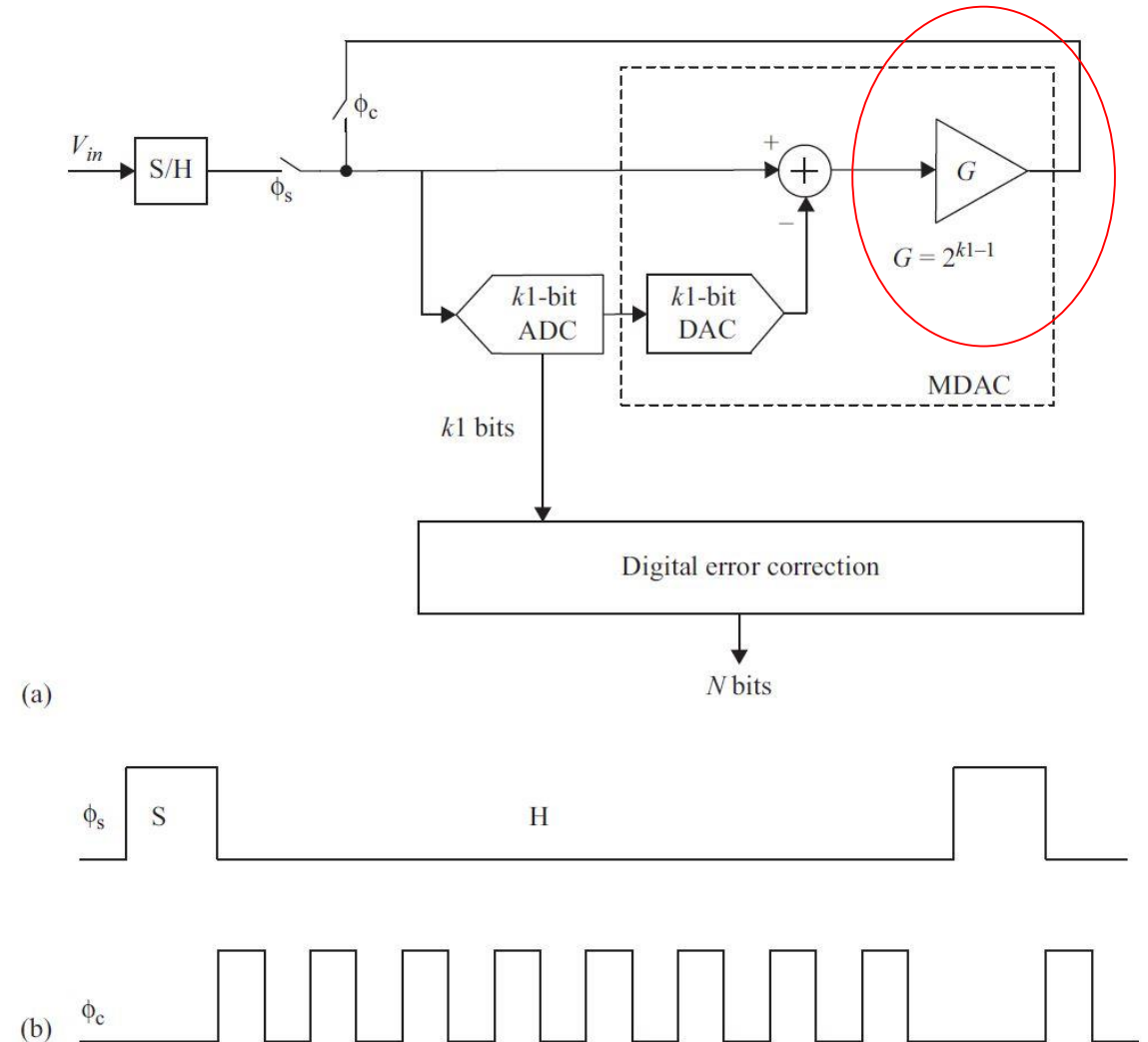
SAR ADC

- ◆ Folding the n pipeline stages into one stage
- ◆ For n cycles $\Rightarrow$ n times slower
- ◆ We need n in parallel to achieve the same speed
- ◆ No inter-stage amplifier is needed



Algorithmic/Cyclic (pipelined) ADC

- ◆ Folding the n pipeline stages into one stage
- ◆ For n cycles $\Rightarrow$ n times slower
- ◆ We need n in parallel to achieve the same speed
- ◆ Inter-stage amplifier is needed
- ◆ If cost of amplifier is reduced substantially
 - $\Rightarrow$ similar to SAR ADC
- ◆ If unfolded into a pipeline
 - $\Rightarrow$ n -times faster
 - $\Rightarrow$ more efficient and higher performance than n parallel SARs



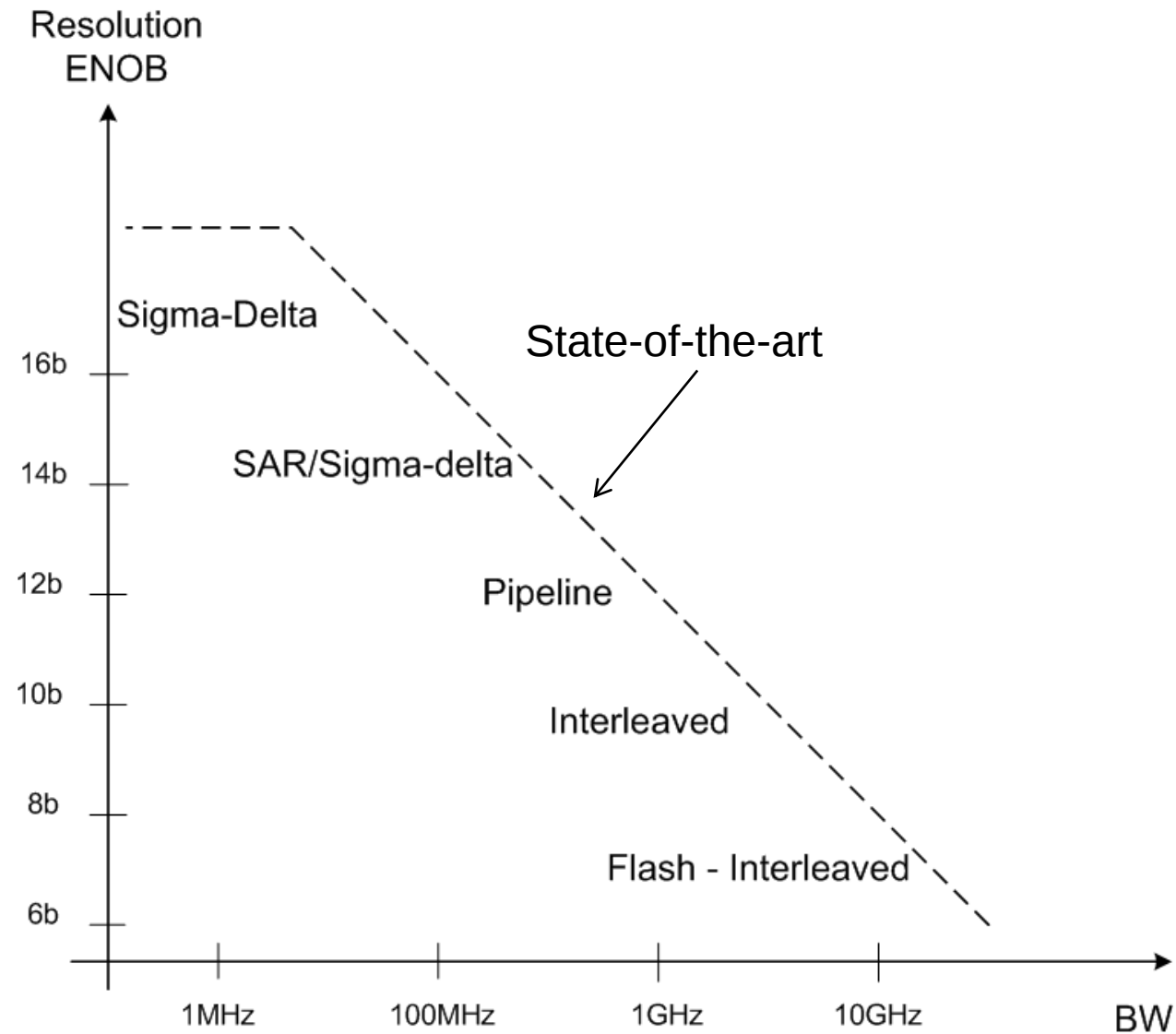
Pipelined A/D Converters

High-Speed and High-Resolution

- ◆ Employs the pipelining approach to work on multiple samples simultaneously
 - => long conversion time (latency) but high throughput (sample rate)
- ◆ Divide-and-conquer: divide the N bits conversion into n stages, each converting N/n bits (more if redundancy is employed)
- ◆ Higher resolution than flash ADCs and faster than SARs
- ◆ Can achieve resolutions up to 14-16 bits and sample rates up to 3GS/s
- ◆ Their algorithmic feed-forward nature makes them amenable to DSP techniques for performance/speed enhancement
 - => compatible with fine geometry CMOS processes and digitally assisted analog
- ◆ Speed extension through interleaving
- ◆ In the high speed space, pipelines in their different incarnations are alive and kicking: interleaved, open-loop, pipe-SARs, continuous-time pipes, etc.

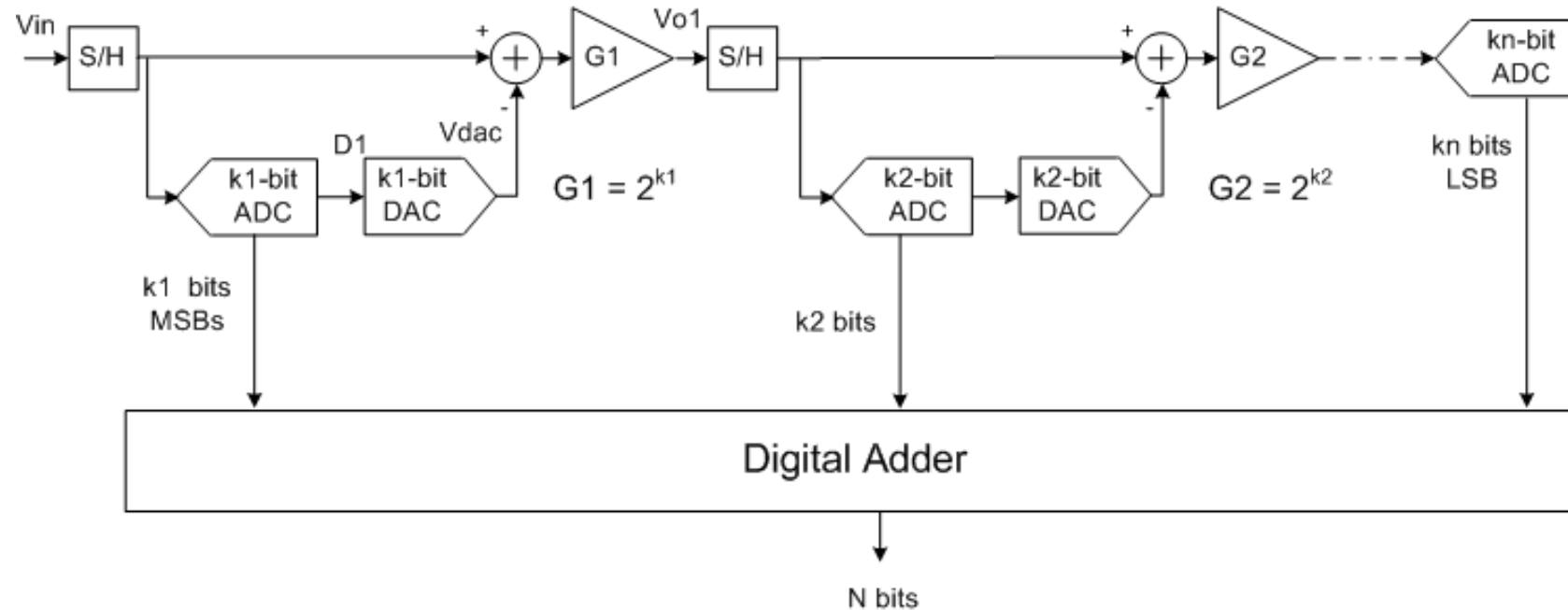
The report of my death was an exaggeration!

A/D Converter Architectures



PIPELINED A/D CONVERTERS

Pipelined A/D Converter



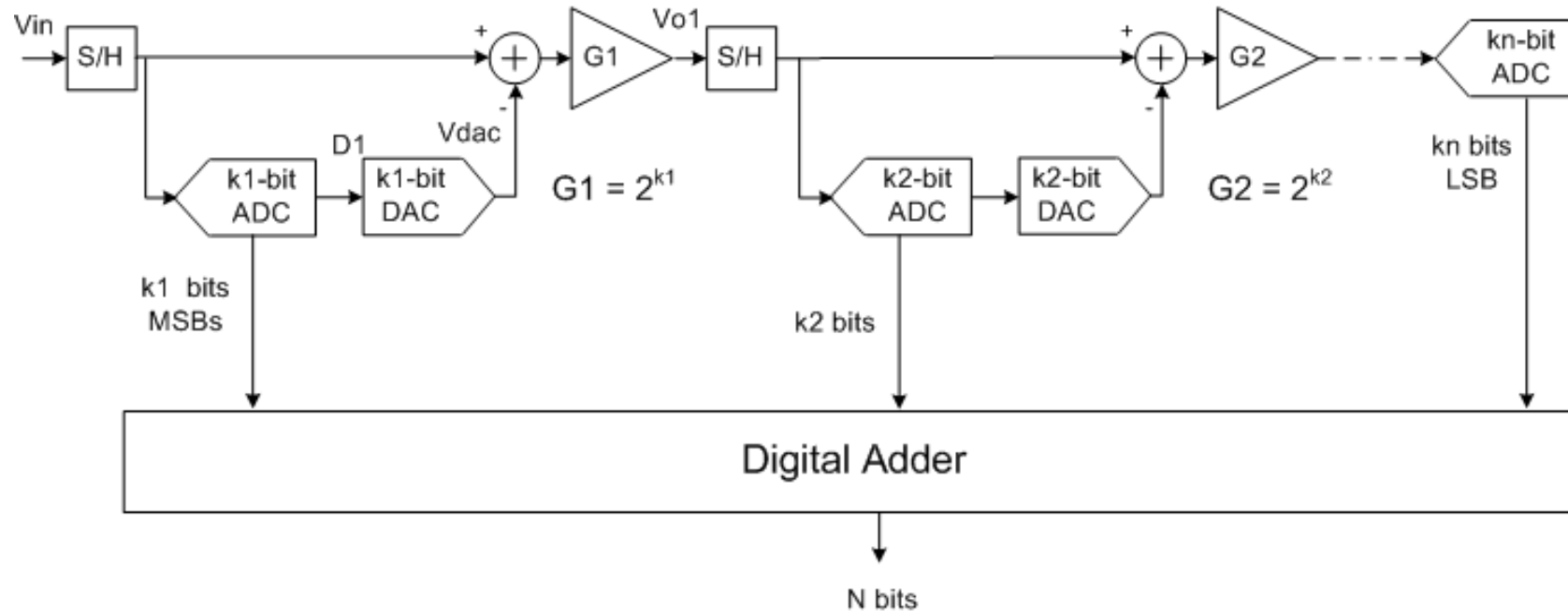
$$D_1 = 0, \pm 1, \pm 2, \pm 3 \dots, \pm 2^{k_1-1}$$

$$V_{o1} = G_1 (V_{in} - V_{dac})$$

$$V_{o1} = G_1 \left(V_{in} - \frac{D_1 \cdot V_{Ref}}{2^{k_1-1}} \right)$$

$$G_1 = 2^{k_1}$$

Pipelined A/D Converter



$$D_1 = 0, \pm 1, \pm 2, \pm 3 \dots, \pm 2^{k_1-1}$$

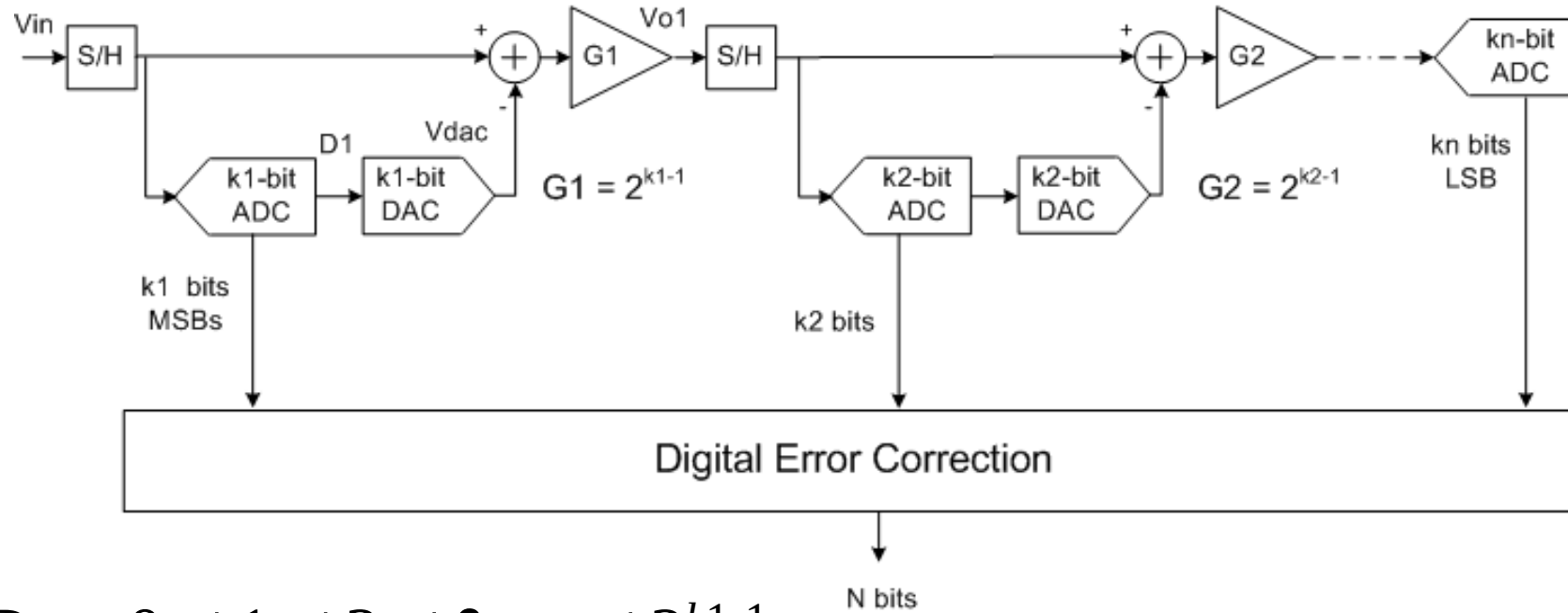
$$V_{o1} = G_1 (V_{in} - V_{dac})$$

$$V_{o1} = G_1 \left(V_{in} - \frac{D_1 \cdot V_{Ref}}{2^{k_1-1}} \right)$$

$$G_1 = 2^{k_1}$$

Not
Practical

Pipelined A/D Converter With Redundancy



$$D_1 = 0, \pm 1, \pm 2, \pm 3 \dots, \pm 2^{k_1-1}$$

$$V_{o1} = G_1 (V_{in} - V_{dac})$$

$$V_{o1} = G_1 \left(V_{in} - \frac{D_1 \cdot V_{Ref}}{2^{k_1-1}} \right)$$

$$G_1 = 2^{k_1-1}$$

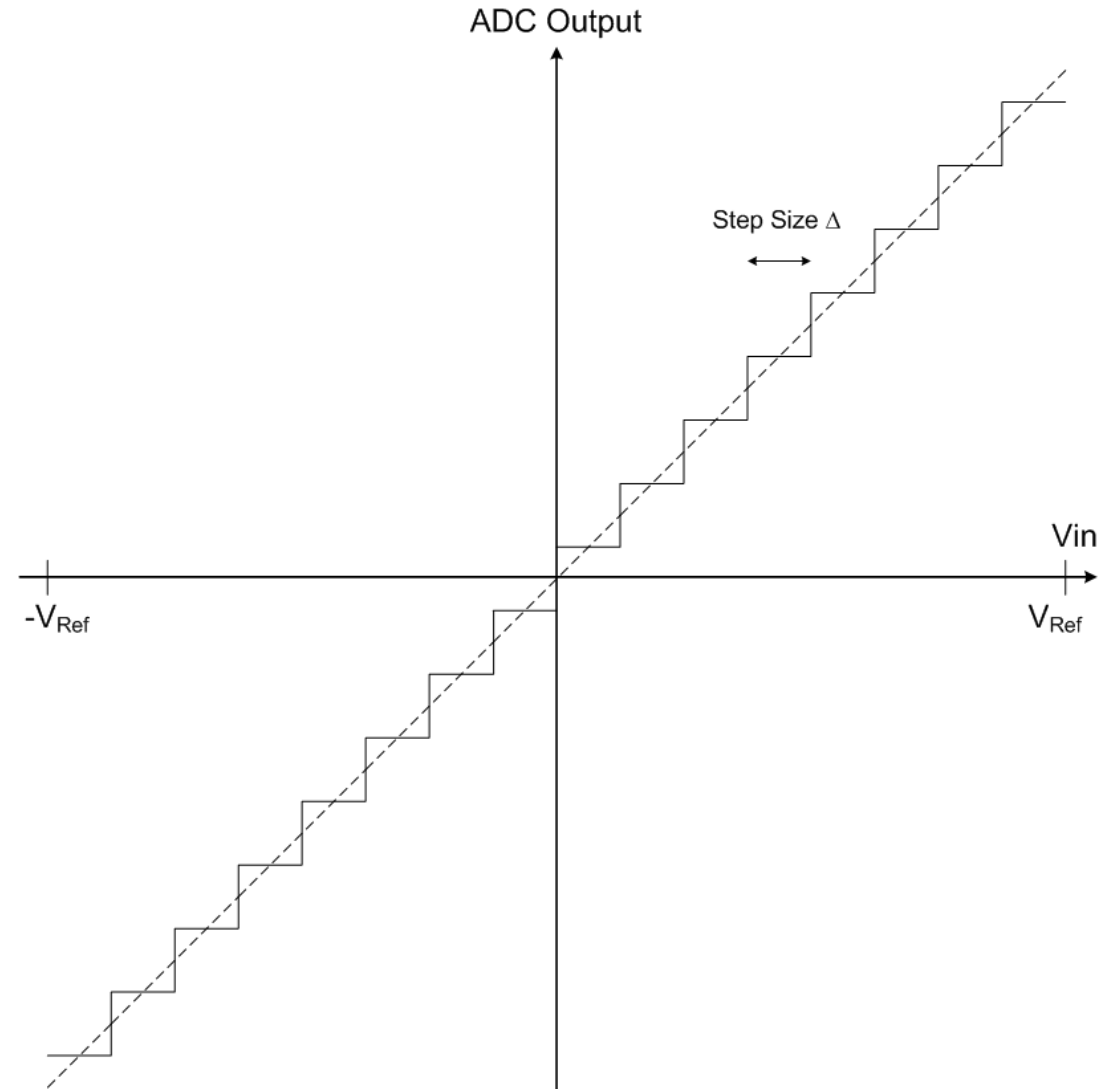
Pipeline Stage Transfer Function

- ◆ **Residue:**

$$V_{o1} = G_1(V_{in} - V_{dac})$$
$$V_{o1} = G_1 \left(V_{in} - \frac{D_1 \cdot V_{Ref}}{2^{k1-1}} \right)$$

- ◆ **The difference between the input signal and the quantized version of it**

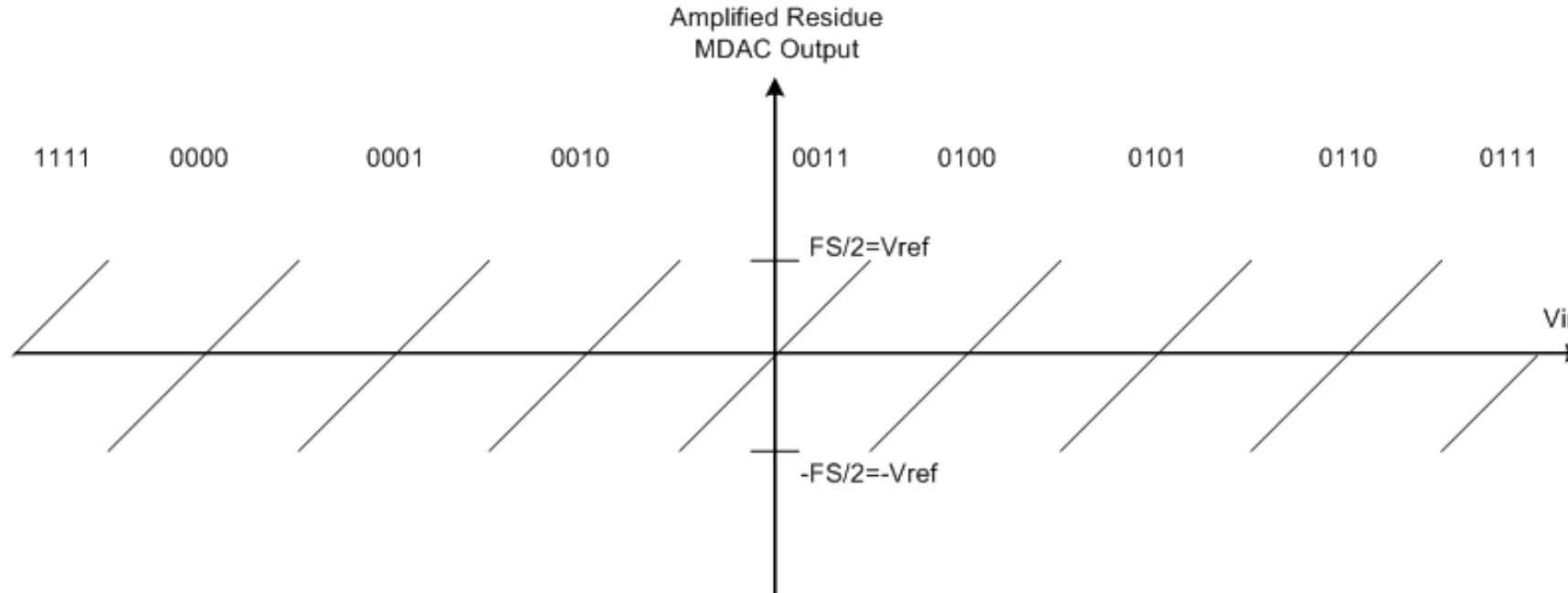
- ◆ **Ideally, the “Transfer Function” looks like a saw-tooth**



First Stage Residue

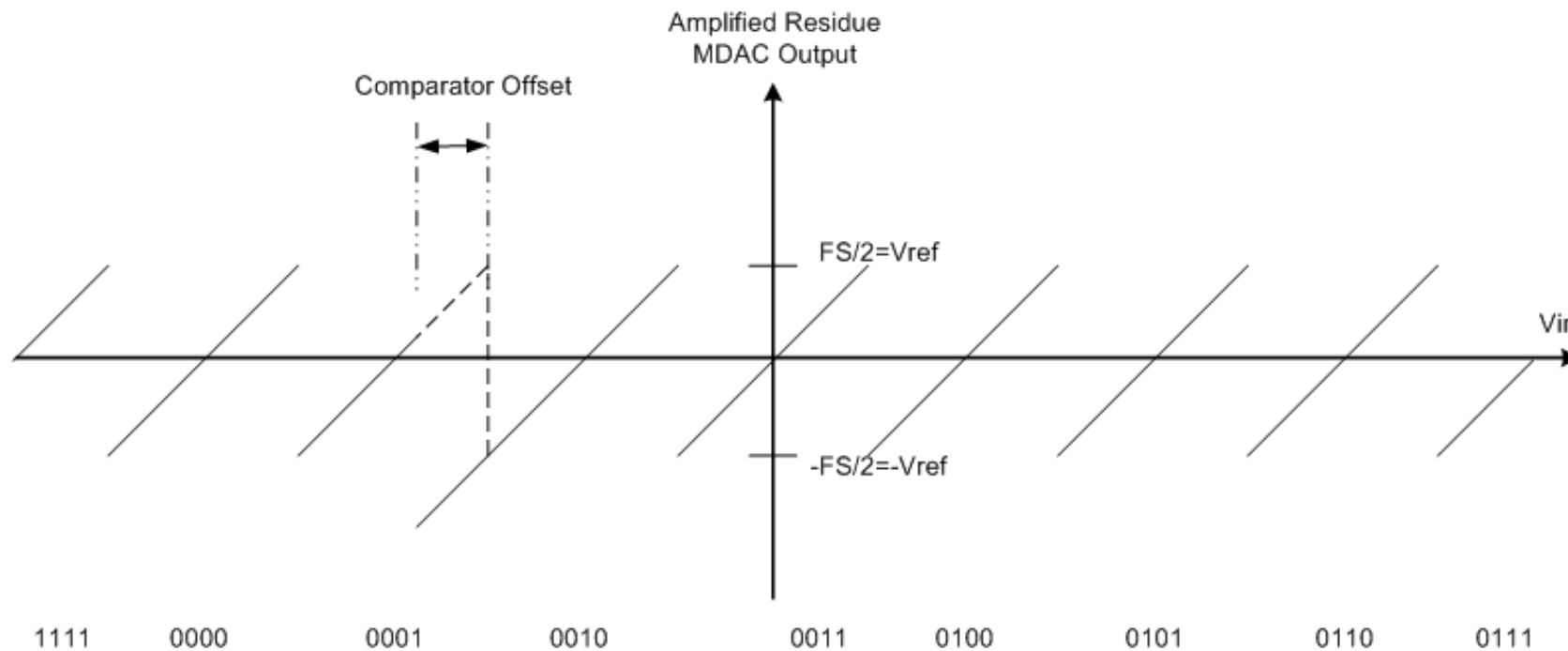
Why We Need Redundancy

- ◆ Ideally, the residue of each stage would be within the range of the back-end ADC



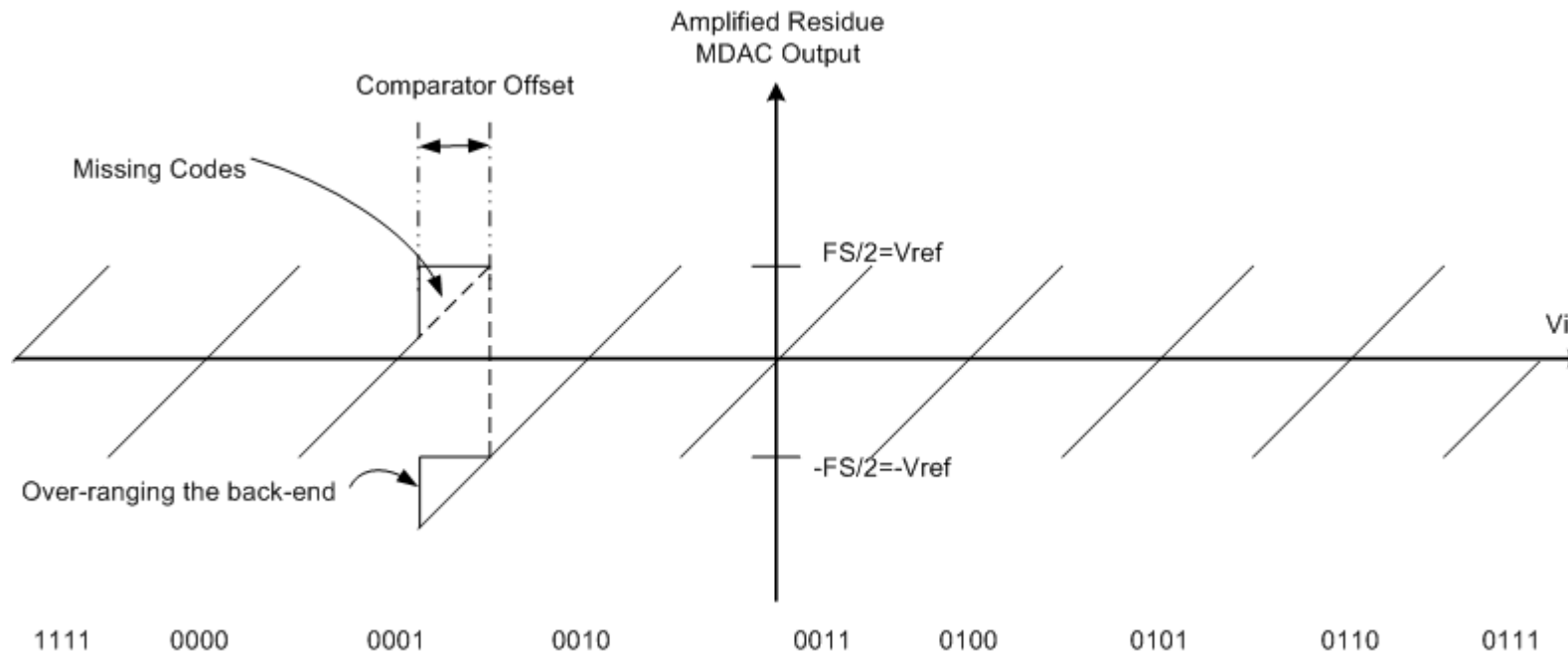
First Stage Residue With Comparator Offsets

- ◆ Offsets in the stage ADC comparators change the residue waveform



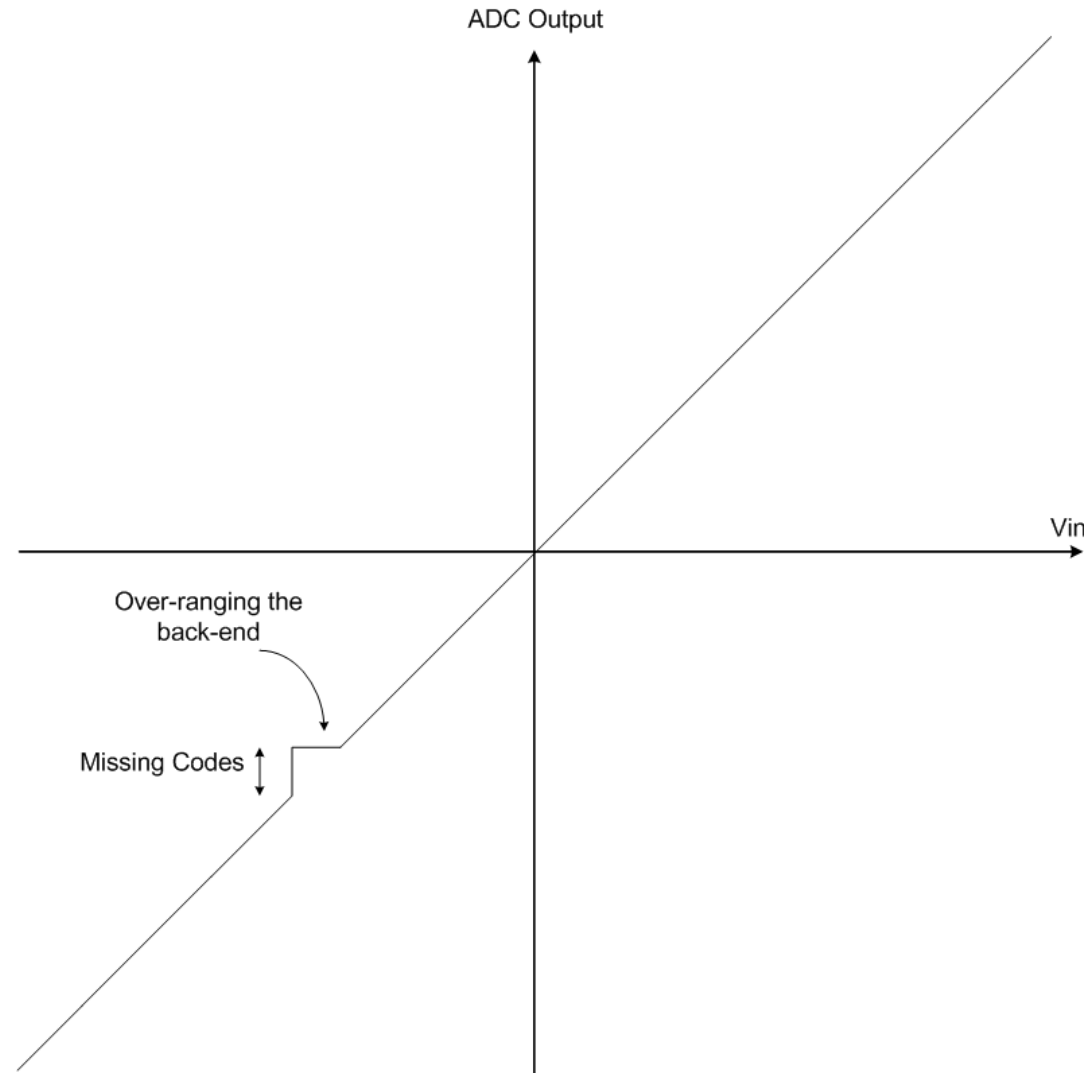
First Stage Residue With Comparator Offsets

- ◆ Offsets would lead to missing codes and over-ranging of the back-end ADC => Redundancy is needed



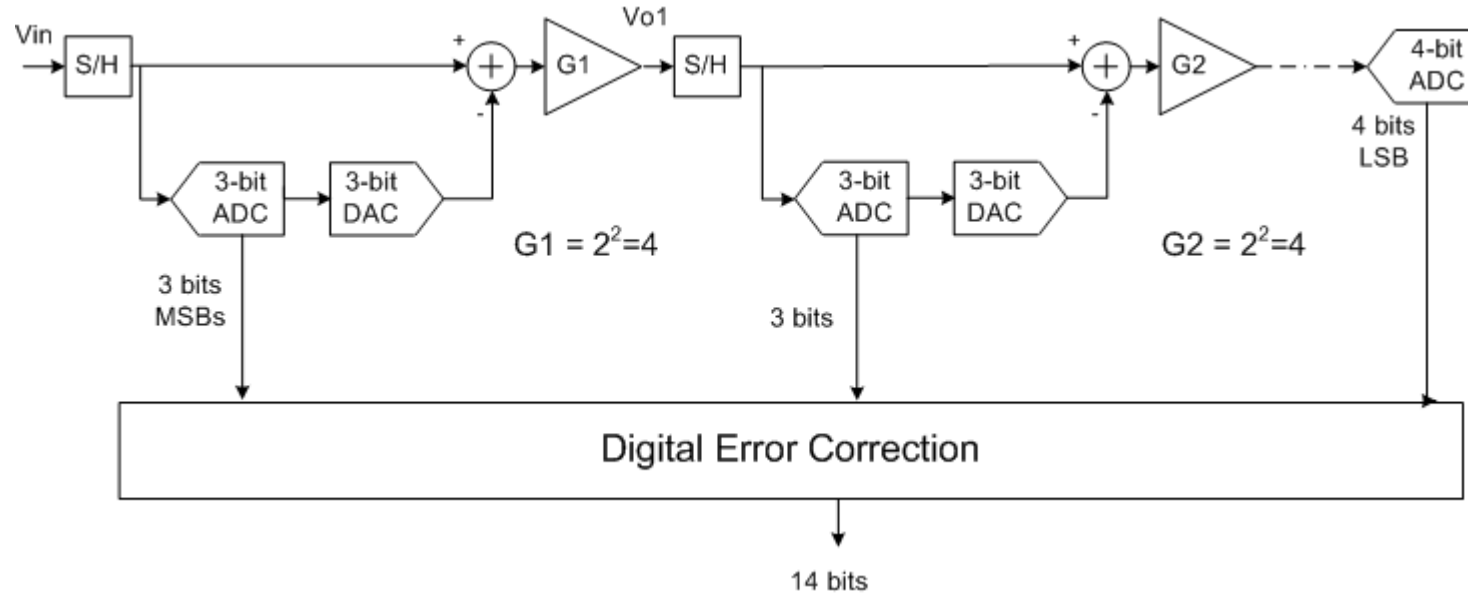
ADC Output With Comparator Offsets

- ◆ Offsets would lead to missing codes and over-ranging of the back-end ADC
- ◆ => Redundancy is needed to de-desensitize the overall ADC to offsets in the ADC's of each stage
- ◆ Typically, one bit of redundancy is employed



Pipelined A/D Converter With Redundancy

$$G_1 = 2^{3-1} = 4$$



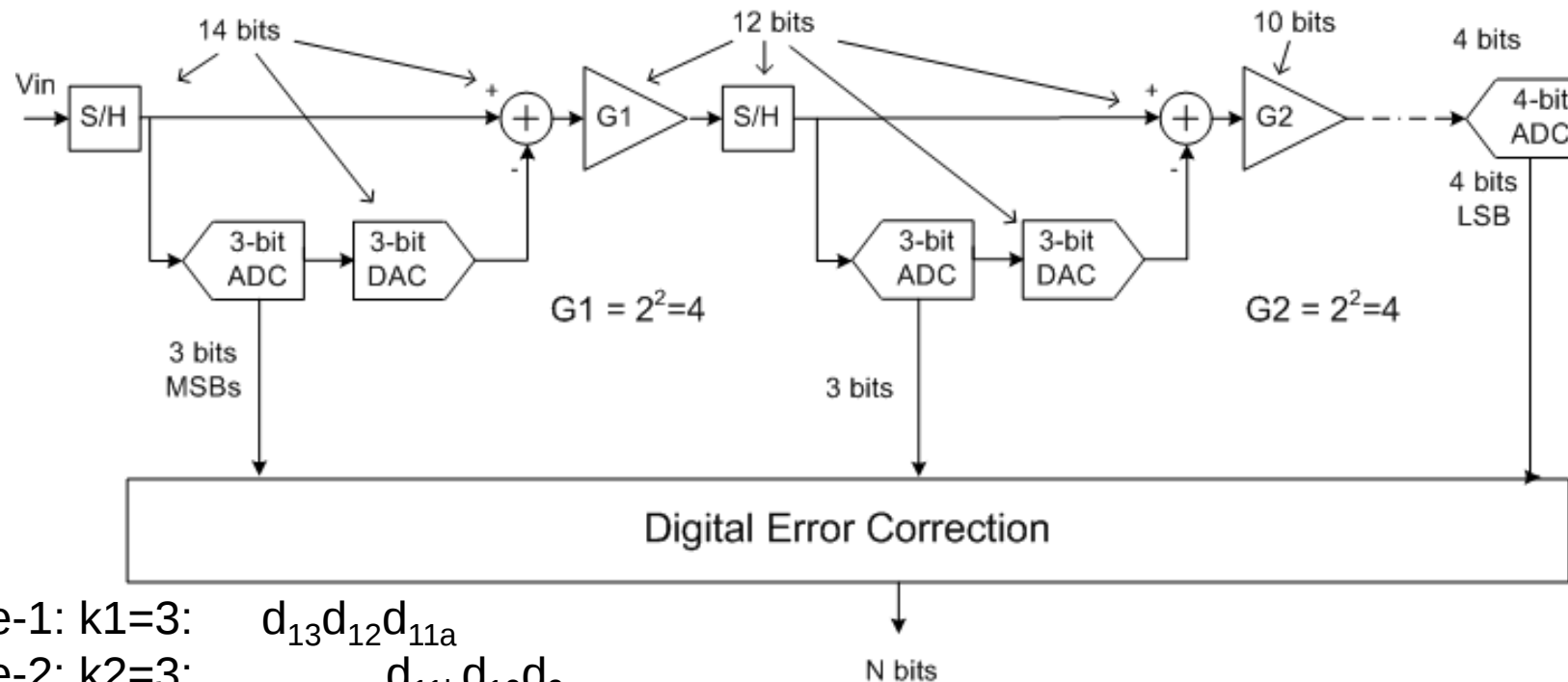
- ◆ $G = 4$
- ◆ Allowed Offset (output referred) = $\pm FS/4$
- ◆ Allowed Offset (input referred) = $\pm FS/16$

$$D_1 = 0, \pm 1, \pm 2, \pm 3 \dots, \pm 2^{k_1-1}$$

$$V_{o1} = G_1 (V_{in} - V_{dac})$$

$$V_{o1} = G_1 \left(V_{in} - \frac{D_1 \cdot V_{Ref}}{4} \right)$$

Pipelined A/D Converter With Redundancy

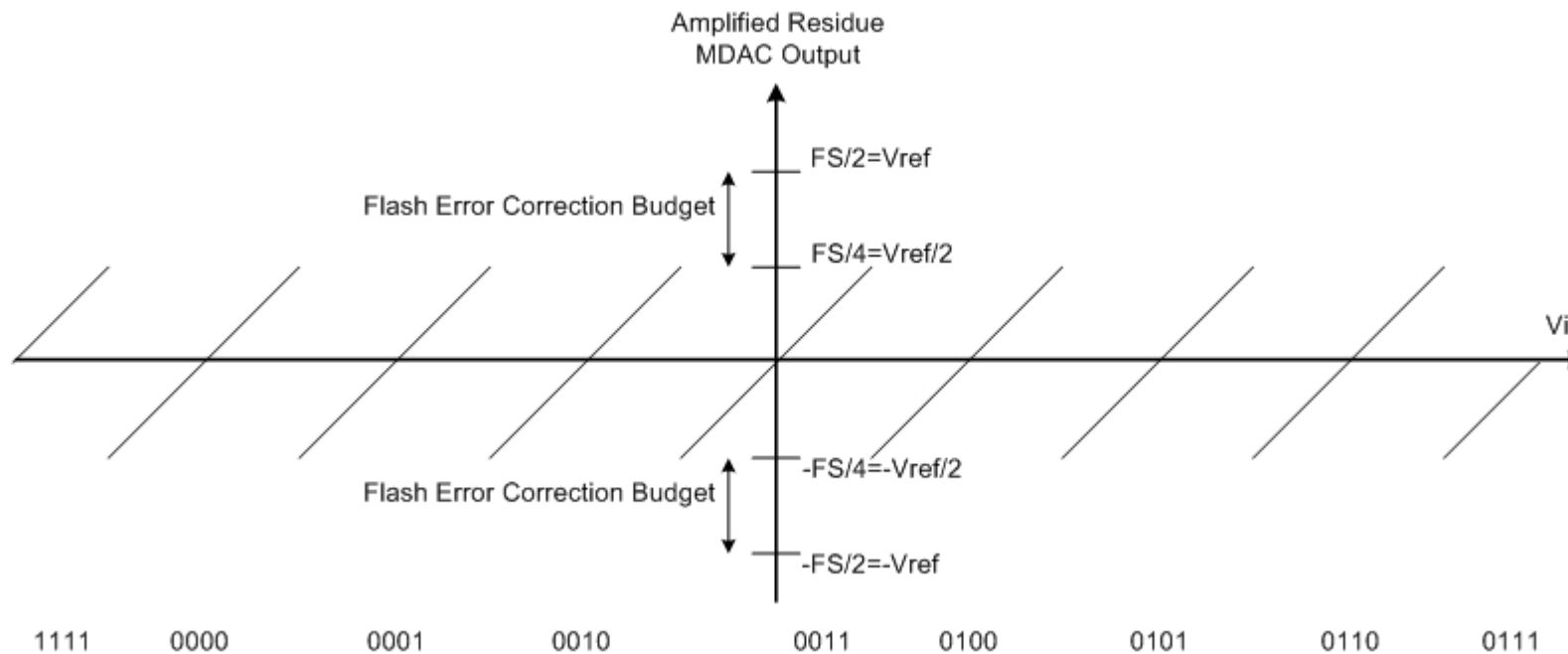


Stage-1: $k_1=3$: $d_{13}d_{12}d_{11a}$
 Stage-2: $k_2=3$: $d_{11b}d_{10}d_{9a}$
 Stage-3: $k_3=3$: $d_{9b}d_8d_{7a}$
 Stage-4: $k_4=3$: $d_{7b}d_6d_{5a}$
 Stage-5: $k_5=3$: $d_{5b}d_4d_{3a}$
 Stage-6: $k_6=4$: $d_{3b}d_2d_1d_0$

Output: 14-bits: $d_{13}d_{12}d_{11}d_{10}d_9d_8d_7d_6d_5d_4d_3d_2d_1d_0$

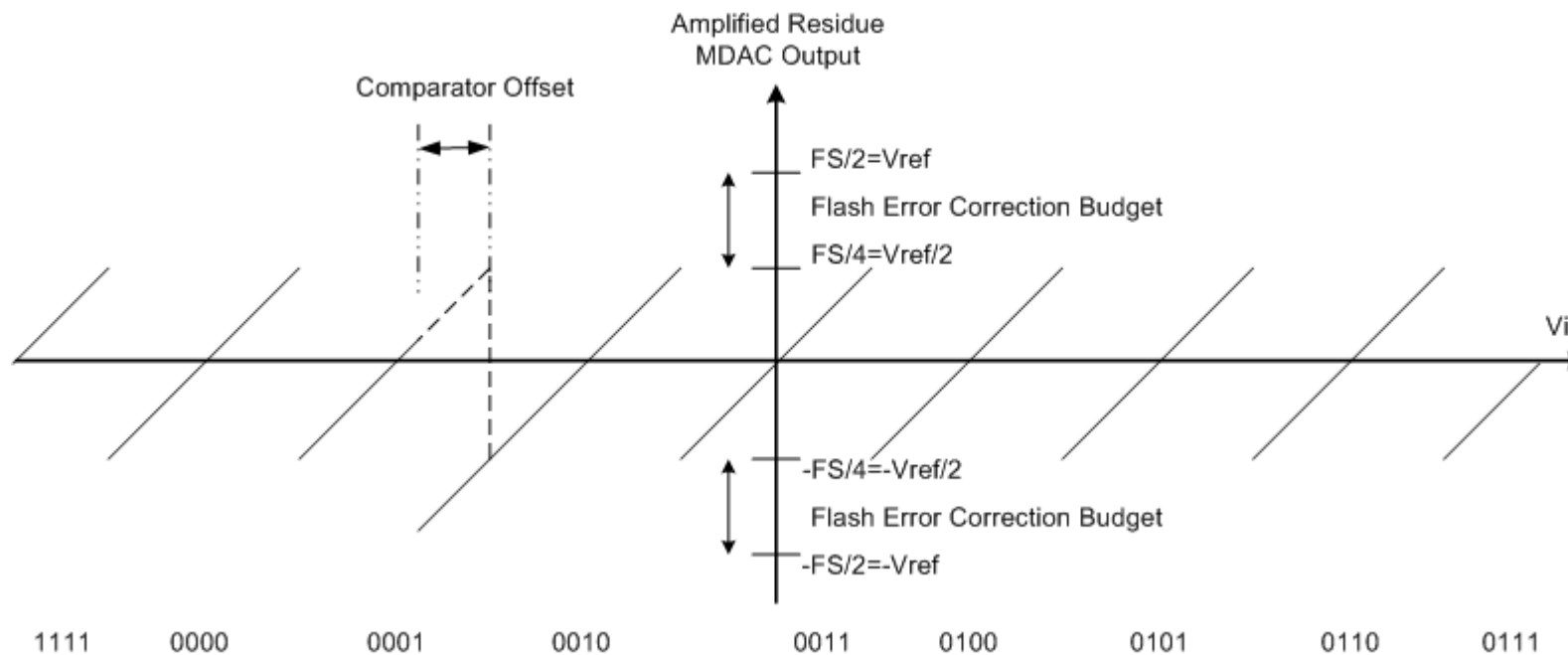
First Stage Residue With Redundancy

- ◆ Ideally, the first stage residue occupies only half of the dynamic range of the back-end ADC



First Stage Residue With Redundancy

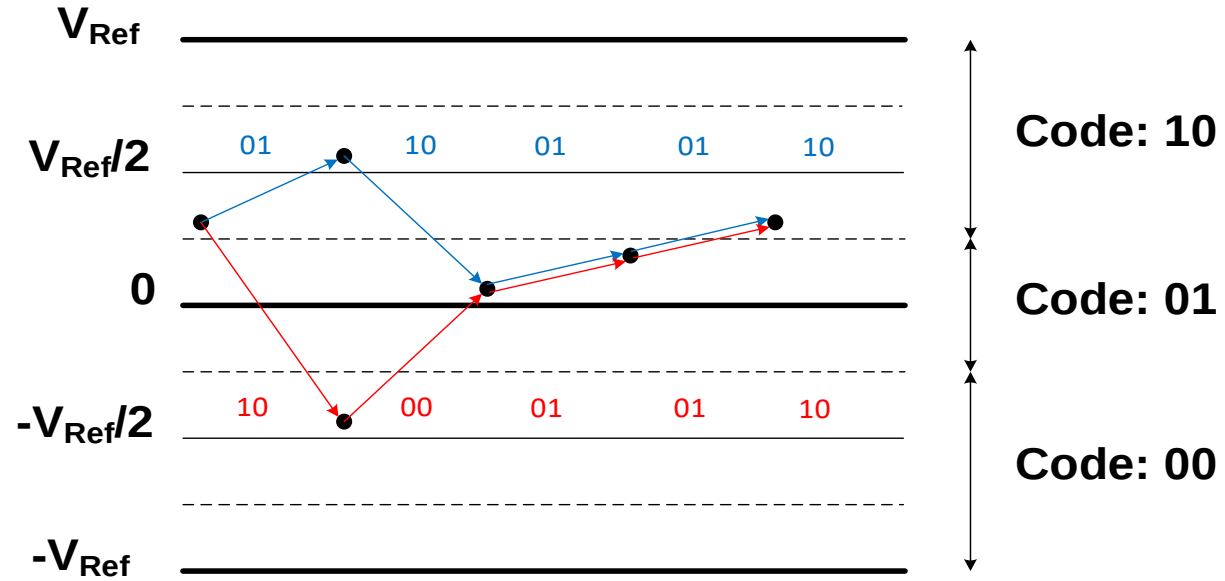
- ◆ Allowed Offset (output referred) = $\pm FS/4$
- ◆ Allowed Offset (input referred) = $\pm FS/4G$



Example

How Redundancy Works

- The answer is the same, independent of the path
- As long as the correction range is not exceeded



Align and sum the codes (raw bits):

Blue Path
01
10
01
01
10

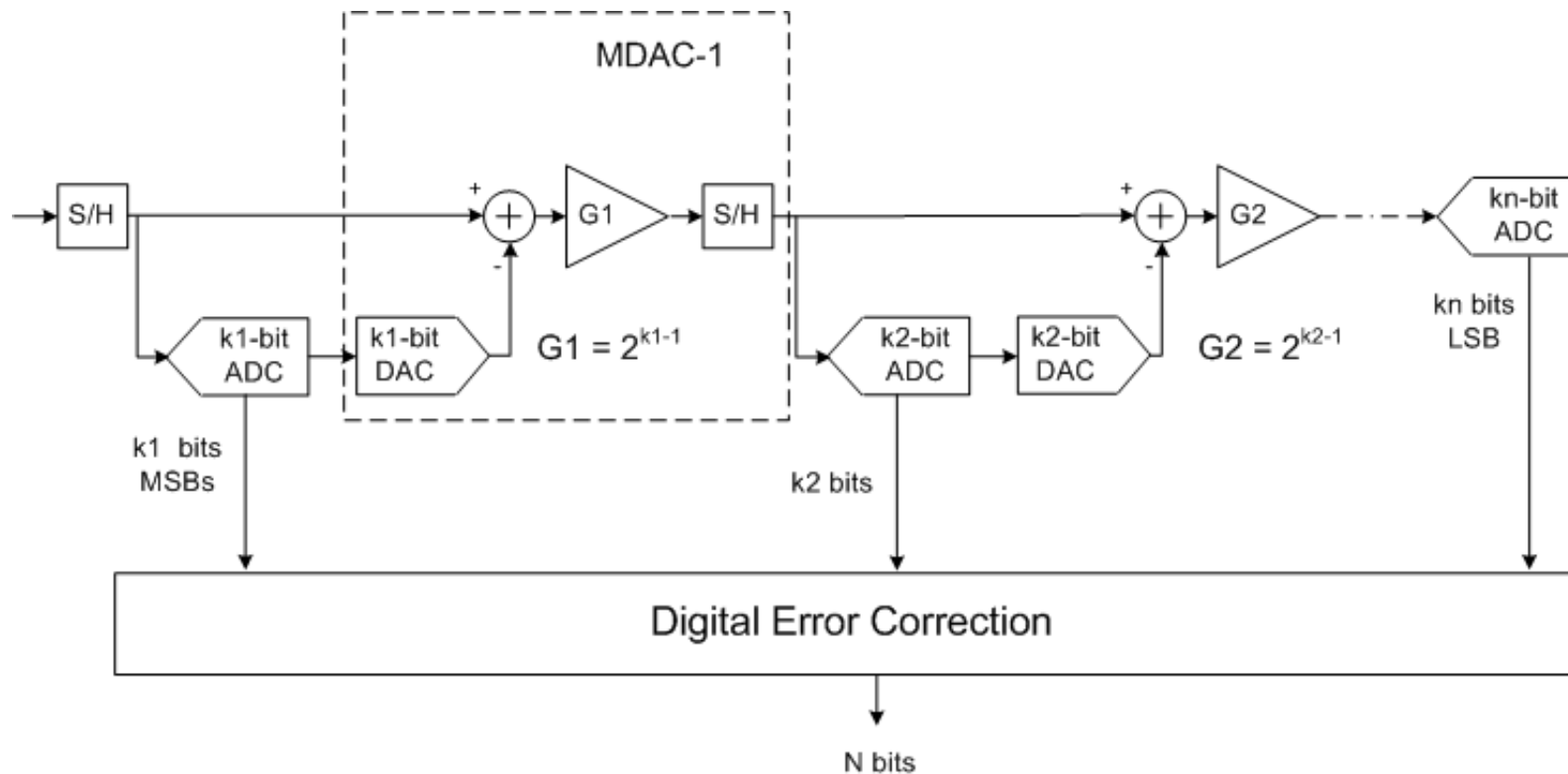
101000

Red Path
10
00
01
01
10

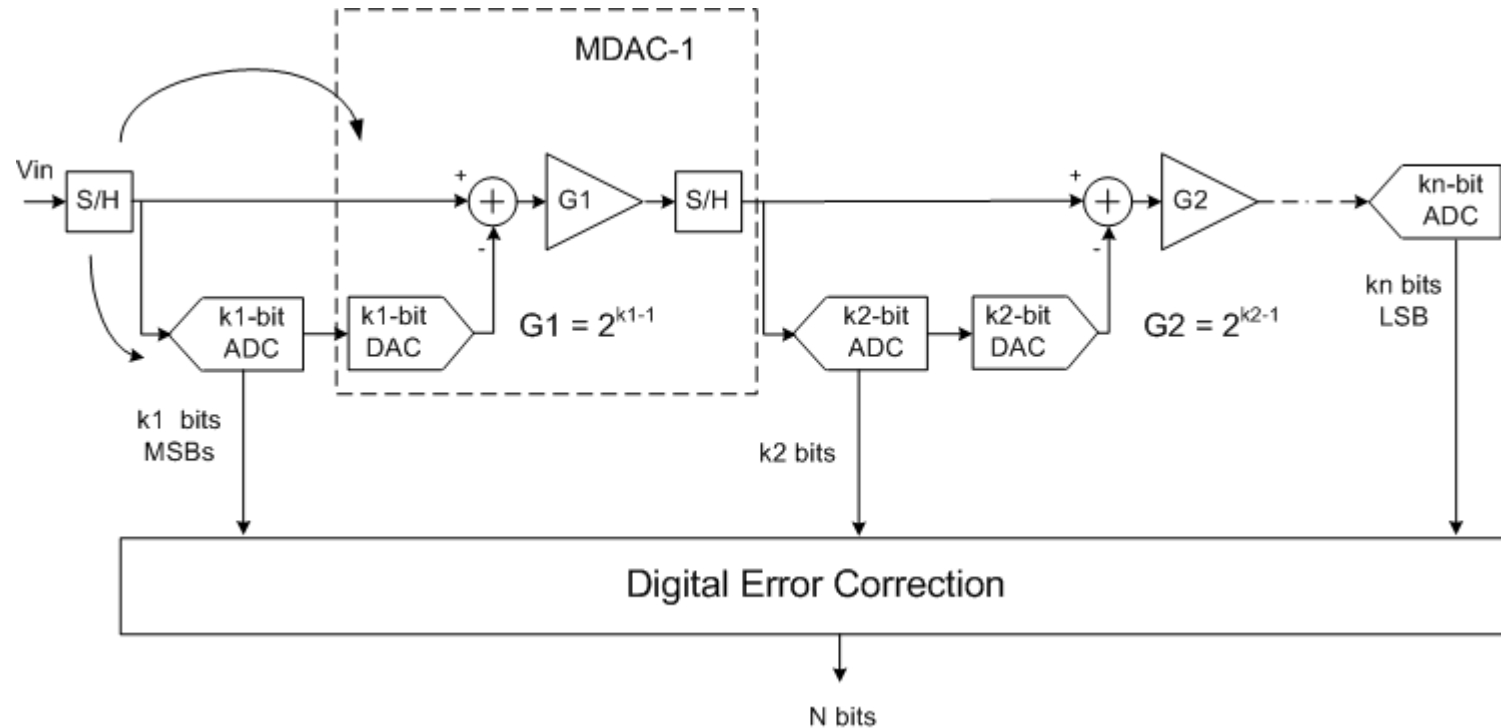
101000

Pipelined A/D Converter

- ◆ The DAC+Subtractor+Amplifier+S/H are called the MDAC

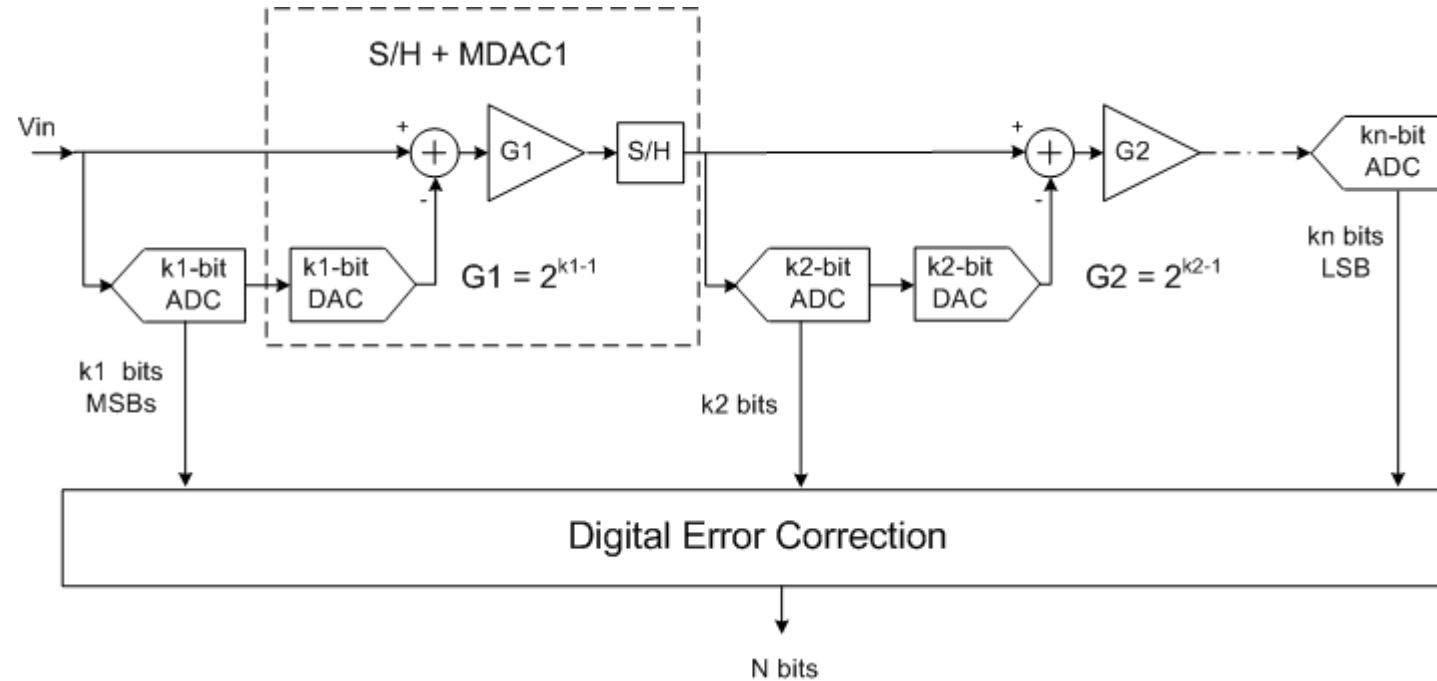


Pipelined A/D Converter



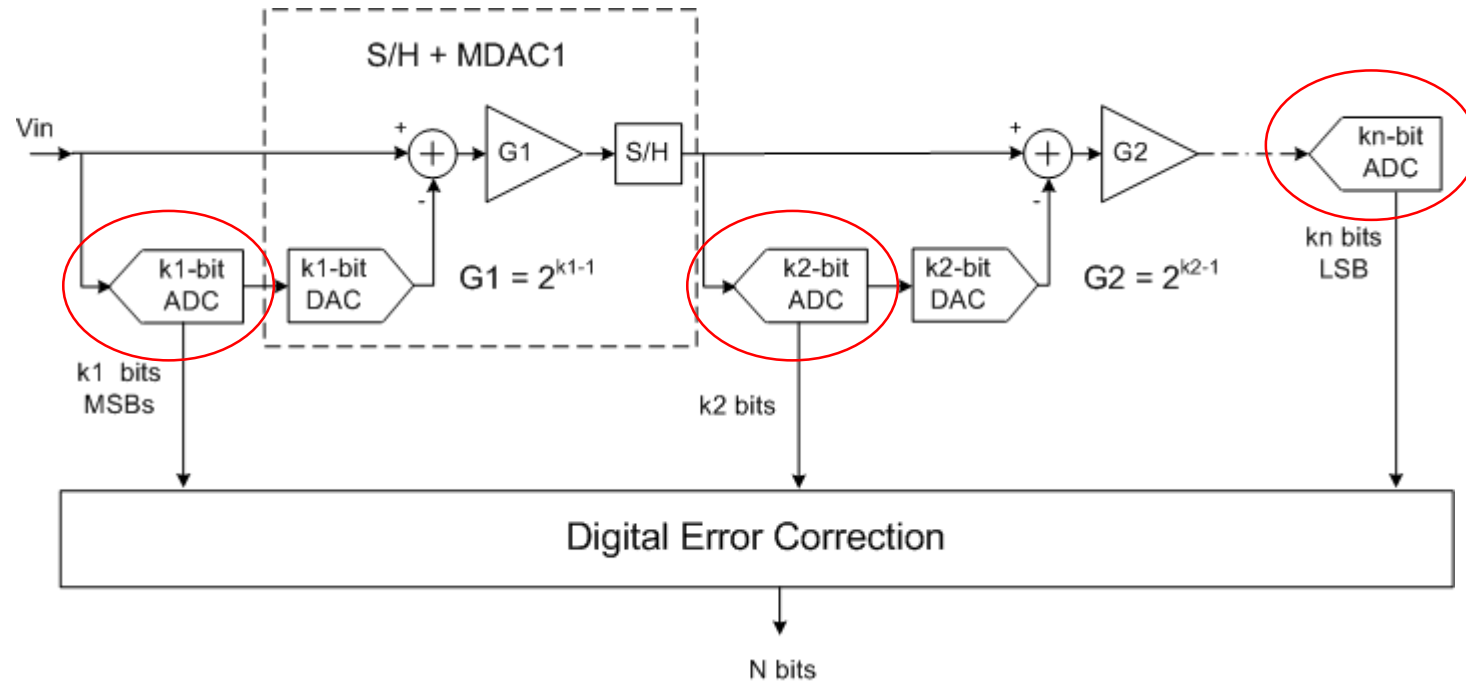
- ◆ The input S/H operation is sometimes performed by the first MDAC to save a dedicated SHA (S/H Amplifier)

Pipelined A/D Converter



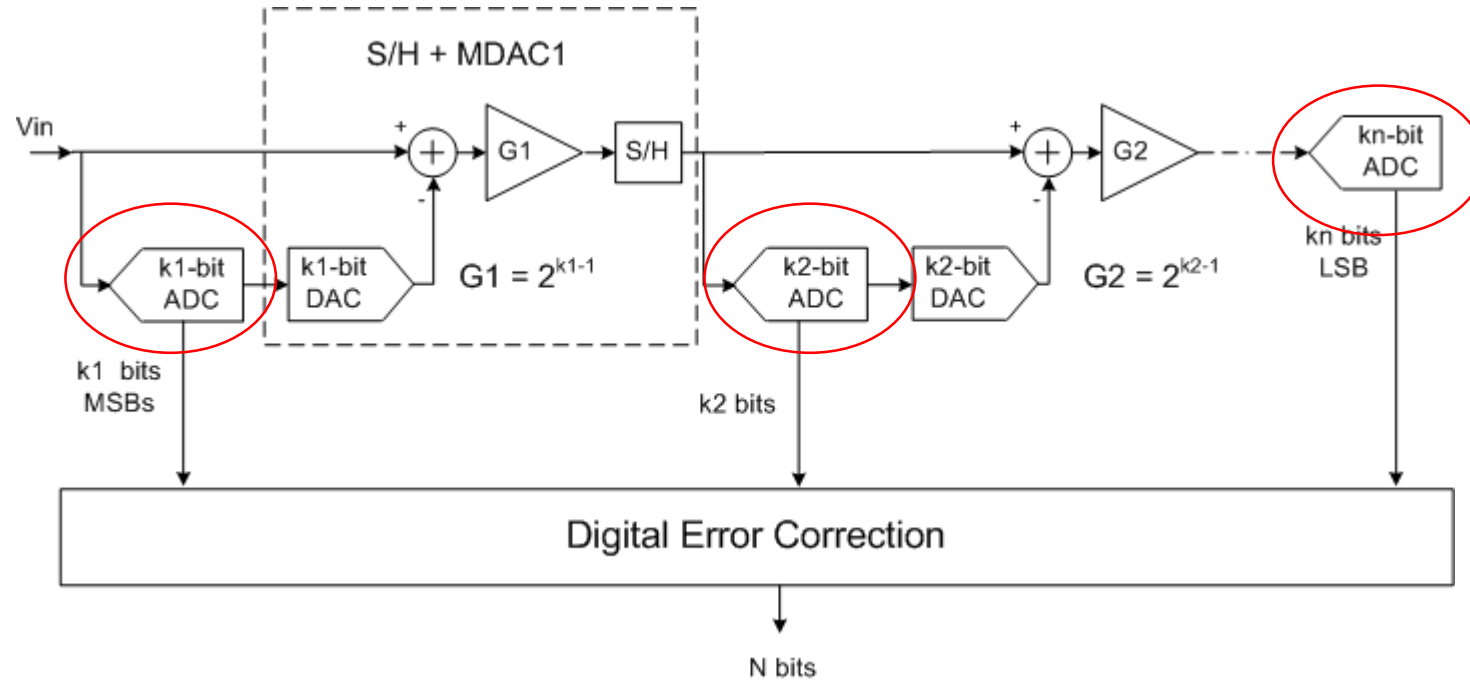
- ◆ The input S/H operation is sometimes performed by the first MDAC to save a dedicated SHA (S/H Amplifier)
- ◆ In this case, matching between the MDAC and flash is important

Pipelined A/D Converter



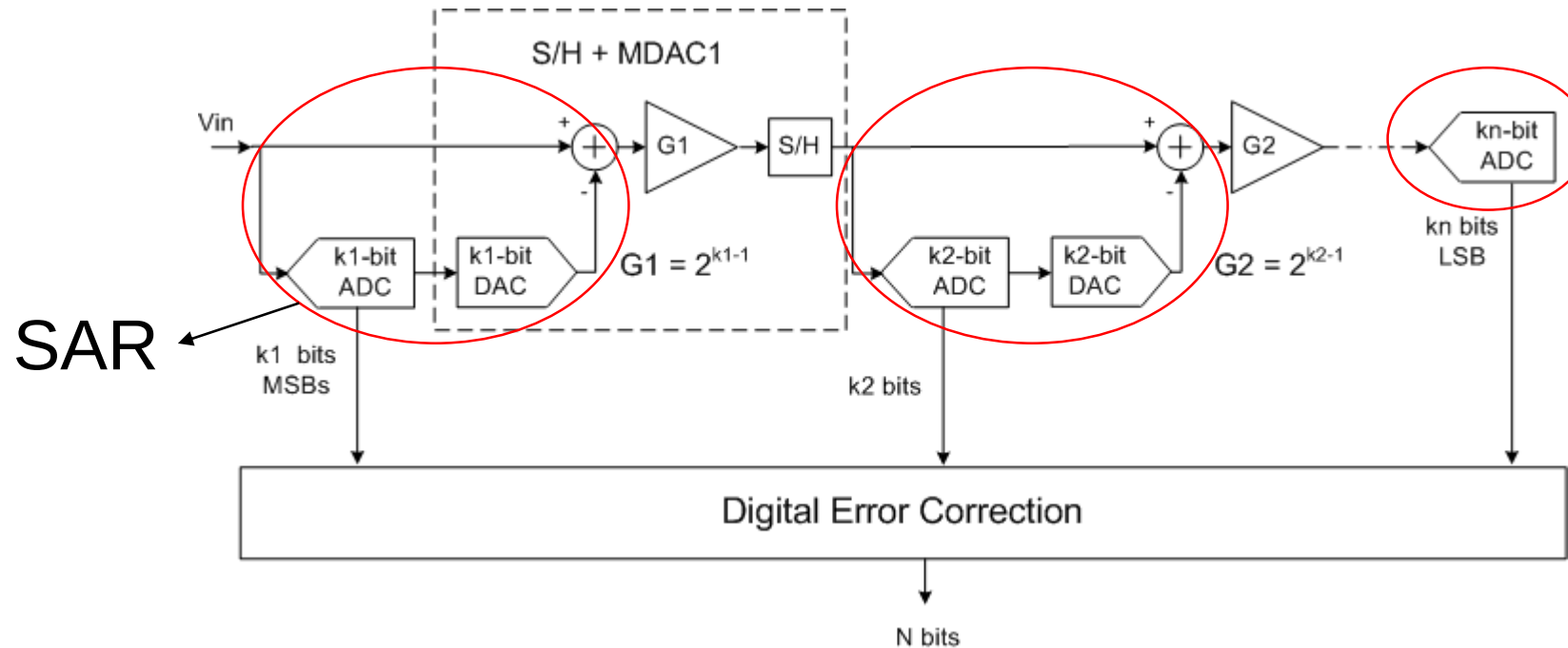
- ◆ The input S/H operation is sometimes performed by the first MDAC to save a dedicated SHA (S/H Amplifier)
- ◆ In this case, matching between the MDAC and flash is important

Pipelined A/D Converter



- ◆ The sub-ADCs were traditionally flash converters
- ◆ Power $\sim 2^N$, so a small number of bits/stage was desirable
- ◆ Optimum for high resolution ADCs was 3-4 bit/stage
- ◆ For low resolution ADCs 1.5 bit/stage

Pipelined-SAR A/D Converter



- ◆ The sub-ADCs can be SAR ADC
- ◆ Power $\sim N$, so a large number of bits/stage is doable (6-7)
- ◆ Lower power and lower speed than flash-based pipeline ADC
- ◆ SARs perform the ADC, DAC and subtraction

Pipelined A/D Converters

Building Blocks

- ◆ S/H and MDAC
- ◆ Comparator
- ◆ Reference
- ◆ Clock
- ◆ Digital processor

Switched Capacitor MDAC Operation

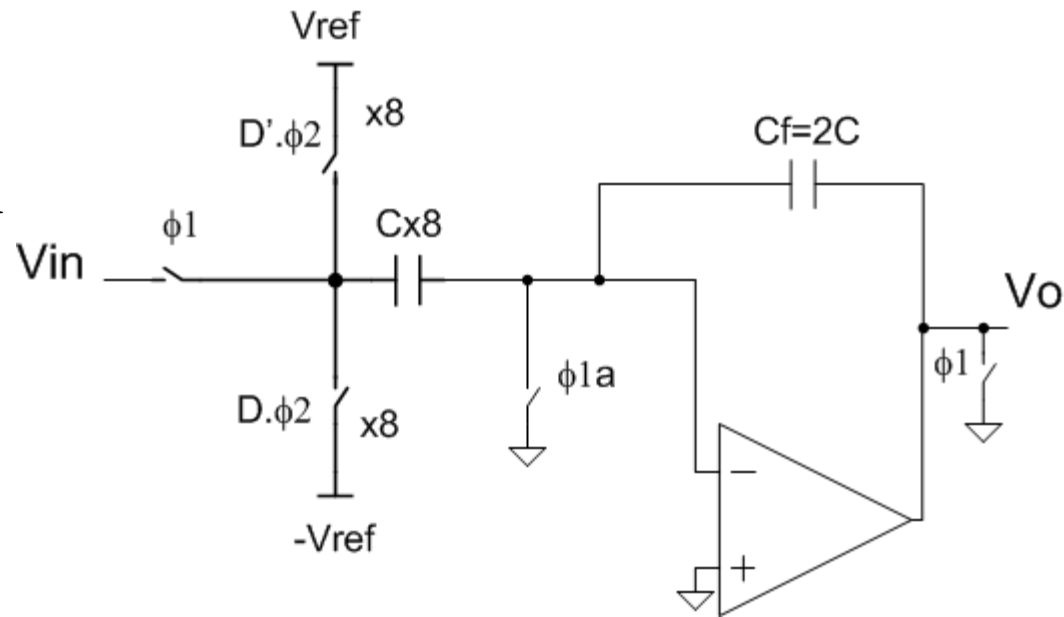
$$V_o = \frac{V_{in} \cdot C_t / C_f}{1 + (C_t + C_f + C_p) / (C_f \cdot A)} - \sum_{i=1}^8 \frac{D_i V_{ref} \cdot C / C_f}{1 + (C_t + C_f + C_p) / (C_f \cdot A)}$$

$$V_o = \frac{V_o | ideal}{1 + (C_t + C_f + C_p) / (C_f \cdot A)}$$

$$V_o = \frac{V_o | ideal}{1 + K / A} = V_o | ideal - V_o K / A$$

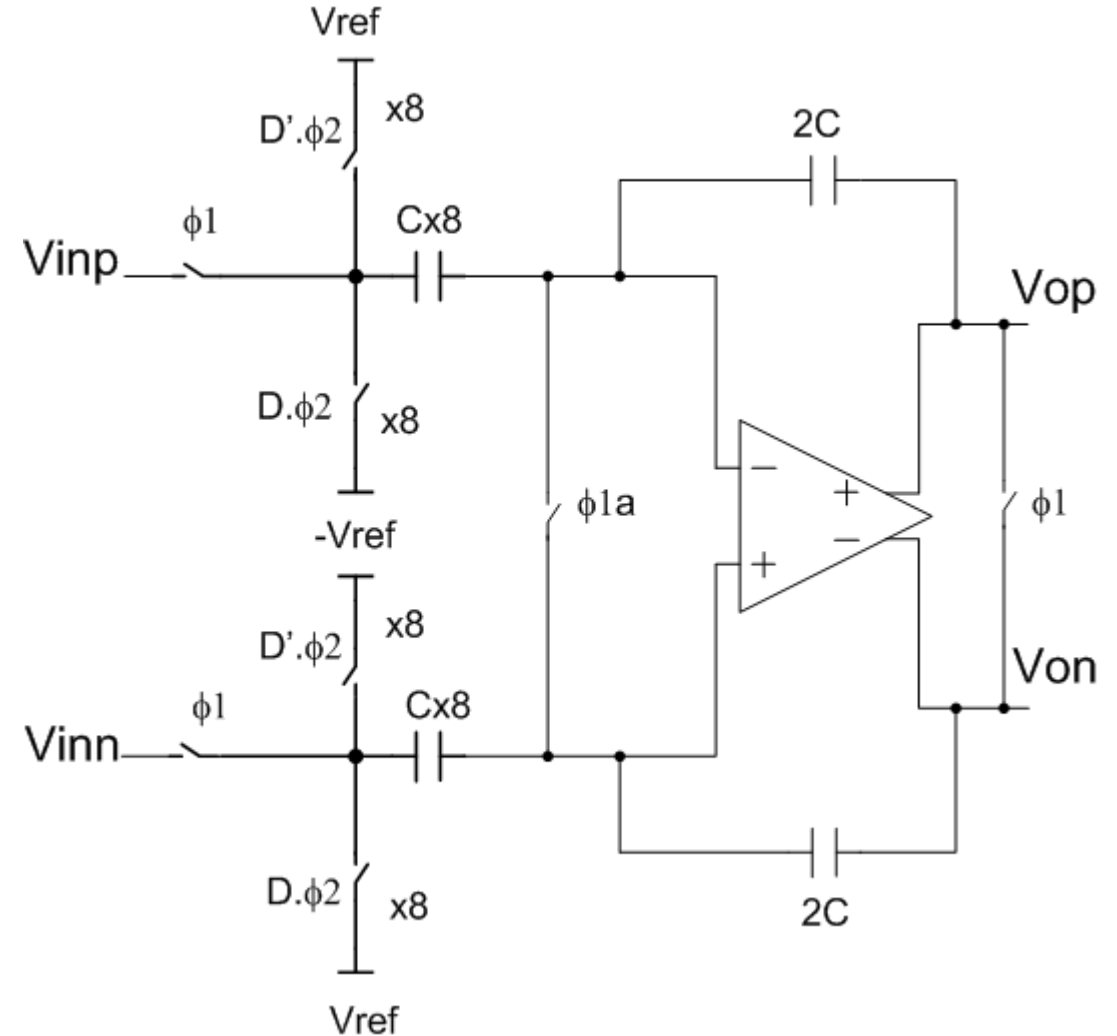
Error Term

$$V_o | ideal = V_o + V_o K / A$$



Switched Capacitor MDAC Operation

- ◆ Fully differential operation helps reduce even order harmonics and common mode sensitivity
- ◆ Input common mode variation is ideally rejected by this circuit
- ◆ Mismatches, common-mode variation in the circuit and the single-ended even-order non-linearity can still limit performance



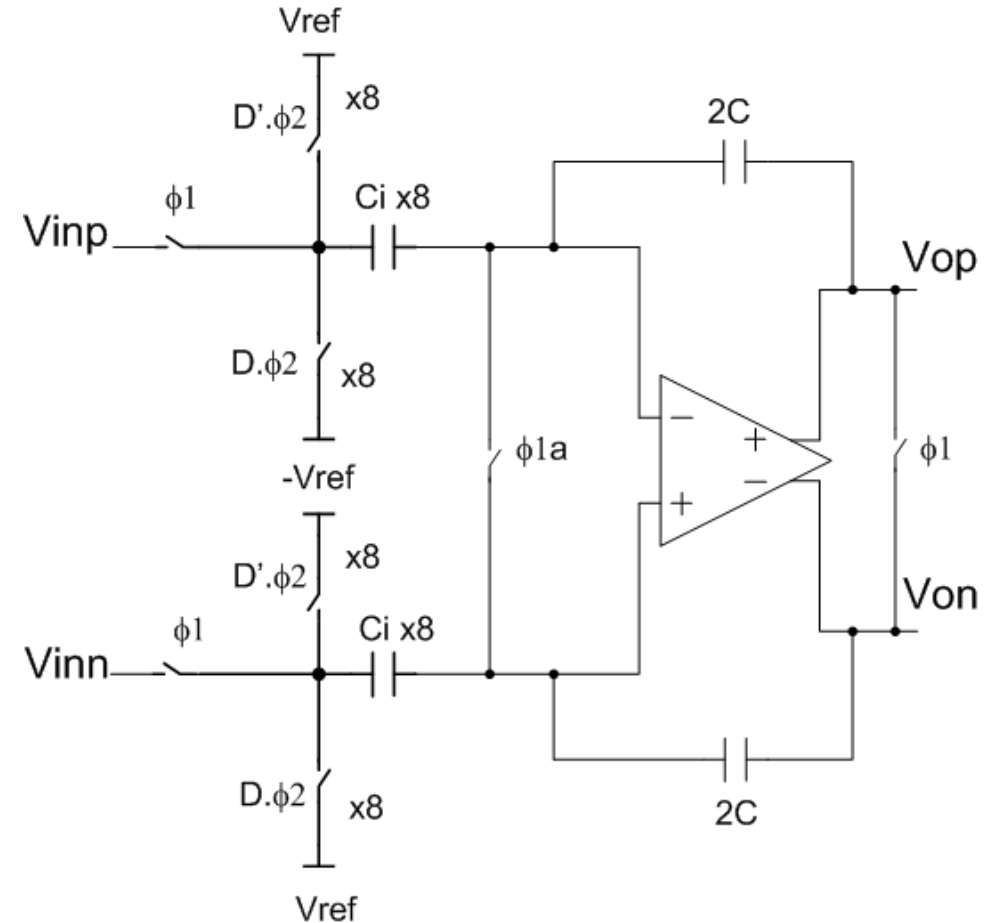
Accuracy Requirements

- ◆ The front stages need to have the highest accuracy
 - The accuracy requirement is relaxed as we go down the pipeline
- ◆ The stage ADC (flash) does not need to be very accurate
 - Usually 4-5 bit accurate
- ◆ The first DAC needs to be as accurate as the whole ADC
- ◆ The first amplifier needs to be as accurate as the back-end ADC
- ◆ The main bottle-neck is designing an MDAC that achieves the required speed (BW) and accuracy (gain)

Accuracy Requirements [1]

$$V_o \approx \left[\frac{V_{in} \cdot C_t / C_f}{1 + K / A} - \sum_{i=1}^8 \frac{D_i V_{Refi} \cdot C_i / C_f}{1 + K / A} \right] (1 - e^{-t_s \omega_t / K})$$

- ◆ If $C_i \neq C_j \Rightarrow$ DAC Errors
- ◆ If $\sum C_i / C_f \neq G \Rightarrow$ Gain Error
- ◆ If $V_{refi} \neq V_{refj} \Rightarrow$ DAC Errors
- ◆ If $V_{ref_stage1} \neq V_{ref_stage2} \Rightarrow$ Gain Error
- ◆ Settling errors cause gain errors



Accuracy Requirements [1]

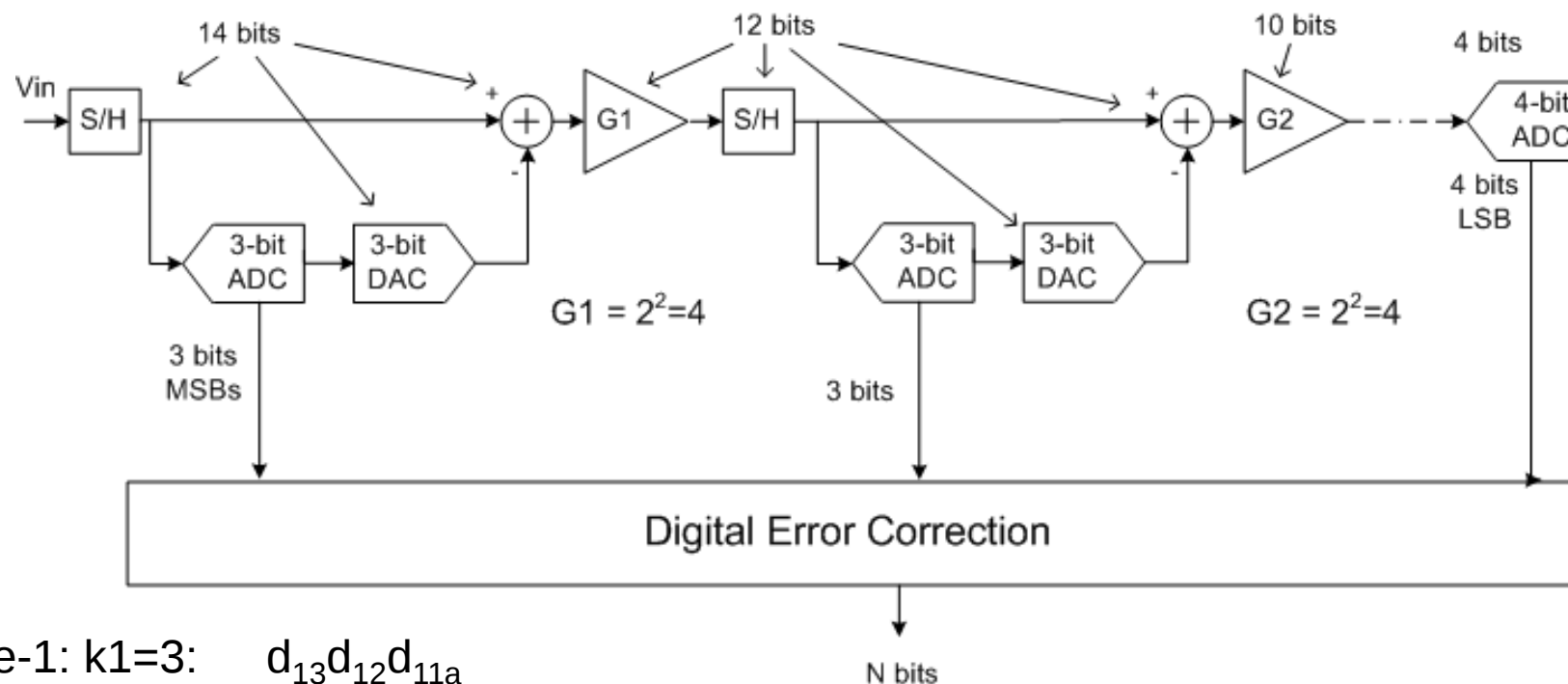
- ◆ **Capacitor matching:** $\Delta_{DAC_i} < 2^{-(N+j_{LS})} \times \prod_{n=0}^{i-1} G_n$
 - Mismatches cause inter-stage gain and DAC errors

- ◆ **Amplifier open loop gain:** $A_i > \frac{2^{-k_i} \times 2^{N+j}}{\beta_i \times \prod_{n=0}^{i-1} G_n}$
 - Low gain causes inter-stage gain error and non-linearity

- ◆ **Settling:** $\varepsilon_{DAC_i} < 2^{-(N+j_{LS})} \times \prod_{n=0}^{i-1} G_n$
 - Settling errors cause inter-stage gain error, DAC errors and non-linearity

- ◆ **Reference:** $\varepsilon_{refDAC_i} < 2^{-(N+j_{LS})} \times \prod_{n=0}^{i-1} G_n$
 - Reference errors cause inter-stage gain and DAC errors

Accuracy Requirements



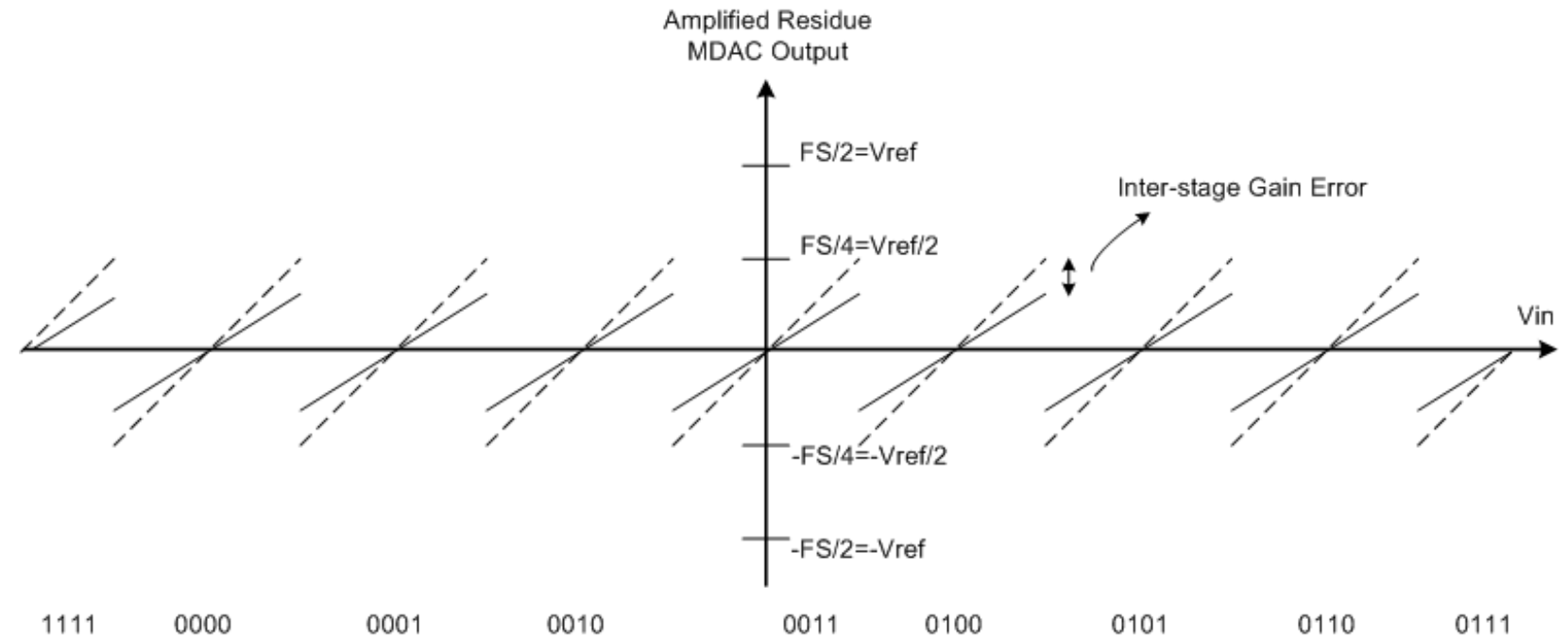
Stage-1: $k_1=3$: $d_{13}d_{12}d_{11a}$
 Stage-2: $k_2=3$: $d_{11b}d_{10}d_{9a}$
 Stage-3: $k_3=3$: $d_{9b}d_8d_{7a}$
 Stage-4: $k_4=3$: $d_{7b}d_6d_{5a}$
 Stage-5: $k_5=3$: $d_{5b}d_4d_{3a}$
 Stage-6: $k_6=4$: $d_{3b}d_2d_1d_0$

Output: 14-bits: $d_{13}d_{12}d_{11}d_{10}d_9d_8d_7d_6d_5d_4d_3d_2d_1d_0$

First Stage Residue With Inter-Stage Gain Error

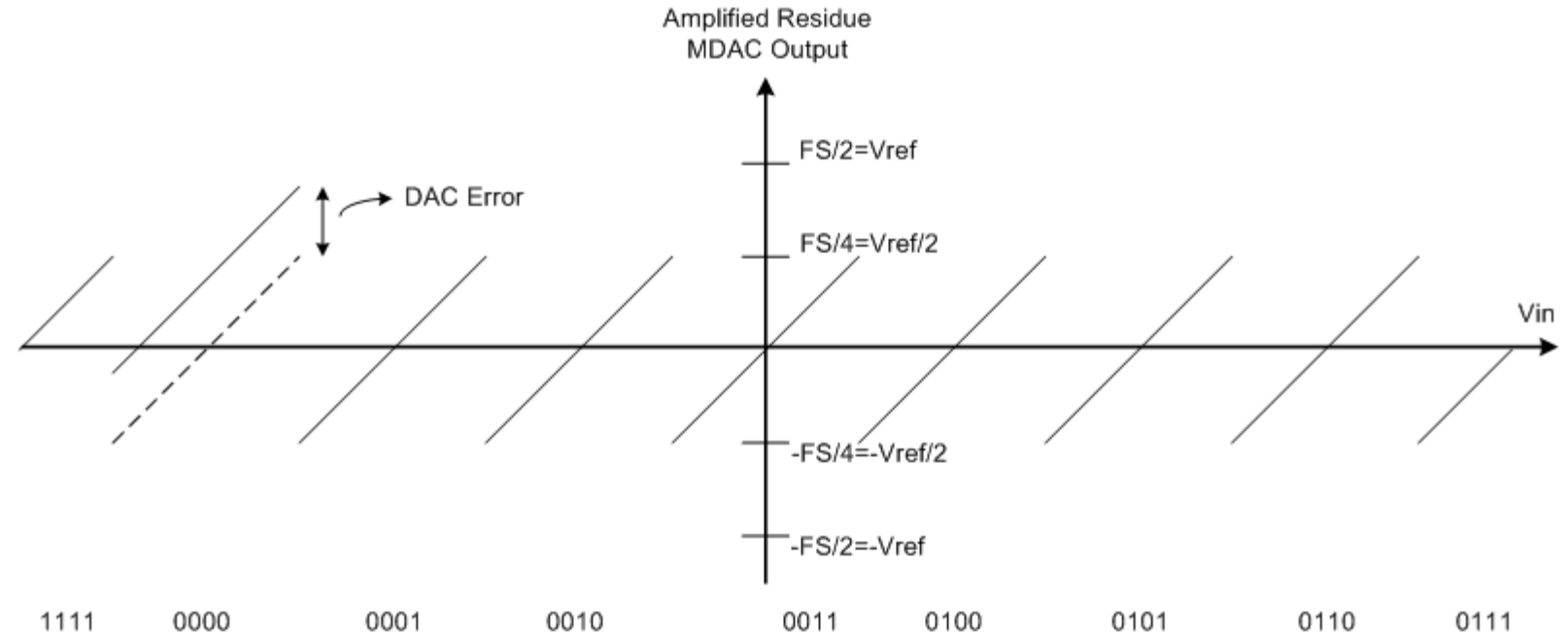
◆ Common sources of IGE:

- Cap mismatch
- Low open loop gain
- Reference error
- Charge injection
- Settling error

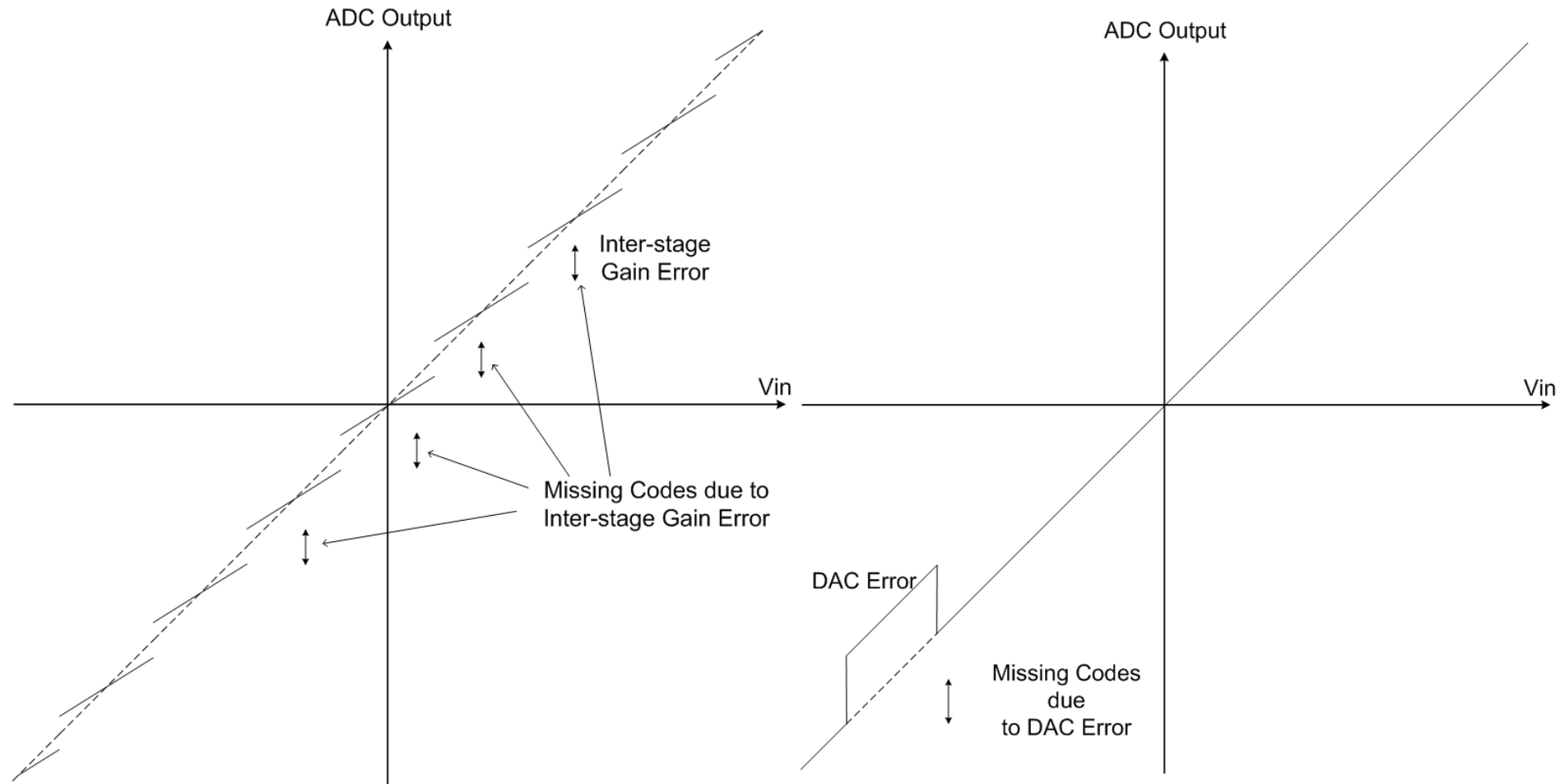


First Stage Residue With Stage DAC Error

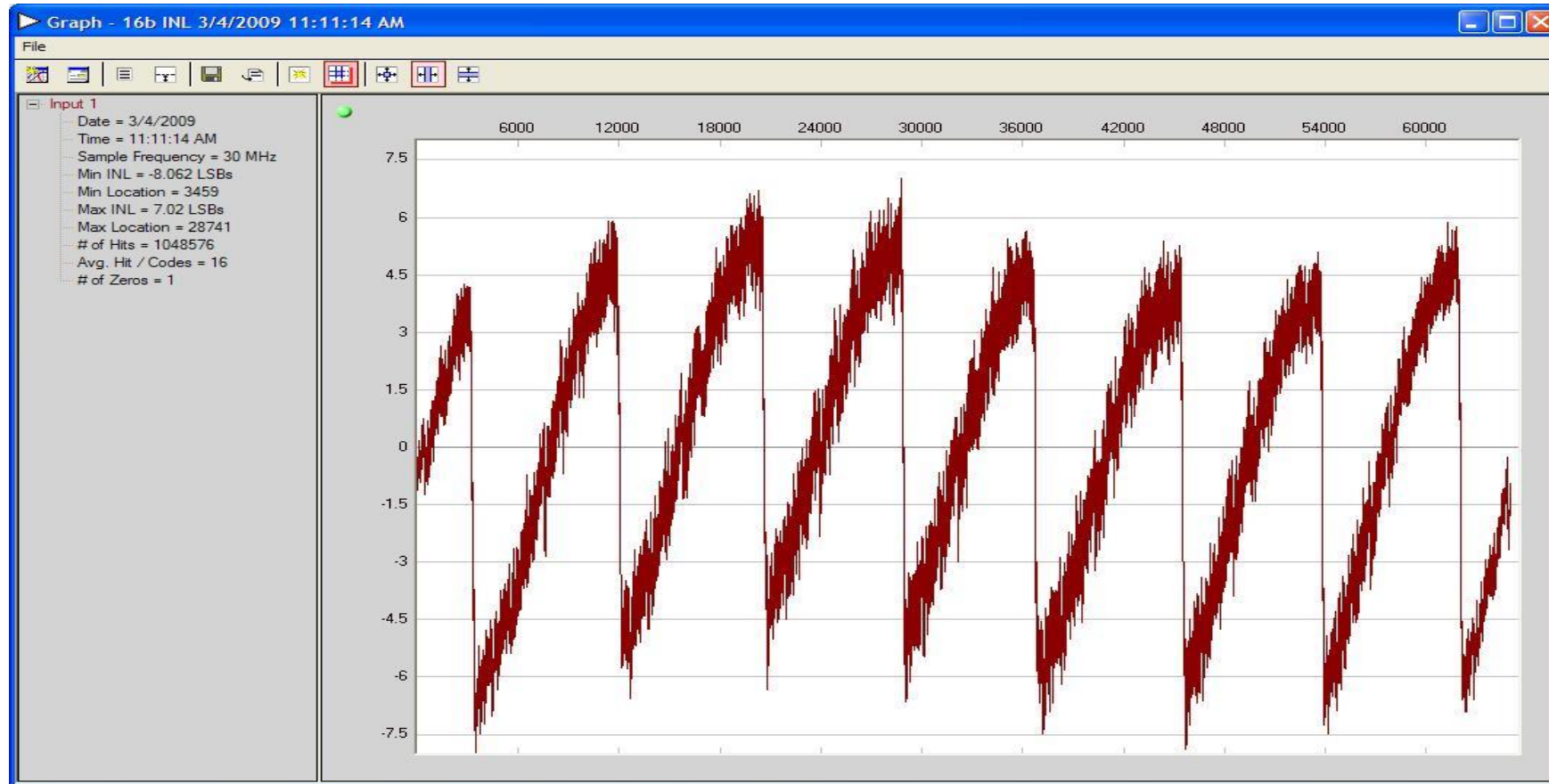
- ◆ Sources of DAC errors:
 - Cap mismatch
 - Reference error
 - Charge injection
 - Settling error



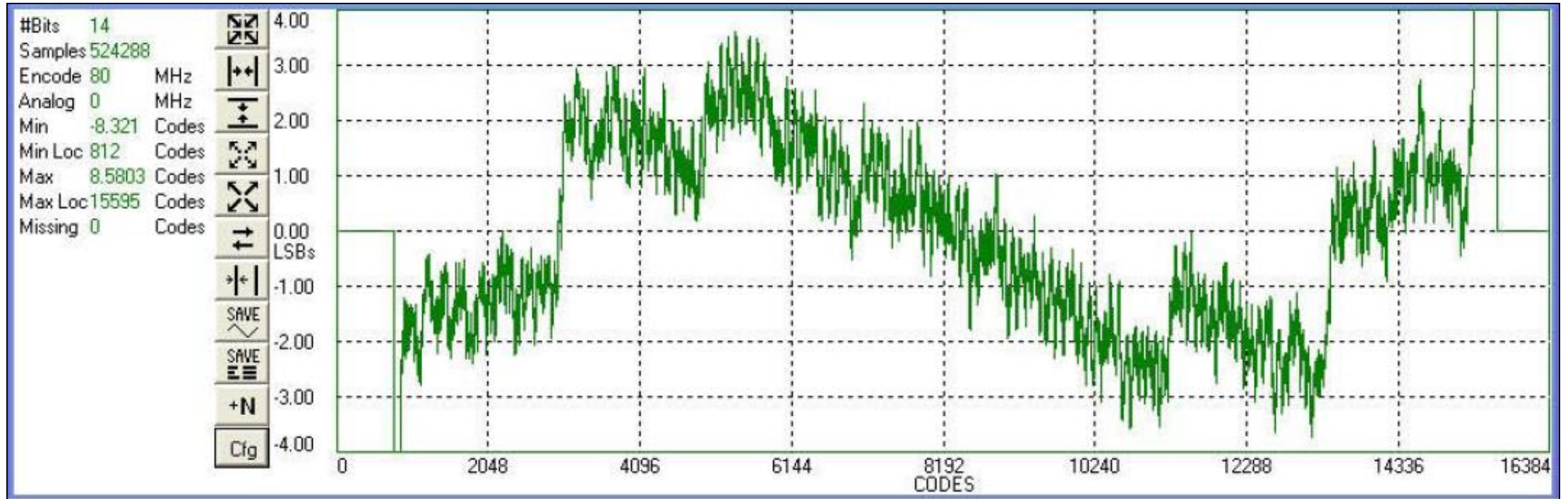
ADC Output With Inter-Stage Gain Error and DAC Error



Example of INL with Inter-stage Gain Errors in the First Stage

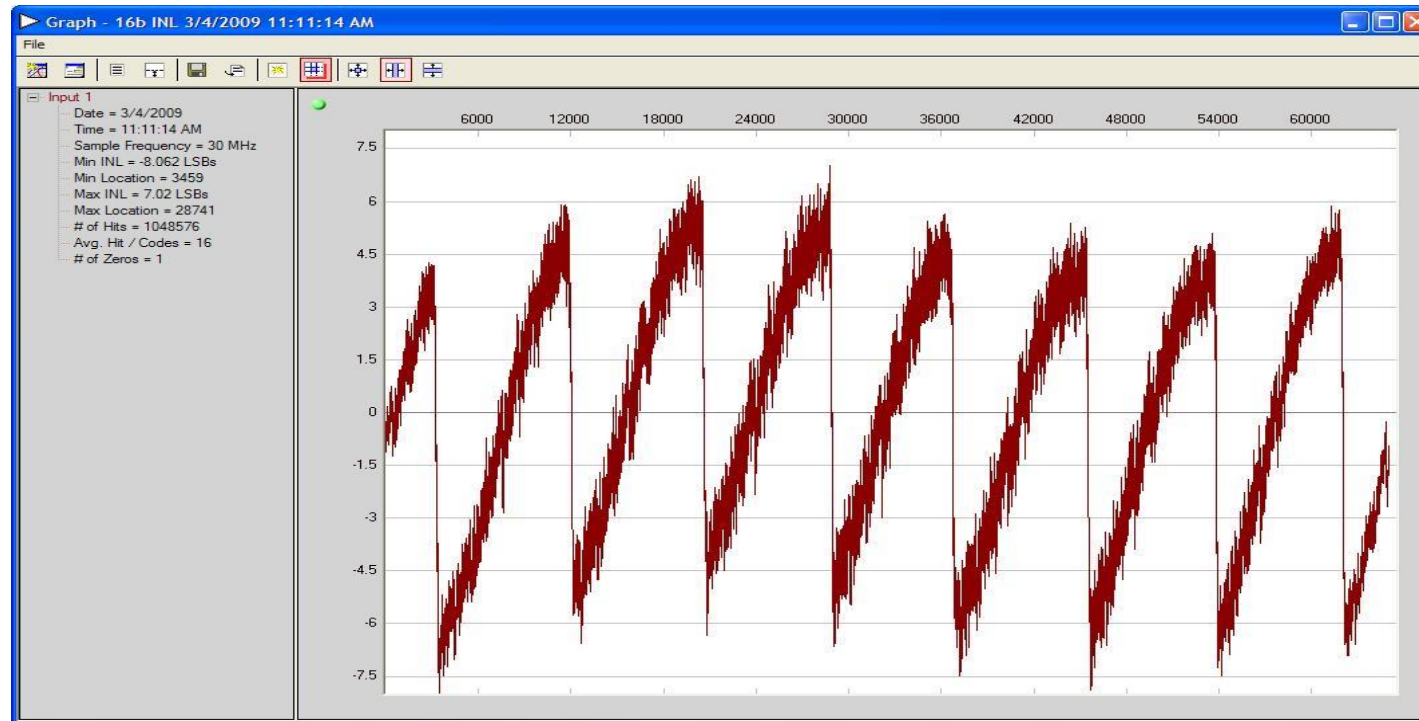


INL Showing Inter-stage Gain Error and DAC Errors

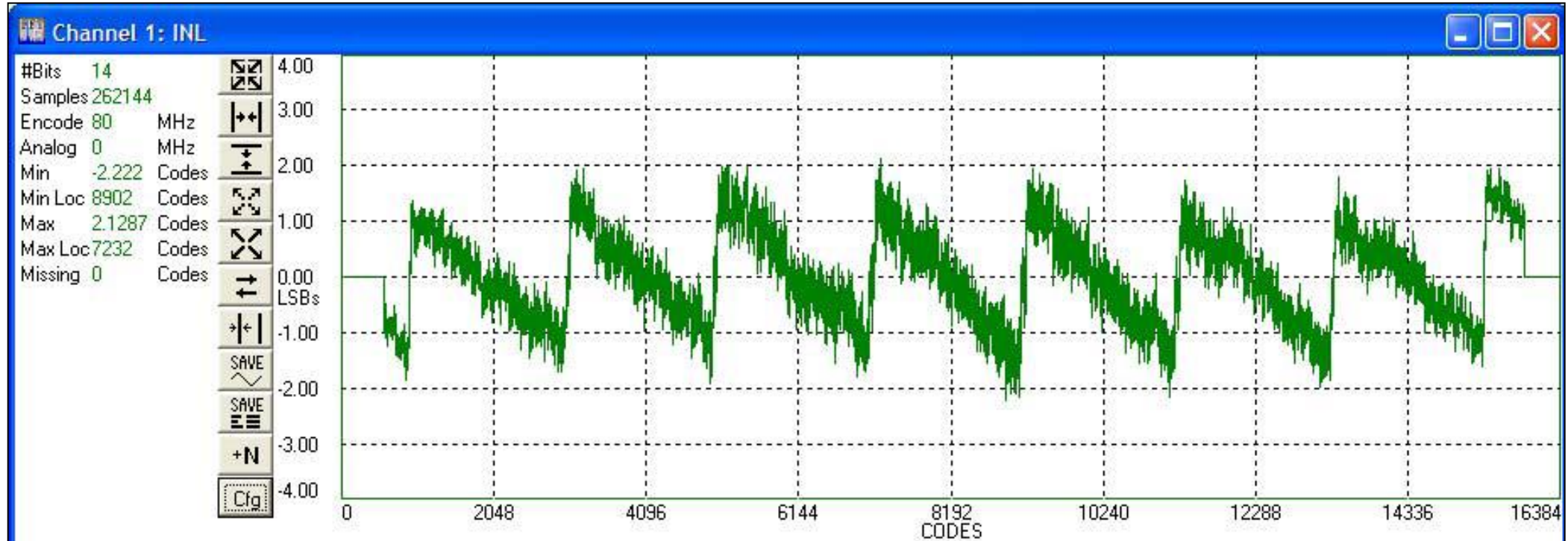


Questions

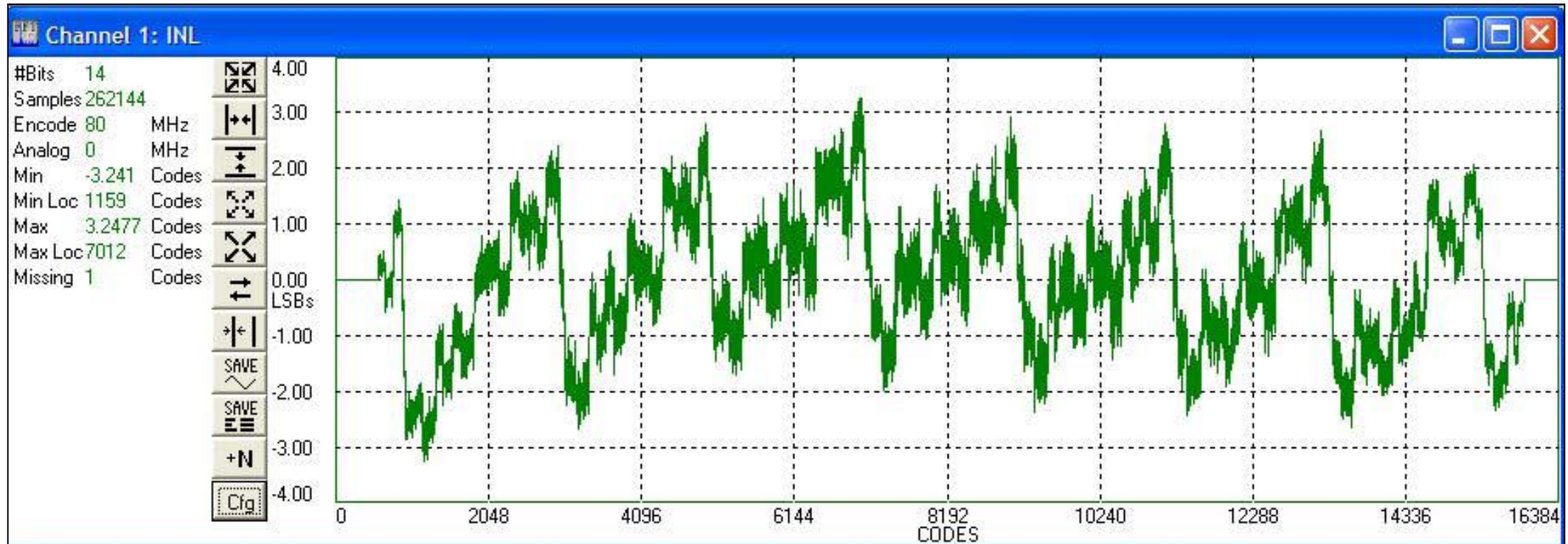
- ◆ Does this INL correspond to a small or large inter-stage gain? Missing codes or not?



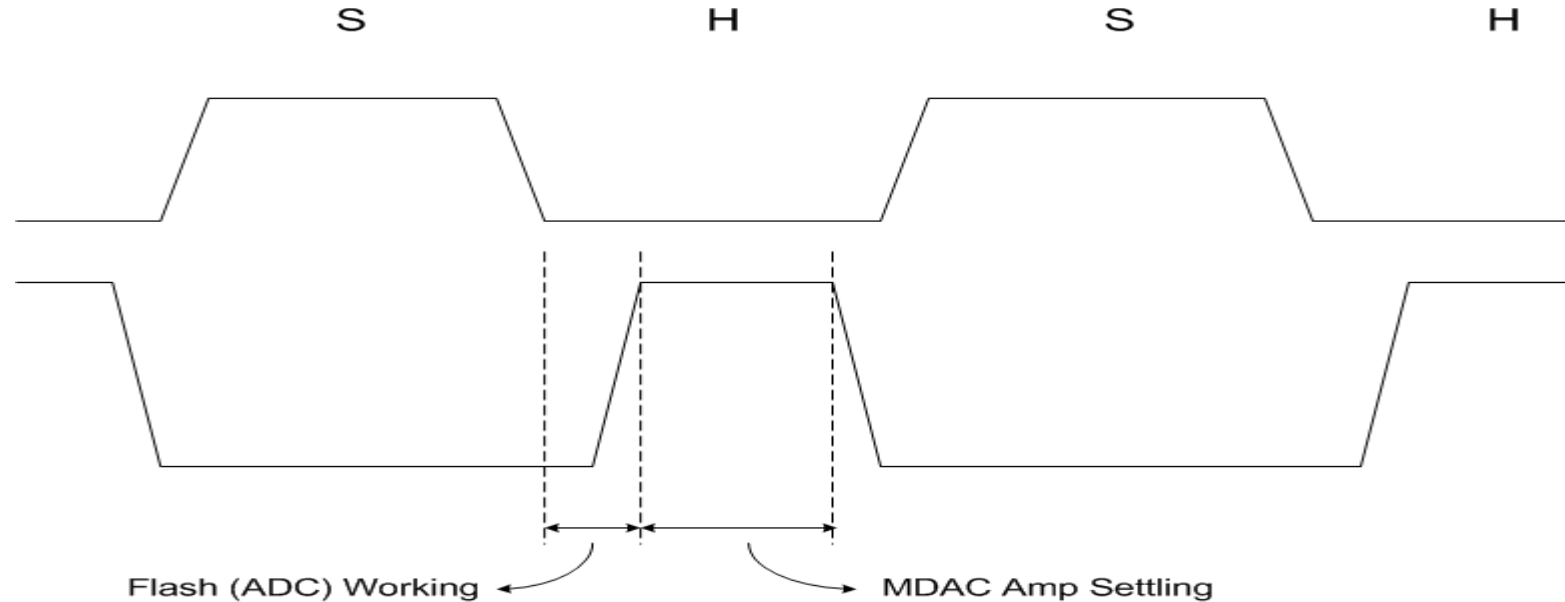
What kind of Errors?



What kind of Errors?



Stage Timing

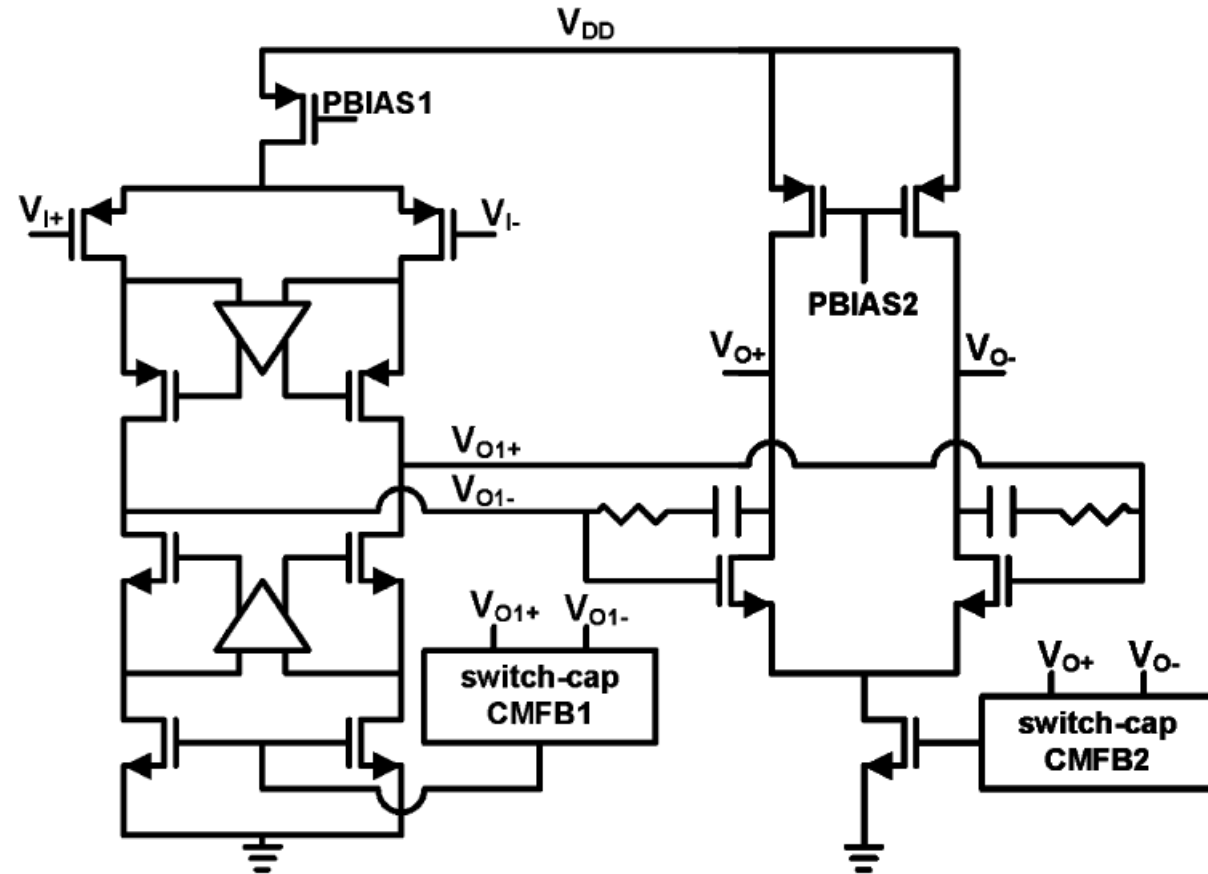


- ◆ The stage ADC usually operates on the input during the same phase as the MDAC generates the output
 - The flash delay consumes portion of the hold/gain phase
 - The MDAC amplifier settling time is the hold phase minus the time consumed by the flash and switches

MDAC Amplifier

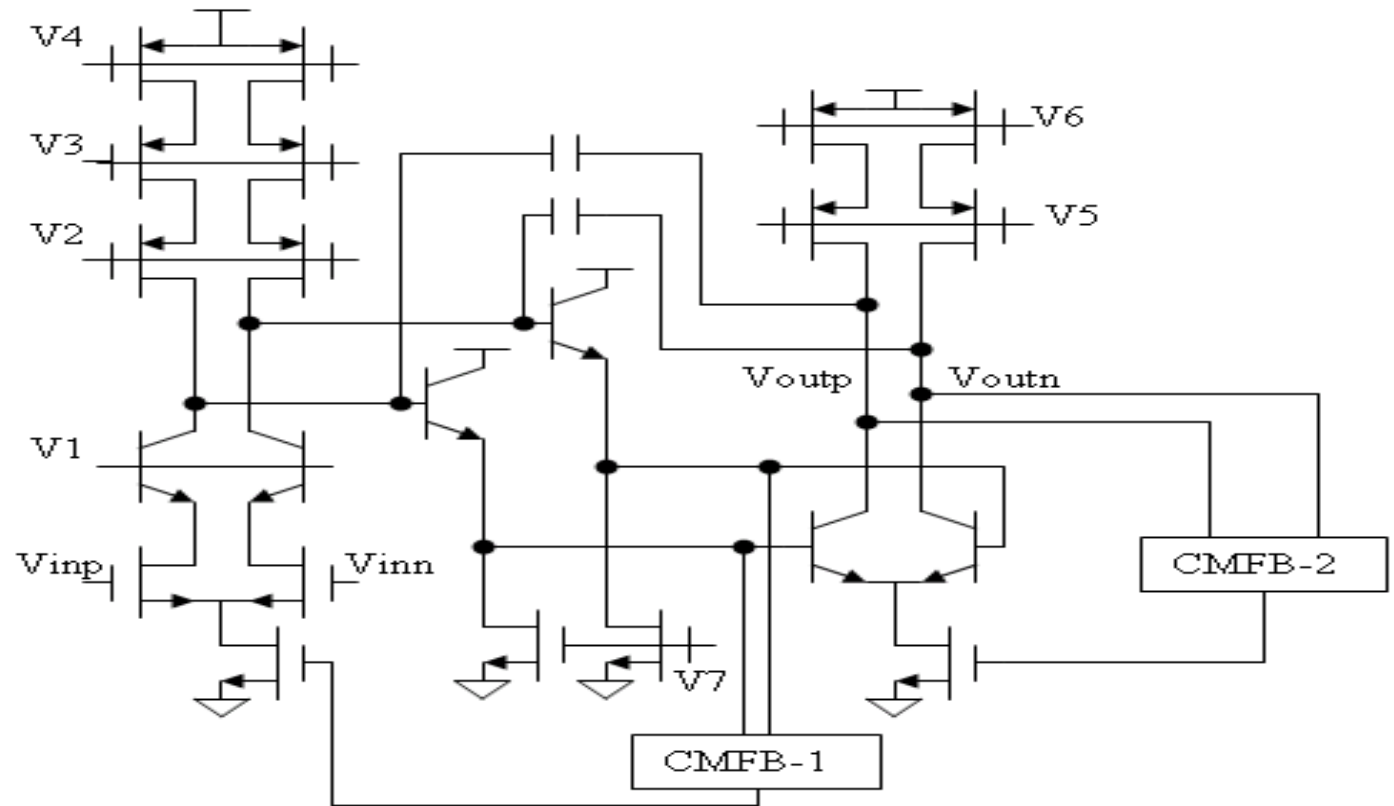
Example [1, 4]

- ◆ Consists of two-stage Miller-compensated amplifiers
- ◆ The first stage is active cascoded
- ◆ Two independent common-mode feedback loops are used to control the output common-mode voltages of the two stages
- ◆ Can achieve high gain



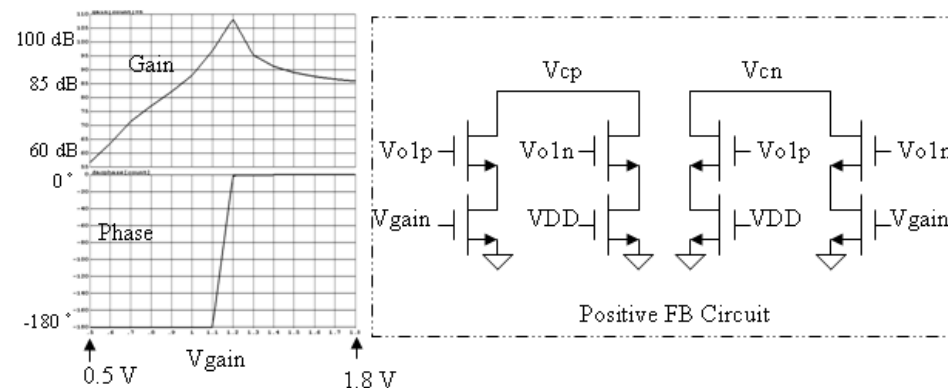
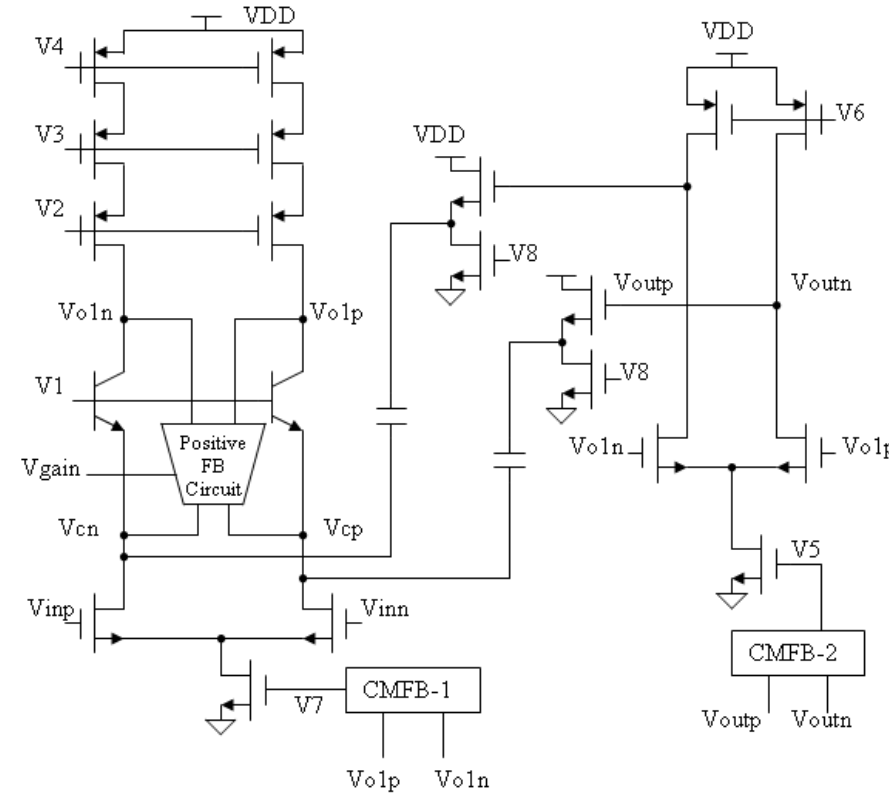
BiCMOS MDAC Amplifier Example [1]

- ◆ Consists of two-stage Miller-compensated BiCMOS cascode amplifiers
- ◆ The two stages are separated by emitter followers
- ◆ Two independent common-mode feedback loops are used to control the output common-mode voltages of the two stages

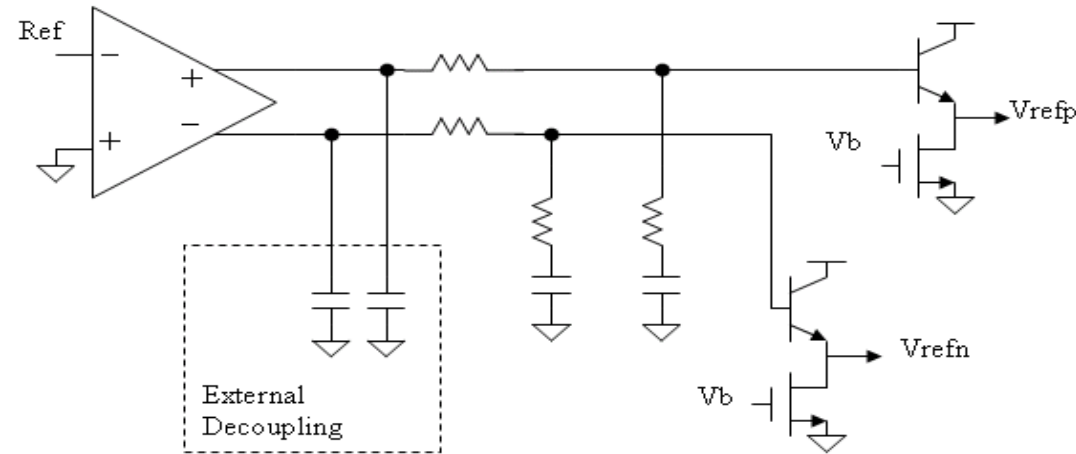


MDAC Amplifier Example [3]

- ◆ Consists of two-stage Miller-compensated amplifiers
- ◆ The first stage is cascoded
- ◆ Two independent common-mode feedback loops are used to control the output common-mode voltages of the two stages
- ◆ Positive feedback is used to increase the gain



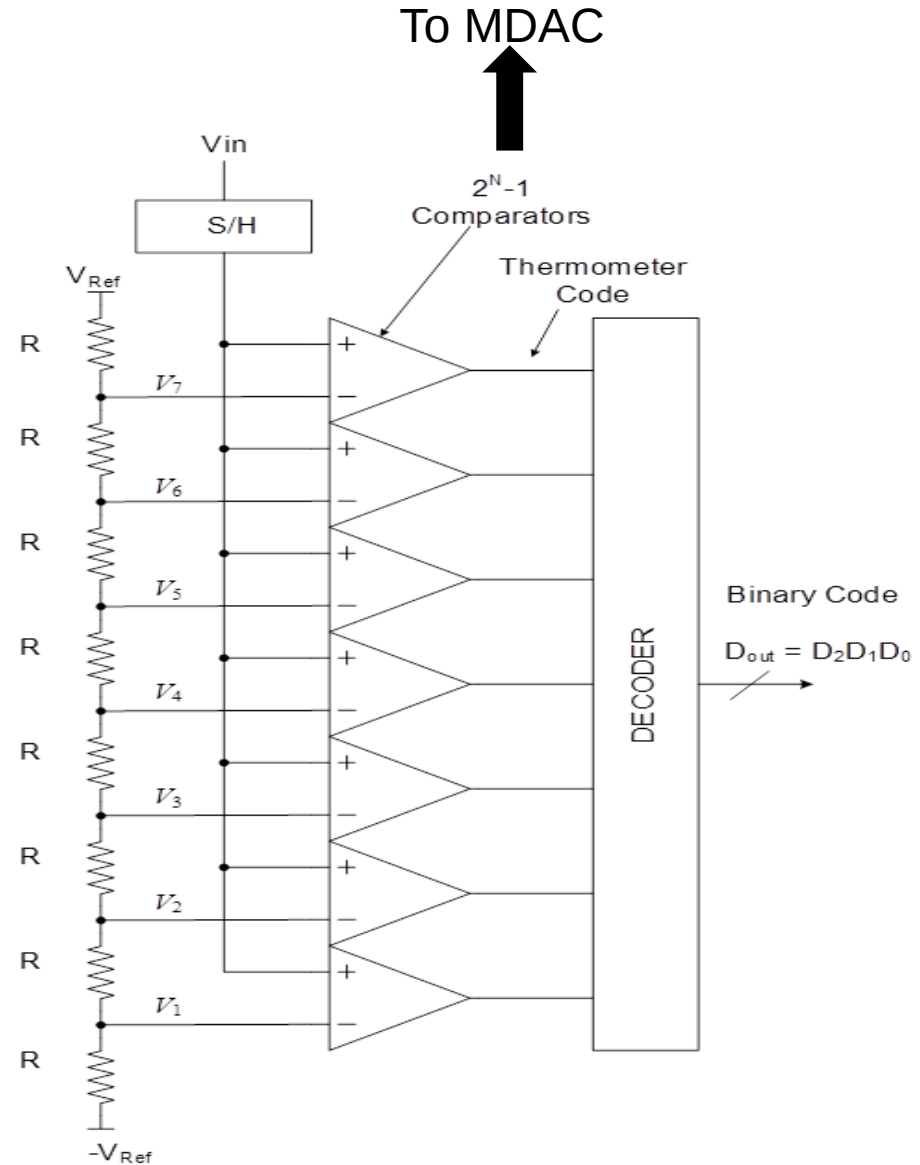
Reference Buffer Example



- ◆ **Fast settling:**
 - Settling errors cause inter-stage gain errors and DAC errors
- ◆ **Good supply and ground rejection**
- ◆ **Good isolation from the input signal**
- ◆ **Good common-mode rejection**
- ◆ **Quiet supplies and grounds**

Stage Flash ADC

- ◆ Generates thermometer and binary codes
- ◆ The thermometer code (or a variation of it) is routed to the MDAC
- ◆ The binary (or 2's complement) output is routed to the digital processor

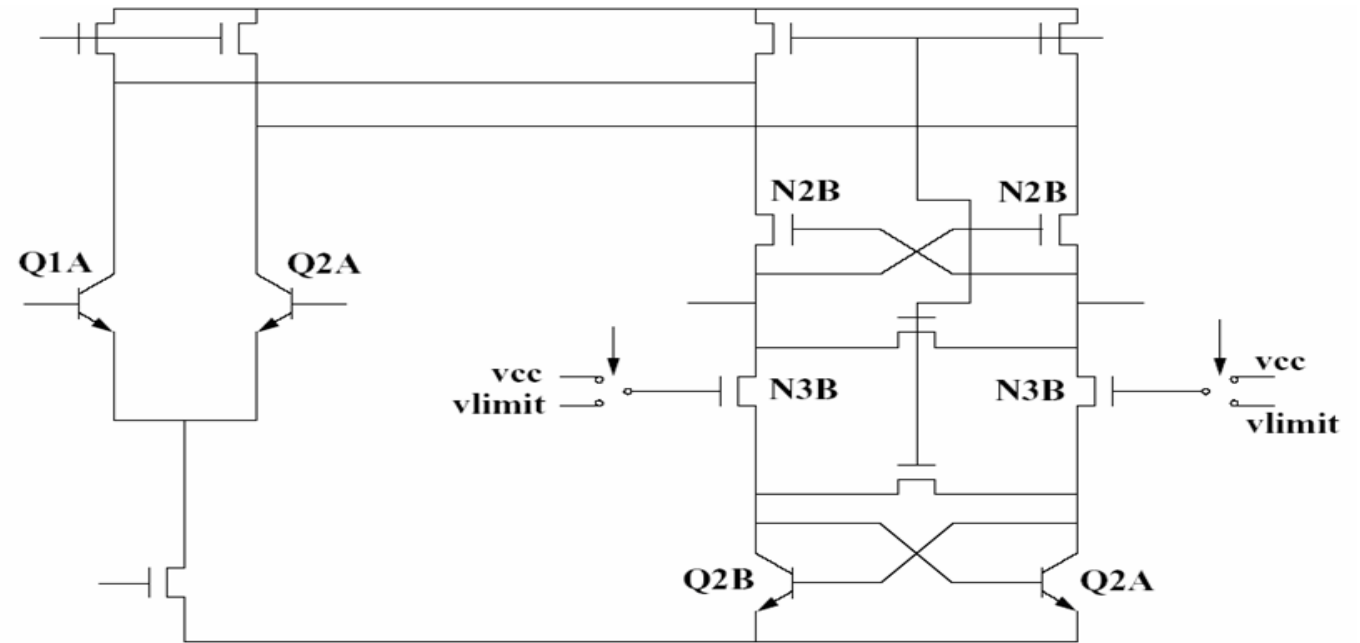


Comparator Design Considerations

- ◆ **Output propagation delay**
- ◆ **Meta-stability (regeneration time constant) determines the Bit-Error-Rate**
- ◆ **Offset needs to be within the correction range of the MDAC**
 - **The smaller the better**
- ◆ **Linearity is not critical**
- ◆ **Flash input BW needs to match the MDAC input BW in SHA-less ADCs**

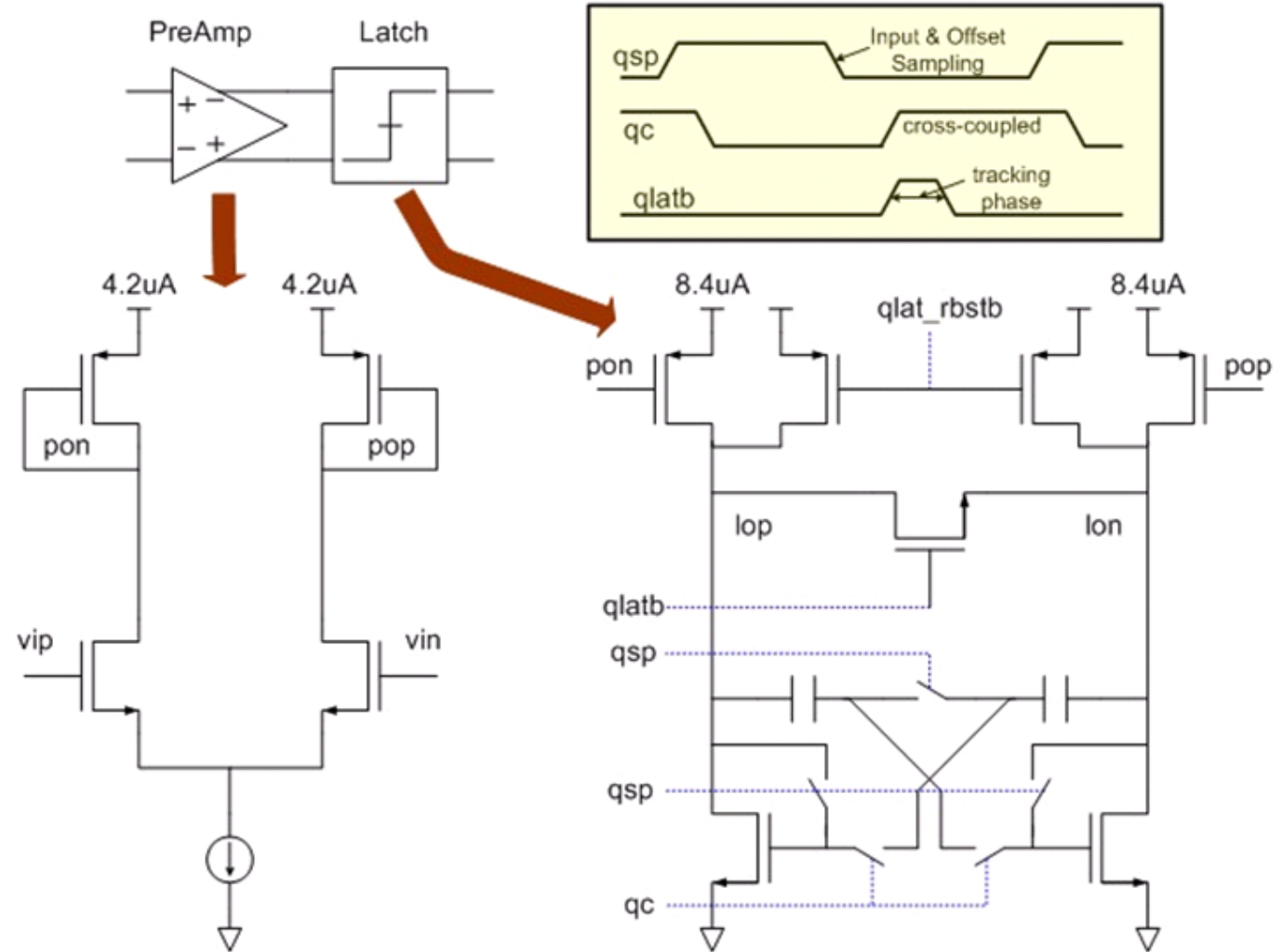
BiCMOS Comparator

- ◆ BJT's are used for their high f_t
- ◆ This leads to small re-generation time and hence good met-stability
- ◆ The pre-amplifier is used for isolation and to improve the offset



CMOS Comparator

- ◆ Relatively low f_t
- ◆ Offset cancellation is employed
- ◆ The pre-amplifier is used for isolation and to improve the offset



DESIGN CHALLENGES

Performance Limitations

Design Challenges

- ◆ Quantization Linearity
- ◆ Input Sampling
- ◆ Noise
- ◆ Jitter

Internal Linearity

◆ DAC accuracy:

- Capacitor matching
- Settling
- Charge injection

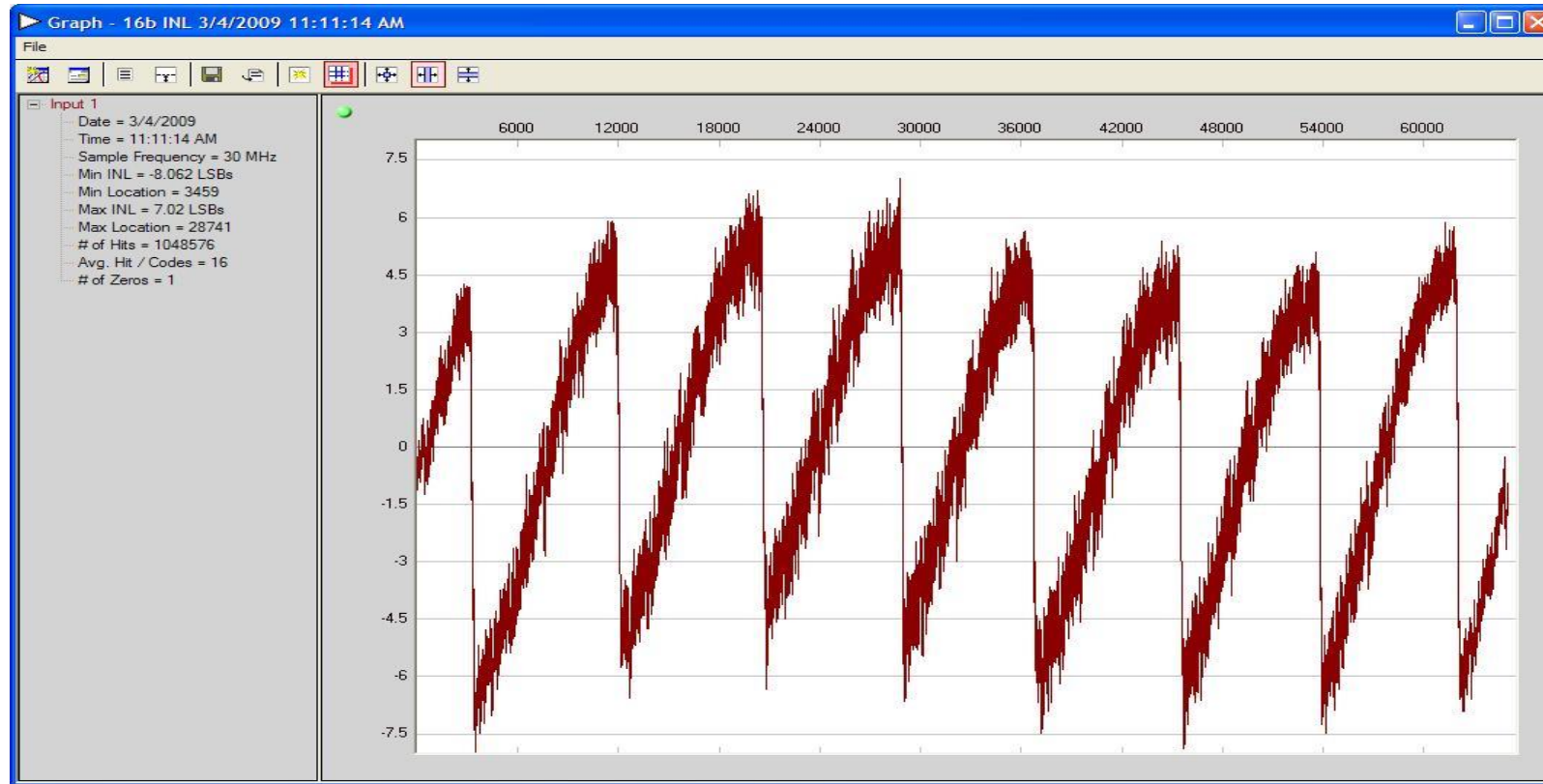
◆ Amplifier accuracy:

- Gain
- Settling
- Charge injection

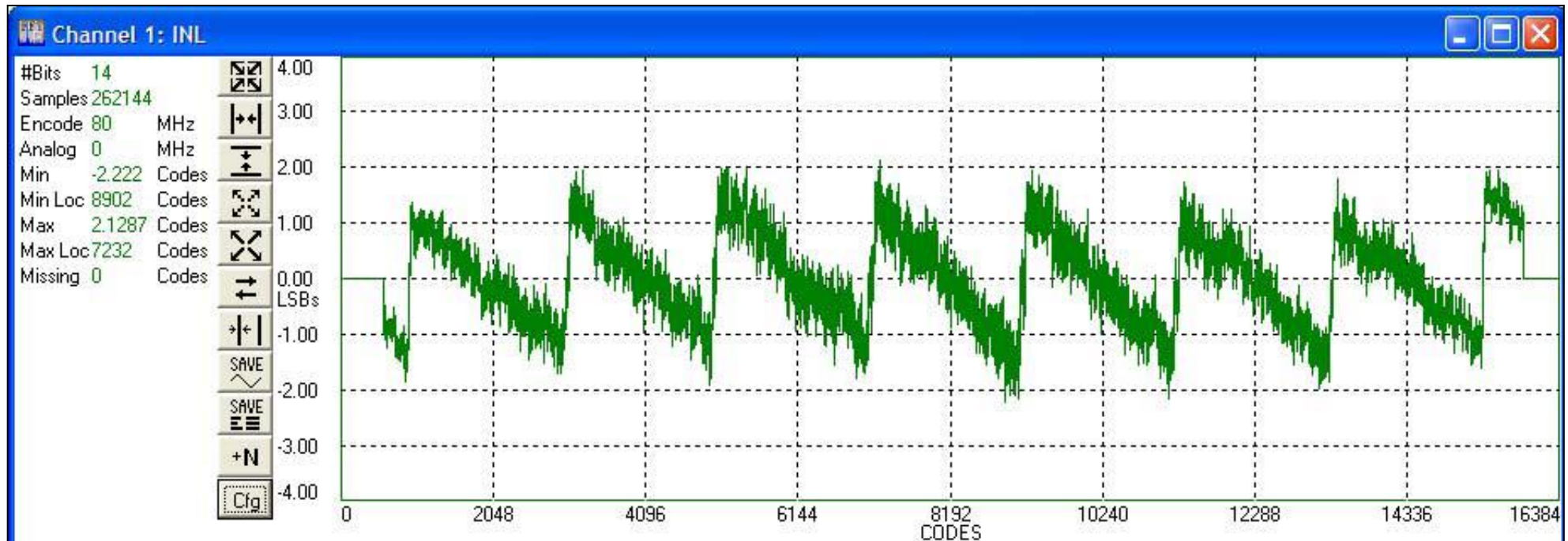
◆ Reference accuracy:

- Settling

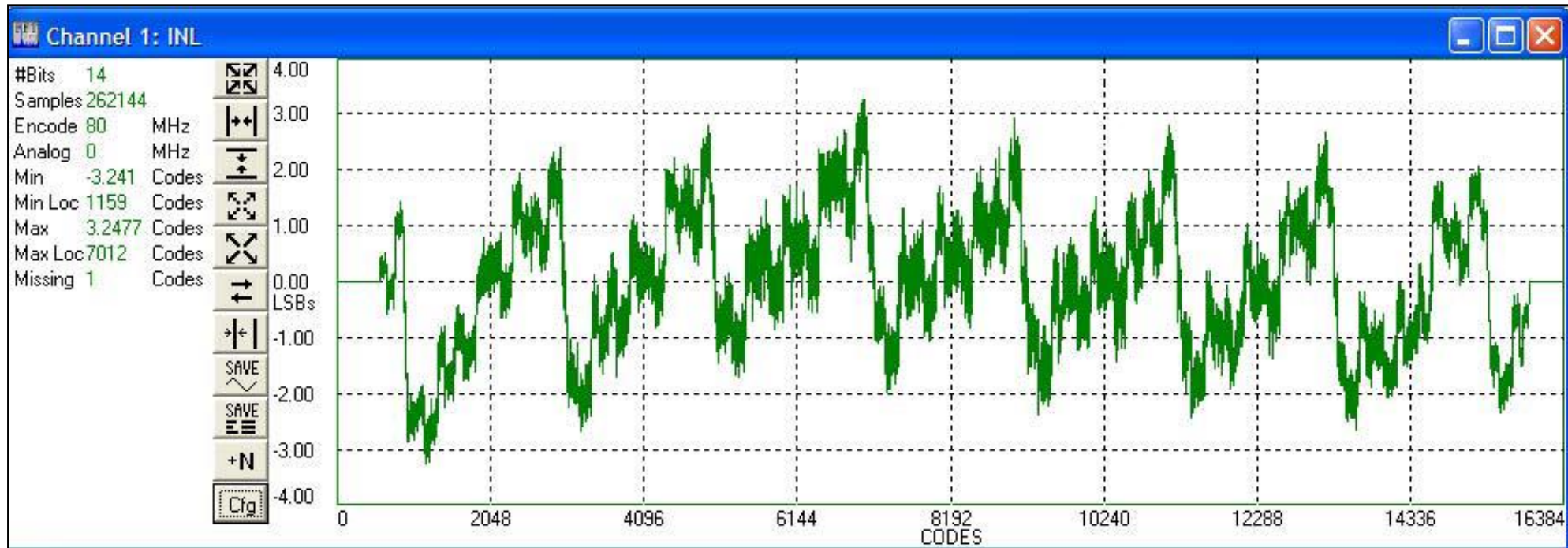
Question: What are the possible circuit causes of this INL?



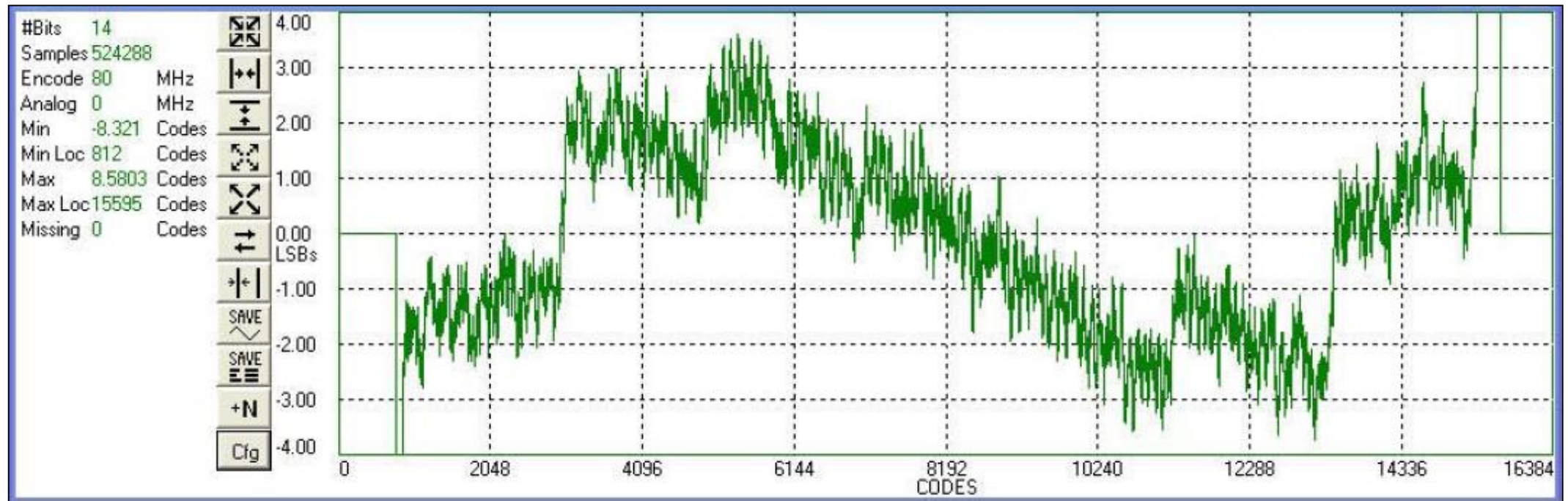
Question: What are the possible circuit causes of this INL?



Question: What are the possible circuit causes of this INL?



Question: What are the possible circuit causes of this INL?



Question

- ◆ A 14-bit pipelined ADC with 4 bits/stage. What is the accuracy requirement for the following?
 - First stage DAC
 - First stage amplifier
 - First Stage ADC
 - Second stage DAC
 - Second stage amplifier
 - Second stage ADC

NOISE

Sources of Noise

- ◆ **Thermal noise in the signal path**
 - Includes noise during both phases (sample and hold)
 - Noise contribution is lower for stages down the pipeline
 - Scaling is usually employed to lower power consumption of back-end stages
- ◆ **Noise due to non-linearity (DNL/INL)**
- ◆ **Noise in the clock path (Jitter)**
- ◆ **Noise due to quantization**

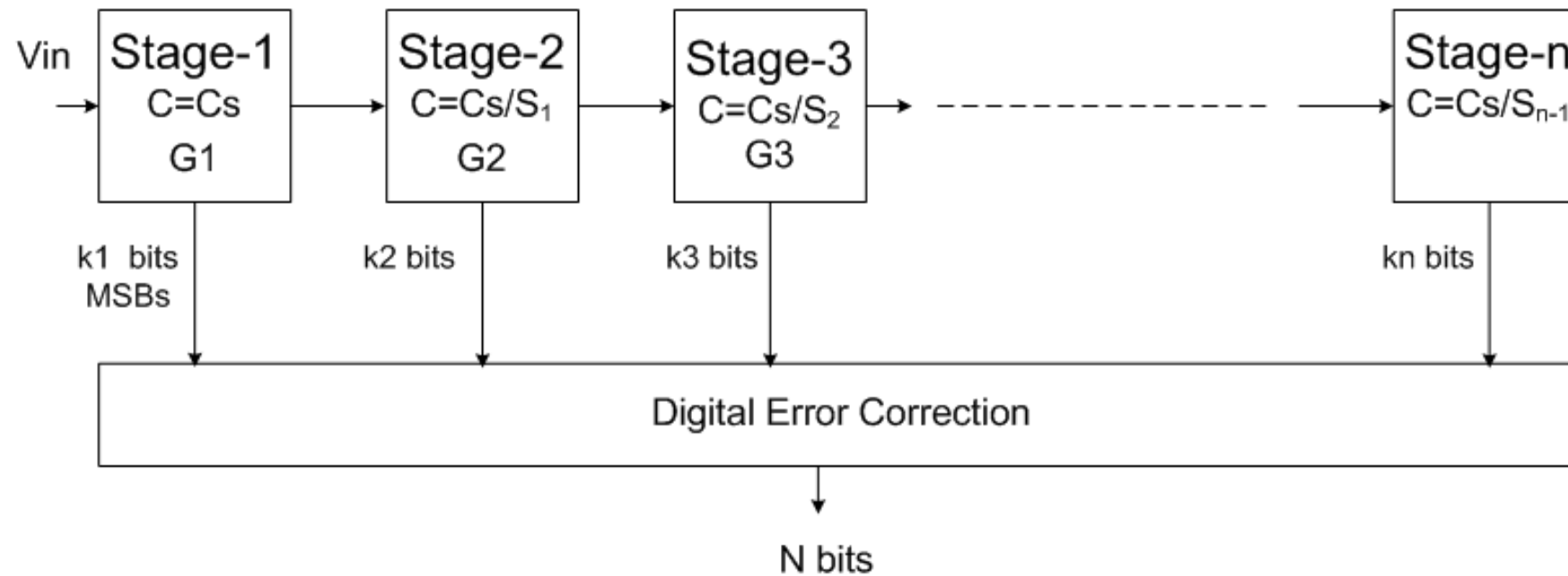
Thermal Noise

- ◆ Sampling noise is proportional to KT/C_s
 - Where C_s is the sampling capacitor
- ◆ If the gain of each stage is G , the noise power of the k th stage is scaled down by $G^2(k-1)$ when referred to the input
- ◆ If a scaling factor S is employed for the capacitor of each stage:
 - The noise power of the k th stage, at the stage (not input referred) is scaled up by S
 - When referred to the input. The noise will be scaled by:
 - ◆ $S(k-1)/G^2(k-1)$

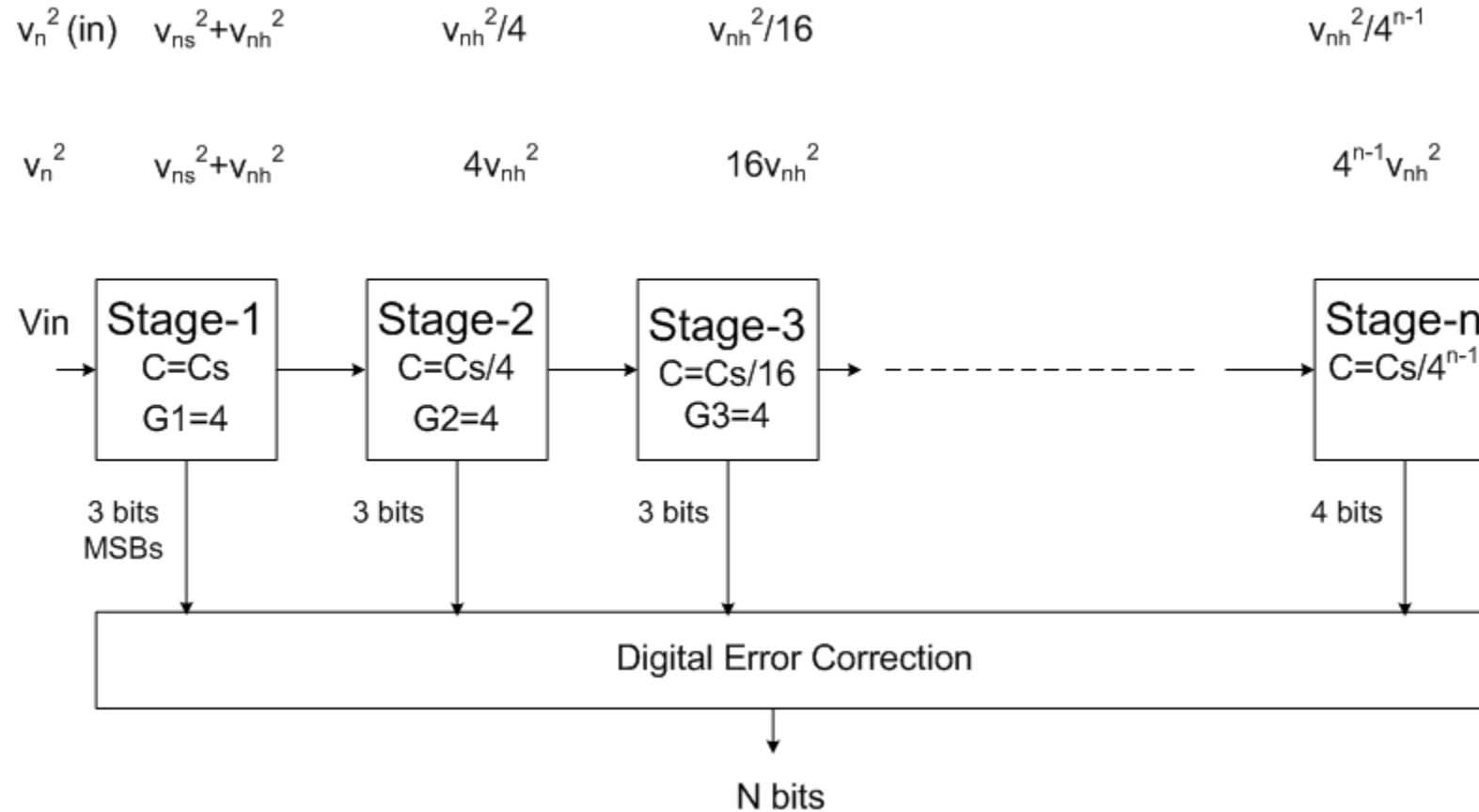
Stage Scaling

$$V_n^2(\text{in}) \quad v_{ns}^2 + v_{nh}^2 \quad S_1 v_{nh}^2 / G_1^2 \quad S_2 v_{nh}^2 / G_1^2 G_2^2 \quad S_{n-1} v_{nh}^2 / G_1^2 G_2^2 \dots G_{n-1}^2$$

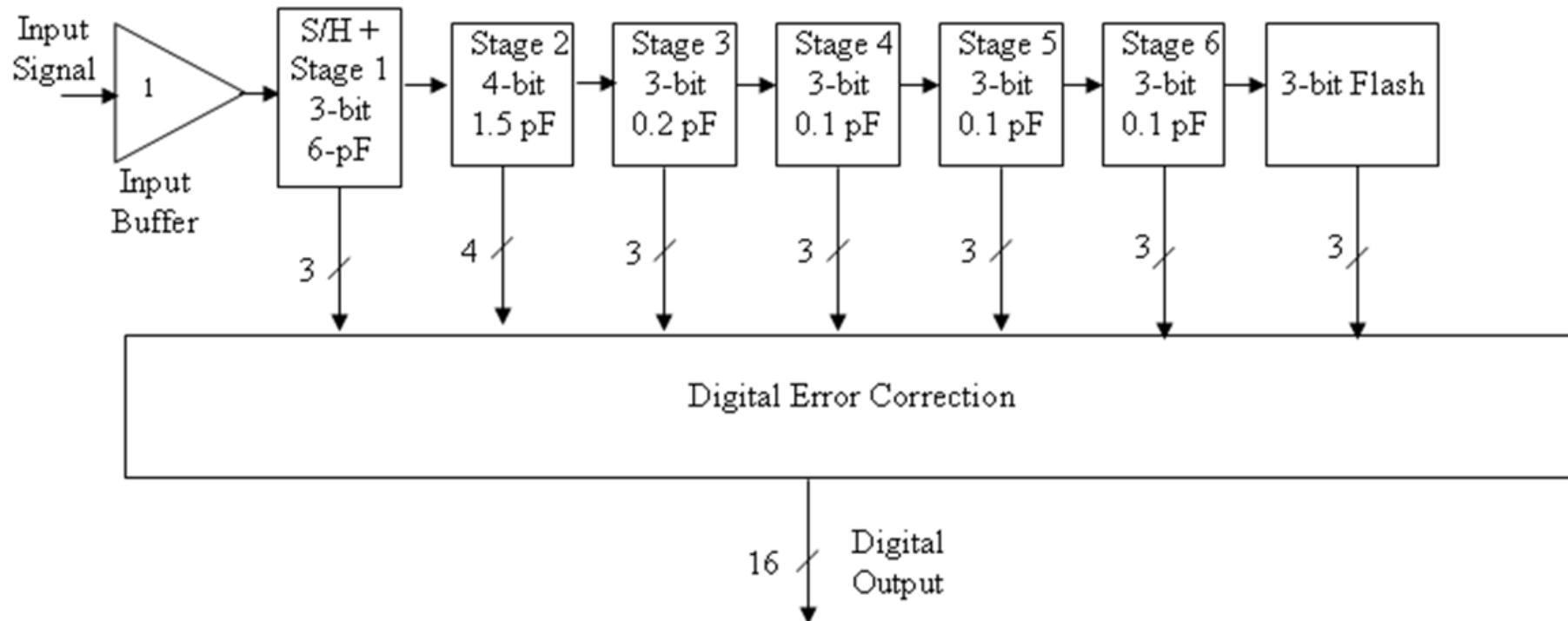
$$V_n^2 \quad v_{ns}^2 + v_{nh}^2 \quad S_1 v_{nh}^2 \quad S_2 v_{nh}^2 \quad S_{n-1} v_{nh}^2$$



Stage Scaling



Example [3]



System Level Design Considerations

- ◆ **Sampling Capacitor Value**
- ◆ **Number of Bits/Stage**
- ◆ **Input Buffer or Not**
- ◆ **Input Span**
- ◆ **Scaling factor**

Number of Bits per Stage

◆ Fewer bits per stage:

- Fewer comparators => lower power in the flash
- Exploit finite gain-bandwidth of the amplifier:
 - ◆ Lower amplifier gain => higher speed
- Less complex
- With 1.5 bit/stage, DAC is inherently linear

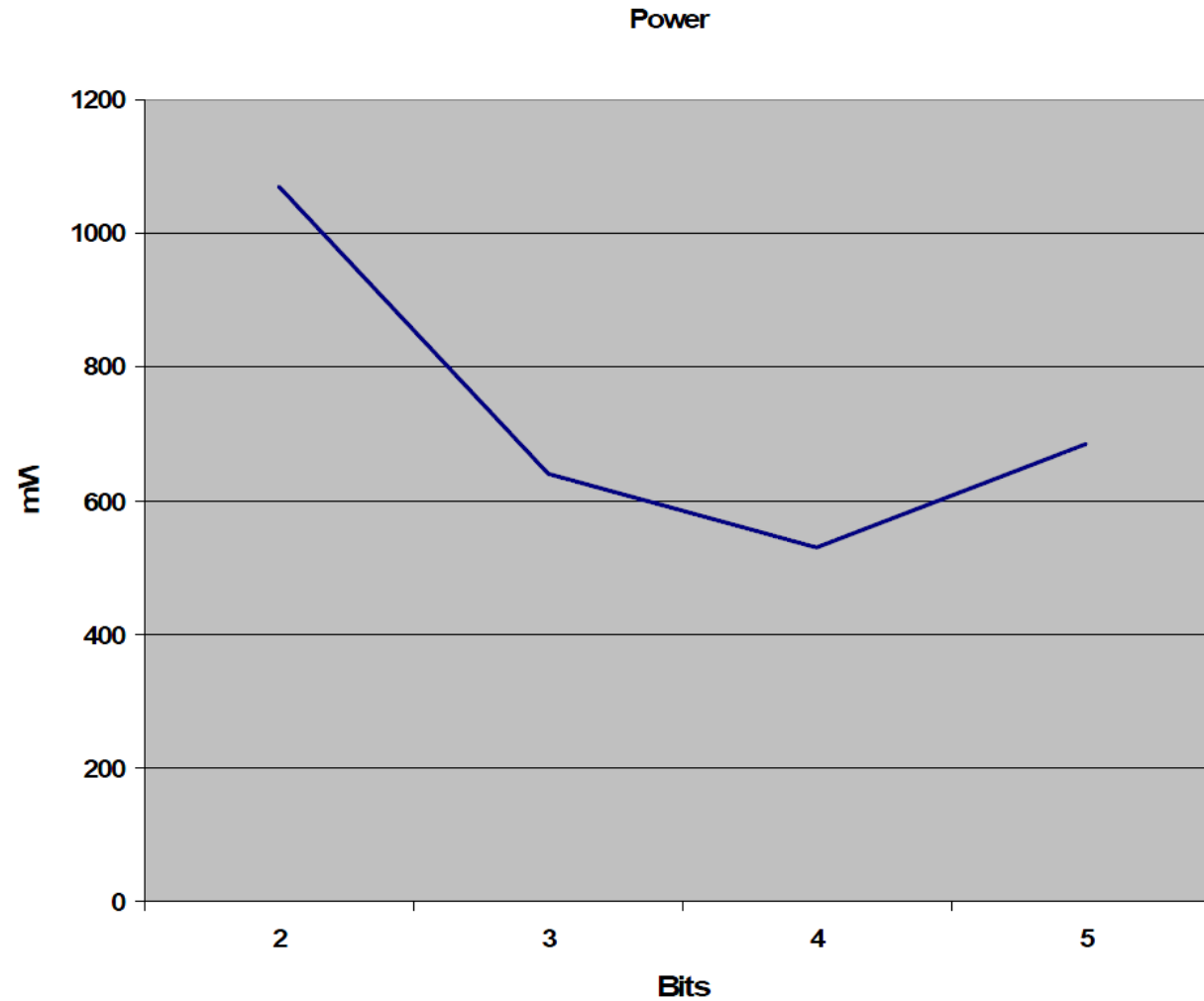
◆ More bits per stage:

- Quickly relaxes the noise and accuracy requirements of the following stages
- The MDAC is more power efficient

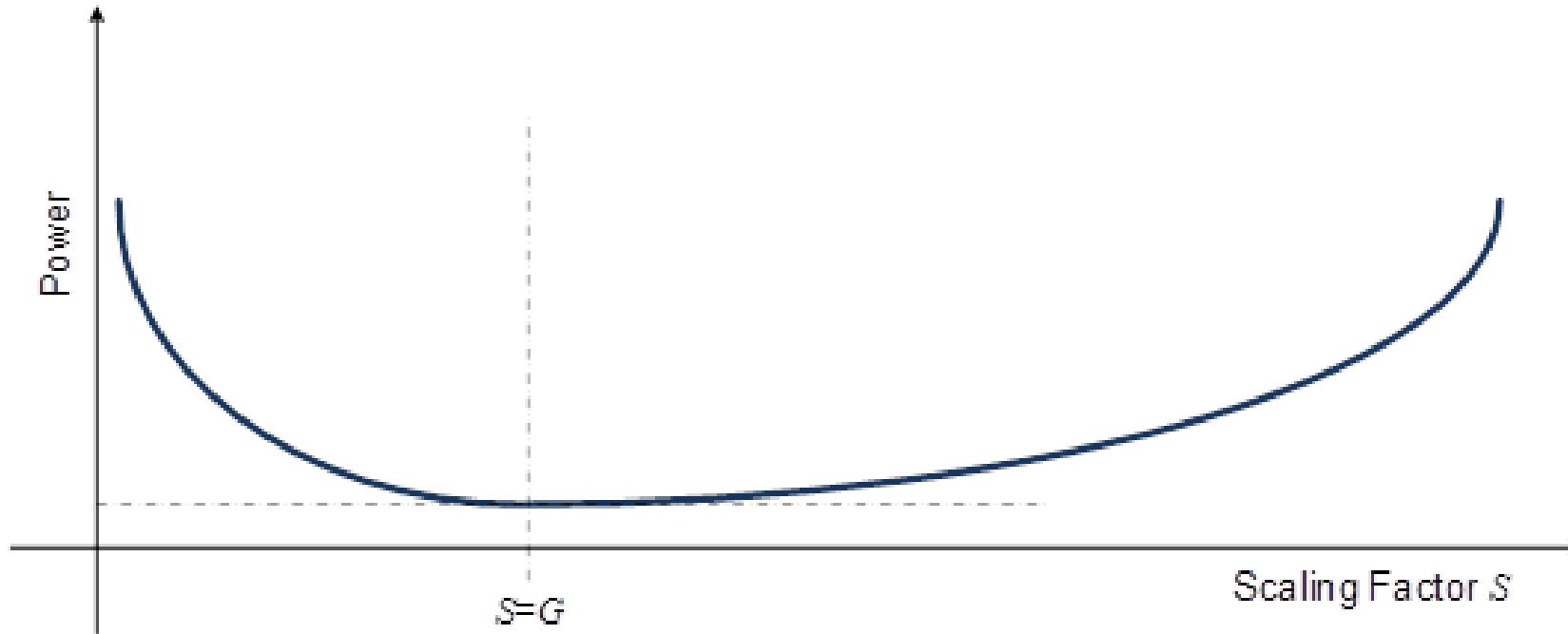
◆ Conclusion:

- There is an optimum resolution per stage to minimize power consumption
- Sometimes the optimum resolution is not feasible => use lower resolution

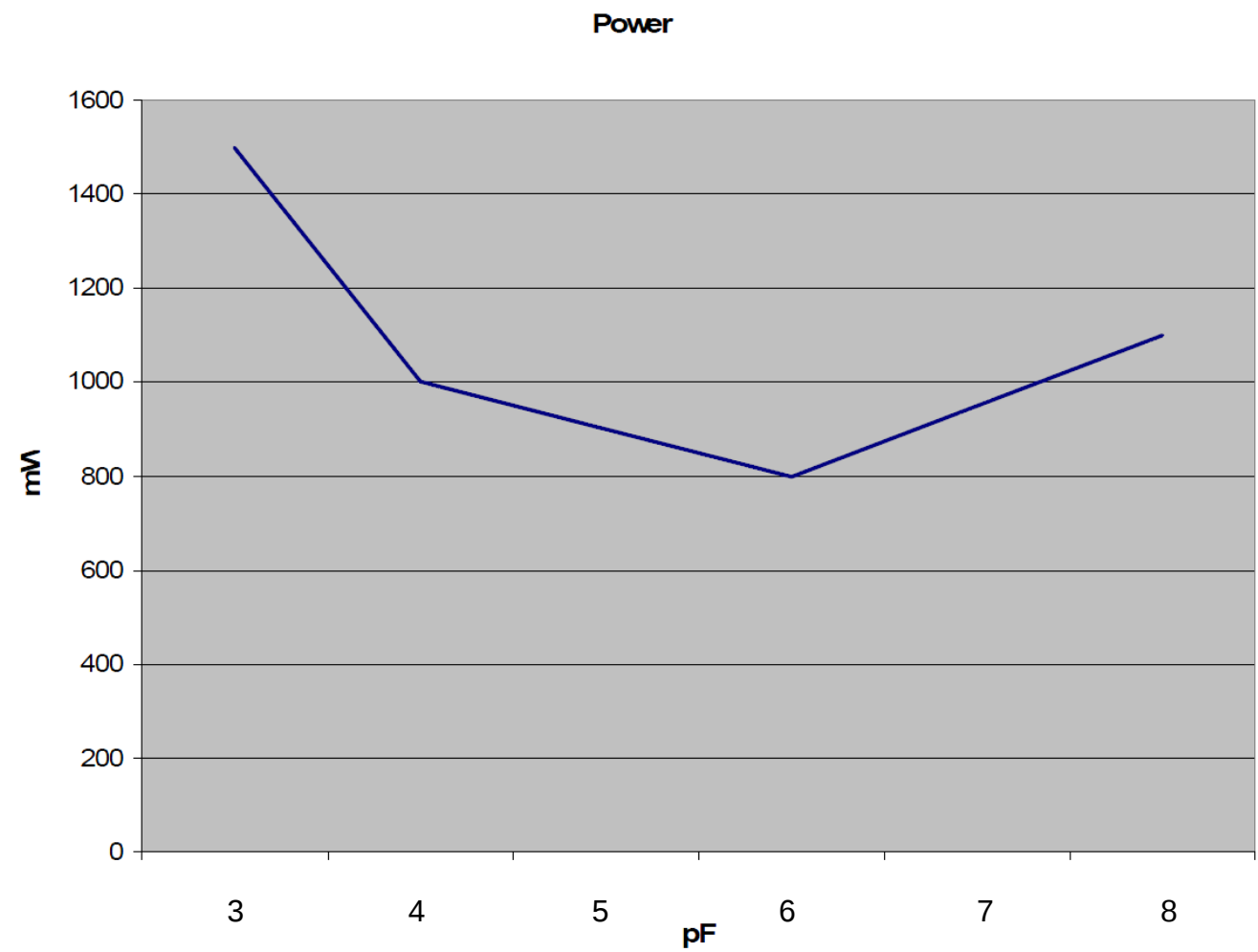
Number of Bits/Stage



Scaling Factor



Capacitance Value



DIGITALLY ASSISTED A/D CONVERTERS

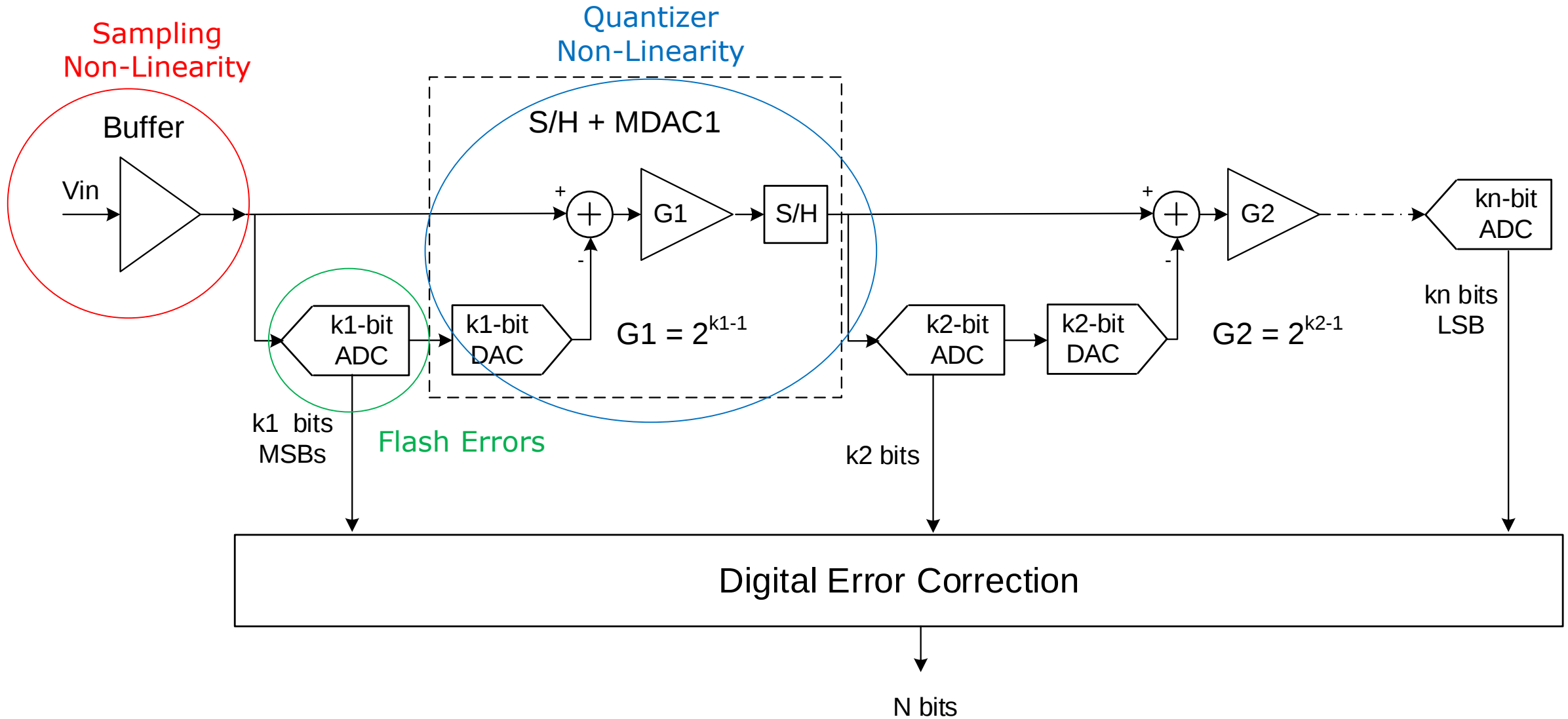
System Challenges

- ◆ **Higher sample rates for larger bandwidths:**
 - Simplifies filtering
 - Frequency planning
 - More information
 - More carriers
 - 14-12 bit at $> 1\text{GS/s}$ ADCs
- ◆ **RF Sampling:**
 - Lower cost
 - More flexibility
- ◆ **Lower power consumption:**
 - More integration
 - More receivers
- ◆ **Higher linearity:**
 - More focus on small signal linearity and high order harmonics

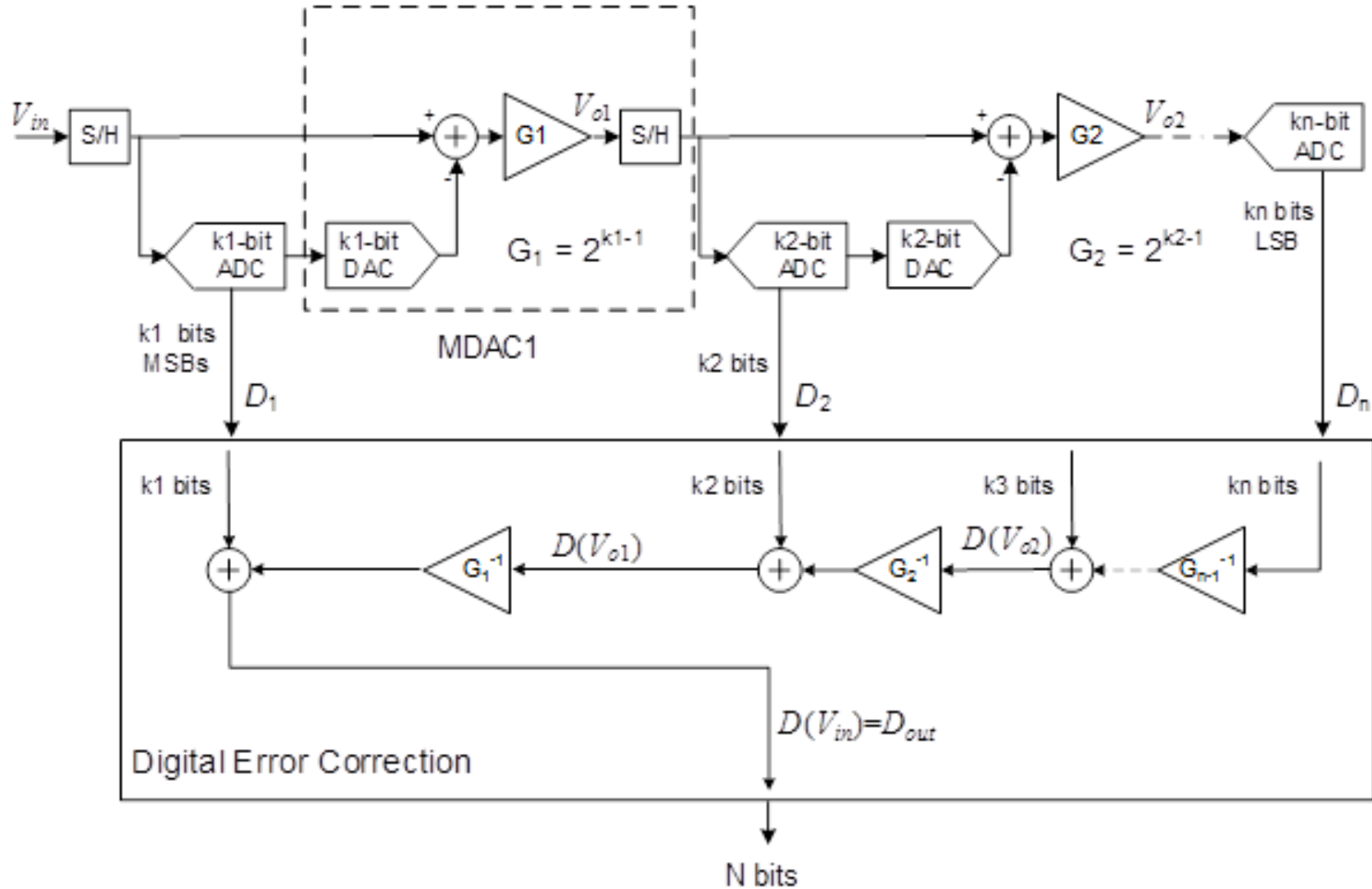
Process Scaling

- ◆ Digital circuits have got more efficient with process scaling
- ◆ Analog circuits get more challenging and can be even less efficient
- ◆ Finer geometry gives:
 - Faster switches
 - Lower parasitics
- ◆ But ...
 - Lower output impedance
 - Lower intrinsic gain
 - Worse dynamic range
- ◆ Digital processing can be used to correct for analog non-idealities
- ◆ Digital assistance has become necessary

Pipelined ADC



Digital Error Correction



Digital Assistance in Pipelined ADCs

◆ Quantizer non-linearity:

- Inter-stage gain and settling errors (IGE)
- DAC errors
- Reference errors
- Inter-stage memory errors (IME)
- MDAC amplifier non-linearity

◆ Sampling non-linearity:

- Kick-back errors
- Sampling non-linearity

◆ Flash errors [5, 6]:

- Offset and gain errors
- BW/timing mismatch errors

Digital Assistance in Pipelined ADCs

◆ Quantizer non-linearity:

- Inter-stage gain and settling errors (IGE)
- DAC errors
- Reference errors
- Inter-stage memory errors (IME)
- MDAC amplifier non-linearity

◆ Sampling non-linearity:

- Kick-back errors
- Sampling non-linearity

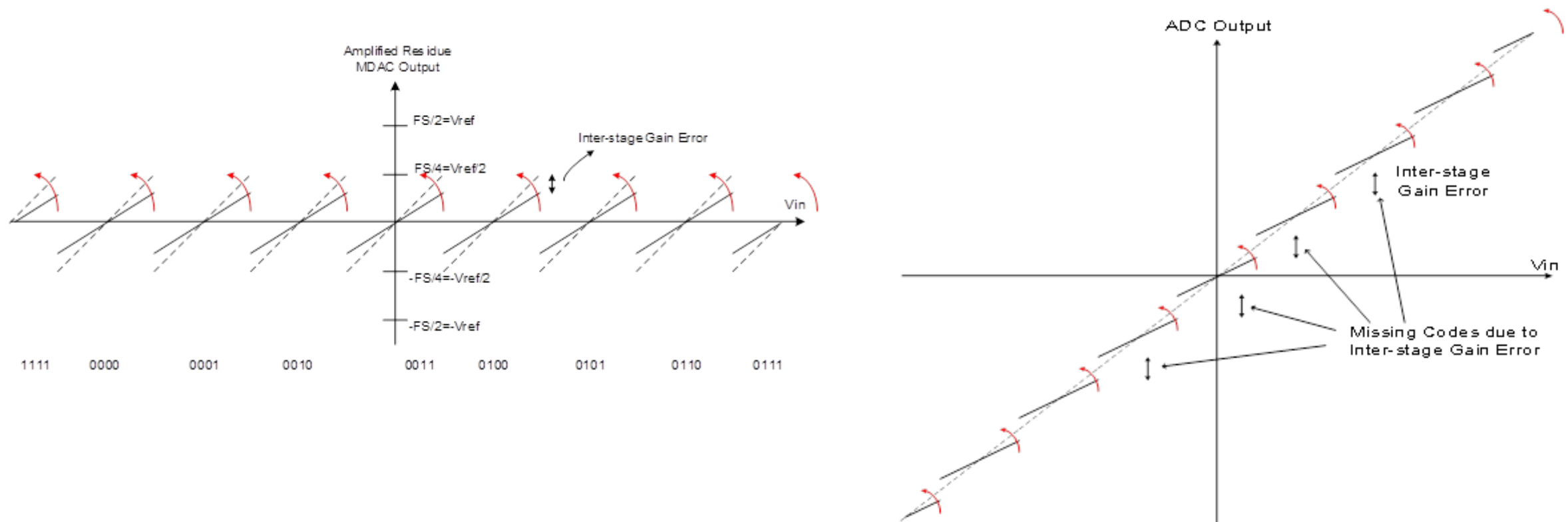
◆ Flash errors [5, 6]:

- Offset and gain errors
- BW/timing mismatch errors

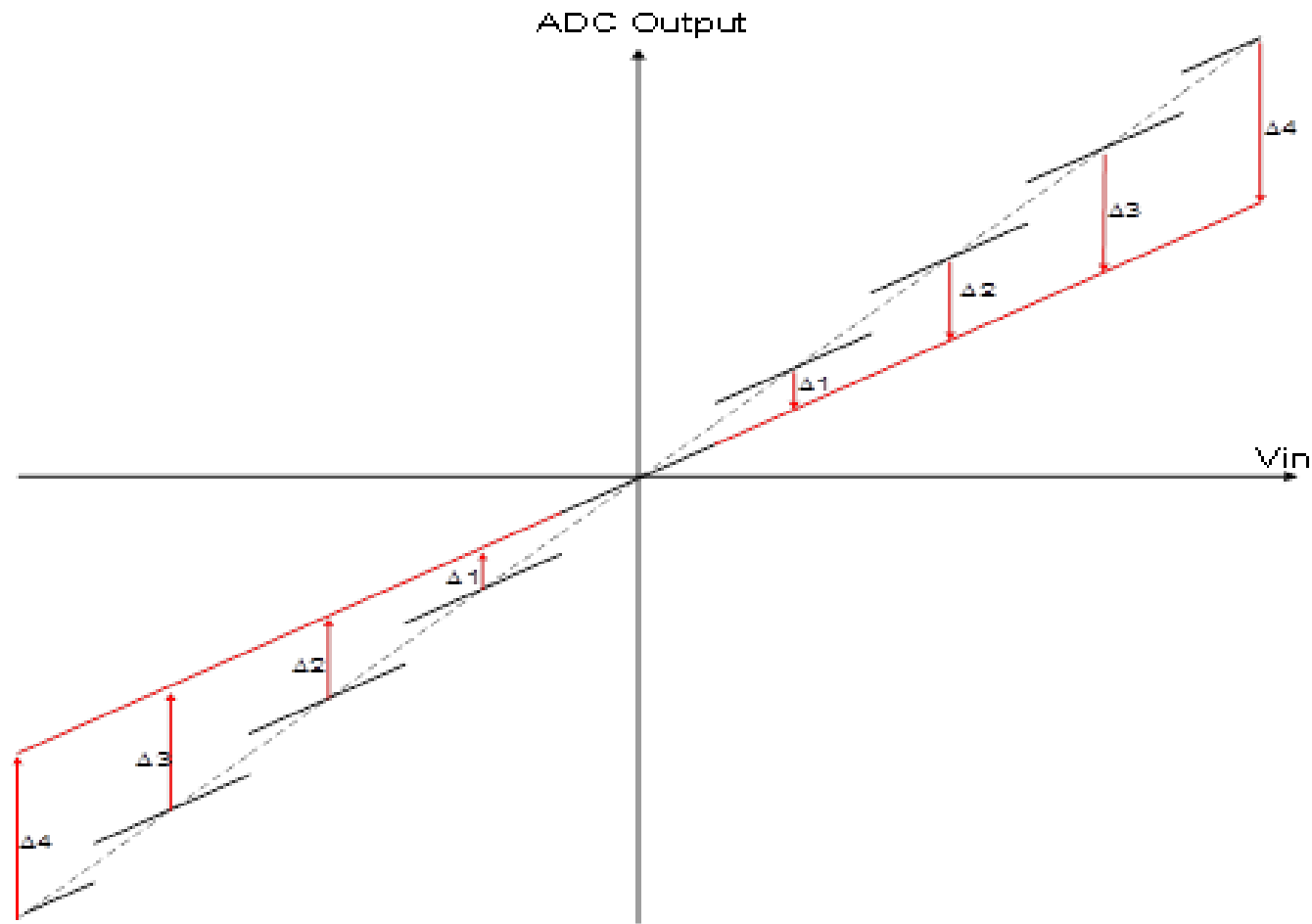
Factory Calibration

- ◆ Capacitor mismatch does not change much with temperature, supply, aging, etc.
 - => It can be factory calibrated
- ◆ This is done using digital coefficients in the backend
- ◆ Factory digital calibration can be employed for some other sources of error, but with less consistency and reliability
- ◆ Background calibration of internal non-linearity is increasingly becoming more important

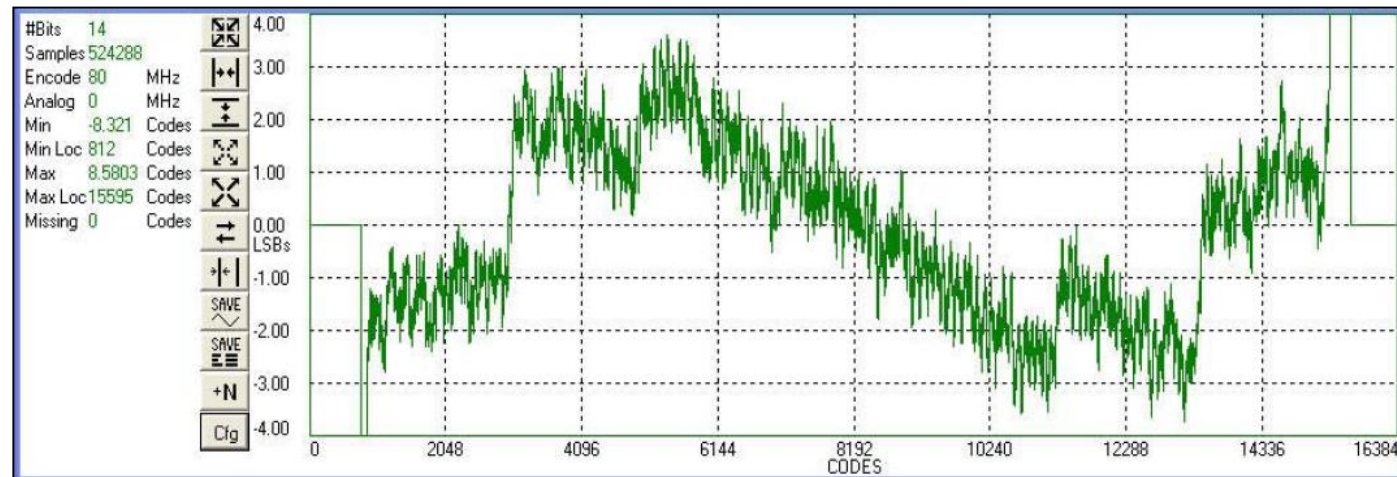
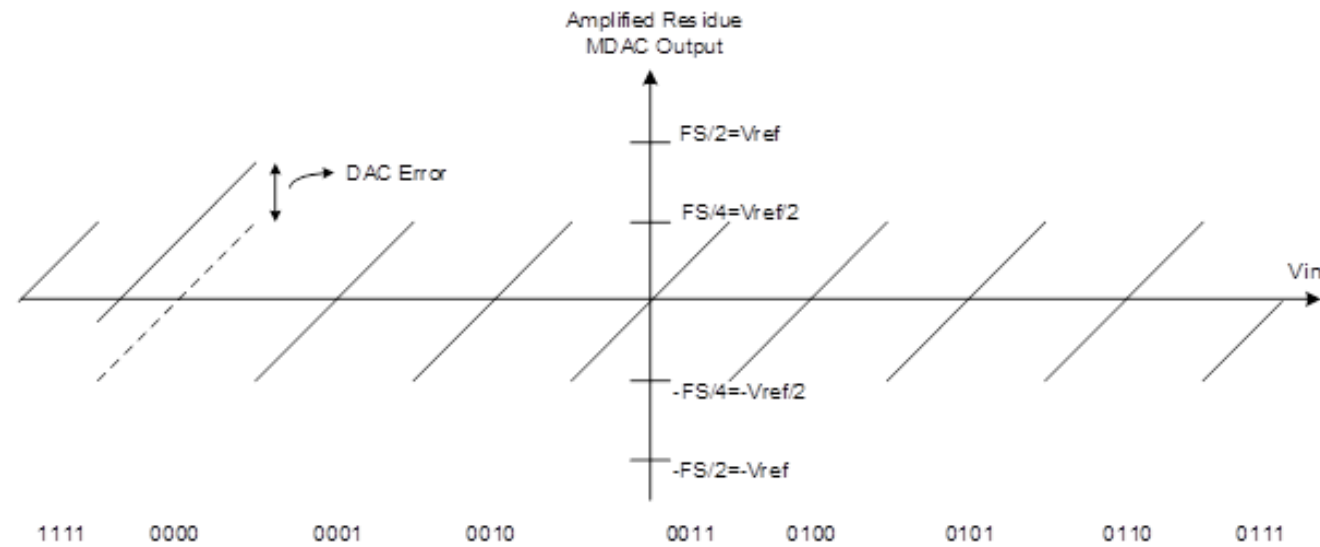
Inter-stage Gain Error Calibration



Inter-stage Gain Error Calibration Using Sub-range Shifting



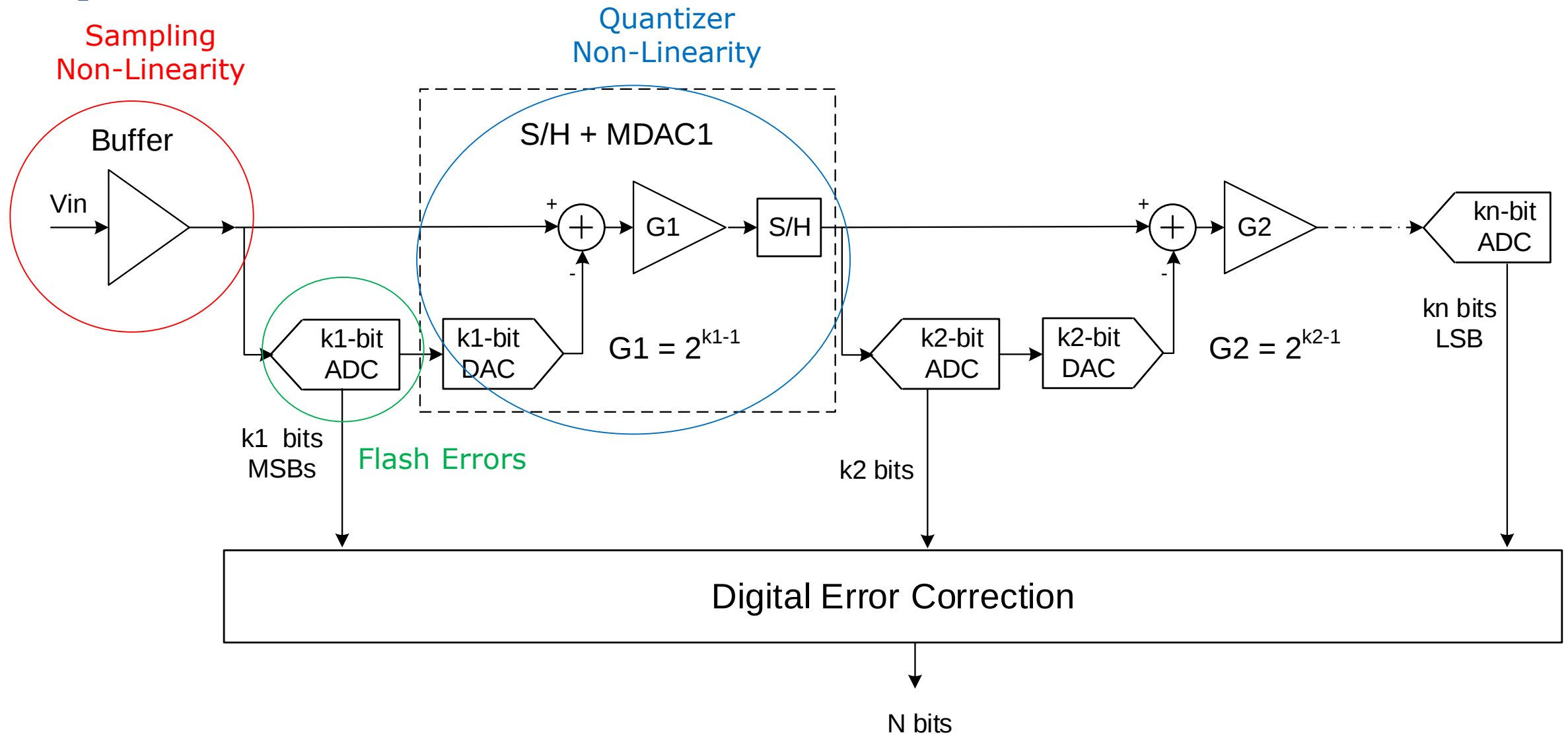
DAC Errors



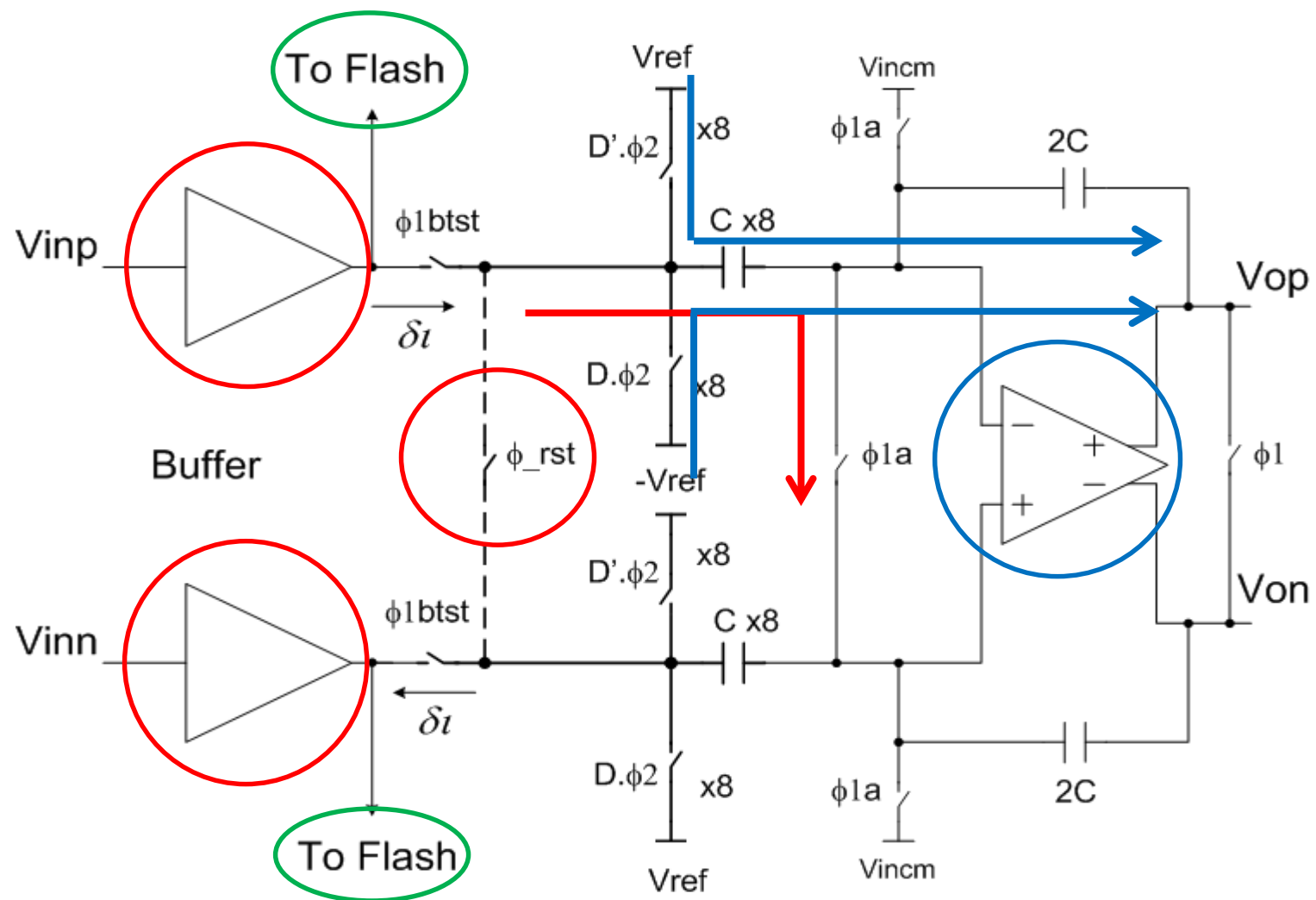
Calibration and Dither

- ◆ Mostly to improve linearity and hence SNDR and SFDR
- ◆ Calibration attempts to fix non-linearity
 - Improves both SNDR and SFDR
- ◆ Dither randomizes non-linearity
 - Spreads harmonics and spurs in the noise floor
 - Improves SFDR, but not SNDR
- ◆ Which one to use?
 - Calibration for large errors that are fixable
 - Dither for small or hard to fix errors
 - Preferably, use both:
 - ◆ Calibration to maximize SNDR
 - ◆ Dither to maximize SFDR

Pipelined ADC



Ahmed Ali



Switched Capacitor MDAC

$$V_o \approx \left[\frac{V_{in} \cdot C_t / C_f}{1 + (C_t + C_f + C_p) / (C_f \cdot A)} - \sum_{i=1}^8 \frac{D_i V_{ref} \cdot C_i / C_f}{1 + (C_t + C_f + C_p) / (C_f \cdot A)} \right] \left[1 - e^{-t 2\pi f_t C_f / (C_t + C_f + C_p)} \right]$$

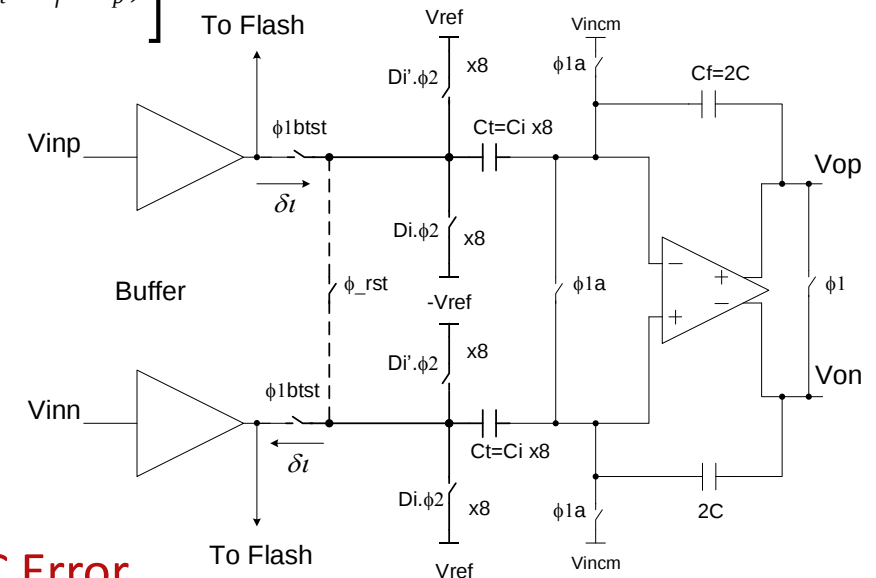
Capacitor Error Terms
Settling Error Term

$$V_o \approx \left[\frac{V_o |_{ideal}}{1 + (C_t + C_f + C_p) / (C_f \cdot A)} \right] \left[1 - e^{-t 2\pi f_t C_f / (C_t + C_f + C_p)} \right]$$

$$V_o \approx \frac{V_o |_{ideal}}{1 + K / A} \left[1 - e^{-t 2\pi f_t / K} \right]$$

$$V_o \approx V_o |_{ideal} - \underbrace{V_o |_{ideal} \cdot e^{-2\pi BW \cdot t}}_{\text{Amplifier Settling Error}} - \underbrace{V_o K / A}_{\text{Amplifier DC Error}}$$

Amplifier Settling Error Amplifier DC Error



DAC Errors

- ◆ **Caused by:**
 - Capacitor mismatch
 - Settling errors: switches can be very fast
 - Input dependent reference errors: can be minimized
- ◆ **Fixed (factory) calibration is usually adequate**
- ◆ **Dynamic element matching (shuffling) can fix any residual errors**
- ◆ **Background calibration can also be employed**
- ◆ **Not a major bottleneck**

Amplifier Gain, Settling and Non-Linearity

- ◆ Realizing a high-gain amplifier at high speeds is a major challenge:
 - For a 14bit, 1GS/s ADC: 100dB with 6GHz BW
- ◆ The amplifier open-loop gain error depends on supply, temperature, and sometimes sample rate (settling errors).
 - => Factory calibration may not be an attractive option
- ◆ The amplifier open-loop gain error can also be input-dependent => Non-linear
- ◆ The amplifier is a major bottle-neck

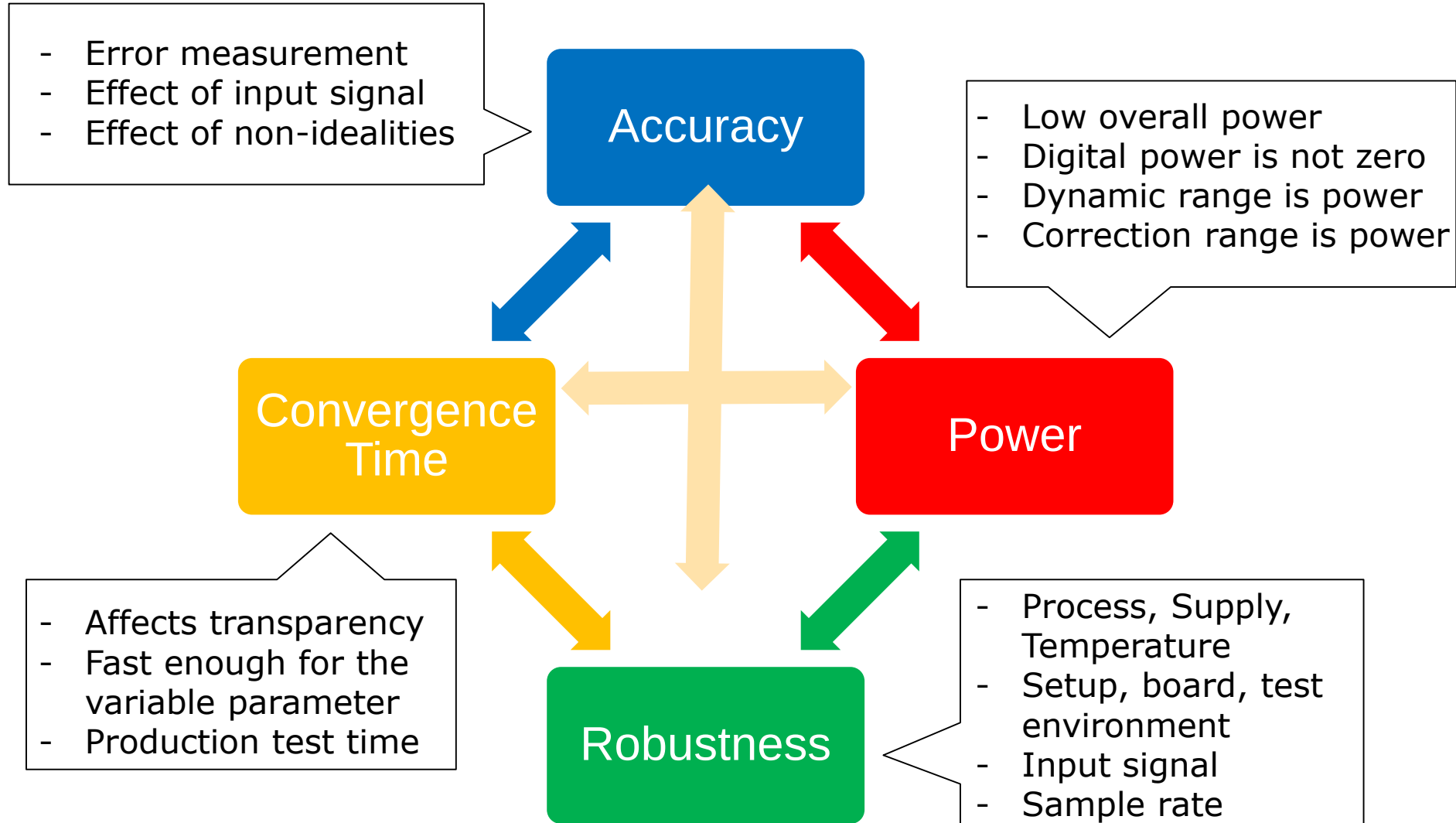
Some Calibration Techniques

- ◆ Correlation-based calibration [15, 2, 11, 12]
- ◆ Summing node calibration [3, 13]
- ◆ Split ADC method [18]
- ◆ Using reference ADC [16, 17]
- ◆ Using statistical methods [19, 20, 21]

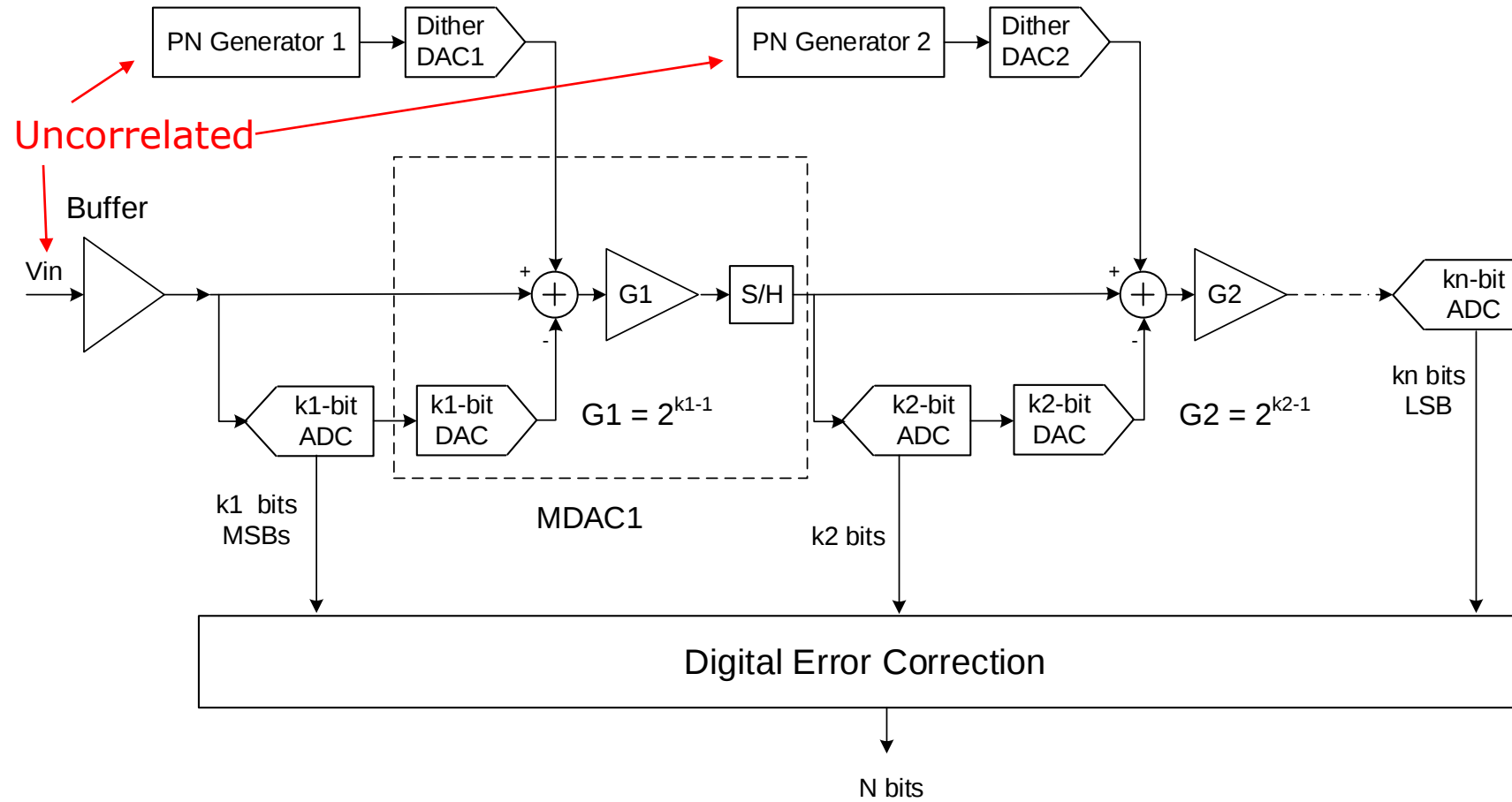
Real World Challenges

- ◆ More than publishing a paper ...
- ◆ Improve performance and lower power consumption
- ◆ No disruption to normal operation => background
- ◆ Robustness with input amplitude, input frequency, sample rate, supply, temperature, etc.
- ◆ Convergence time
- ◆ Manufacturability:
 - Reasonable test time
 - Tester to bench correlation

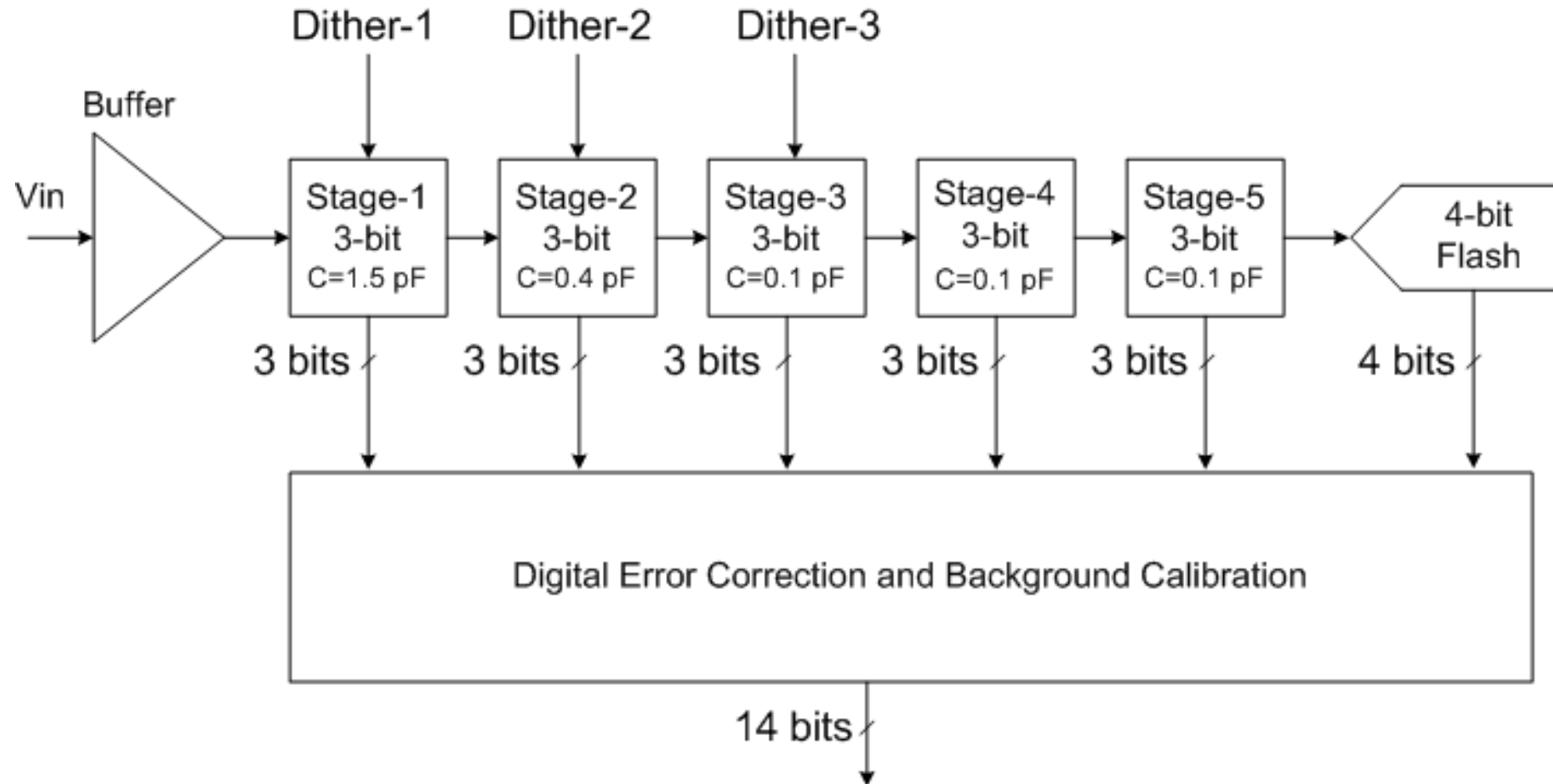
Some Design Trade-offs



Correlation-Based Calibration



Pipelined ADC Architecture [2]



IGE and Settling Calibration

- ◆ **IGE calibration can fix gain errors and linear settling errors**
- ◆ **Correction of settling errors can be limited by:**
 - Non-linearity due to slewing, reference settling, DAC settling
 - Memory errors
 - Kick-back from the following stage
- ◆ **The design needs to be optimized to take advantage of IGE calibration**
- ◆ **Memory and kick-back calibration helps improve settling error correction**

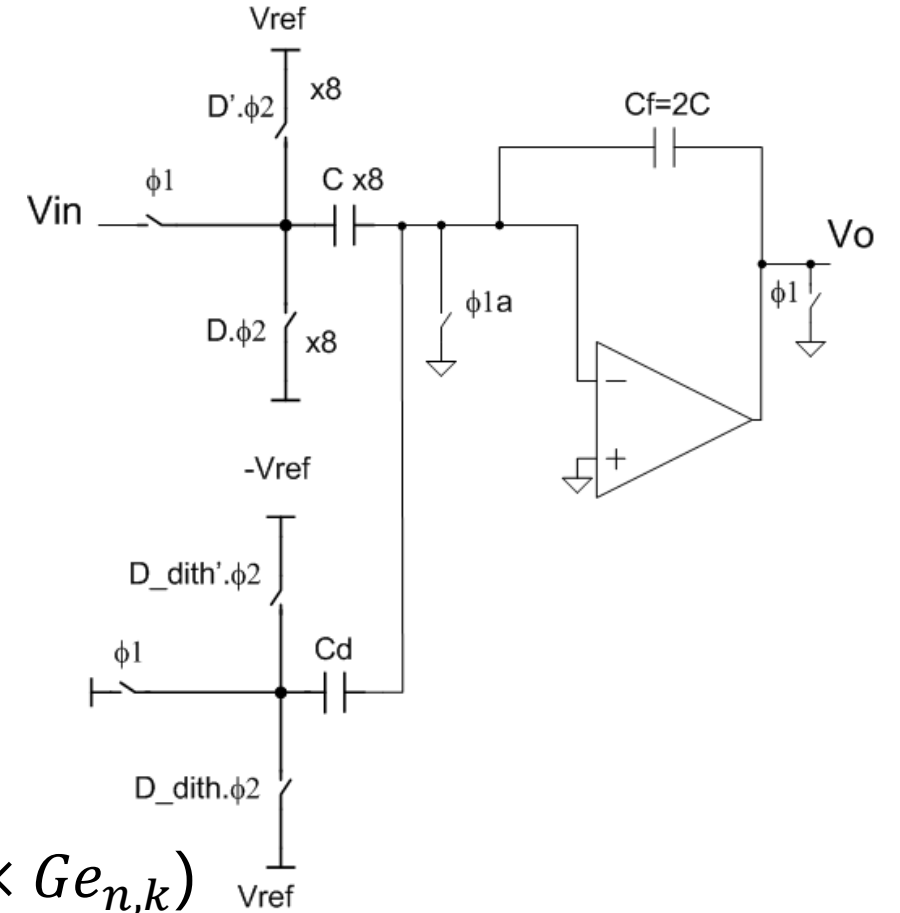
The IGE/IME Calibration [1, 2, 3, 9, 10]

- Correction:

$$V_{R_cal} = \sum_{k=0}^M V_R[n-k] \times 1/Ge_{n+1,k}$$

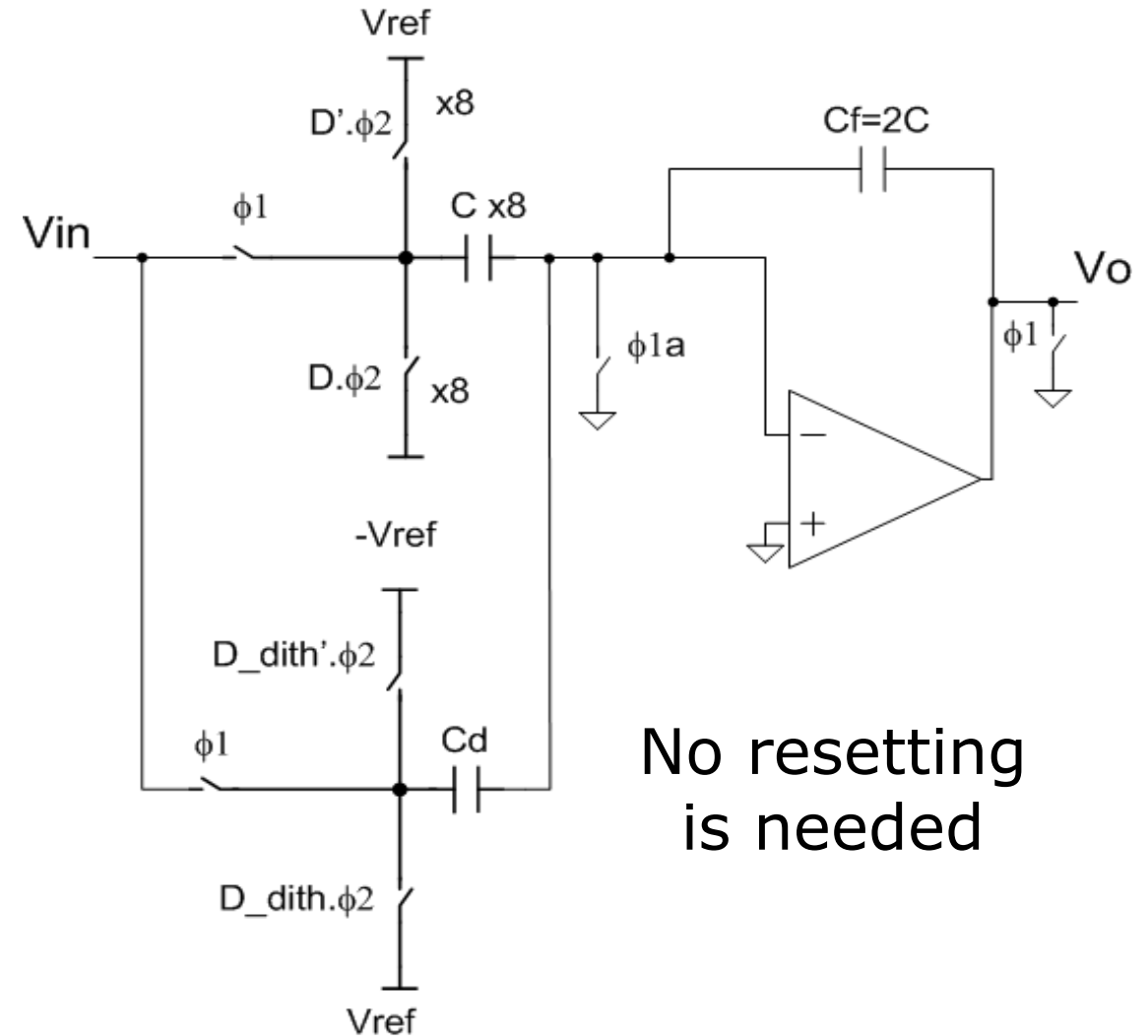
- Estimation:

$$Ge_{n+1,k} = Ge_{n,k} + \mu \cdot V_d[n-k] \cdot (V_R[n] - V_d[n-k] \times Ge_{n,k})$$



Kick-back Background Calibration [1, 2]

- ◆ The kick-back dither caps kick the input similar to the sampling capacitances' kick
- ◆ The dither's kick is sampled on the total capacitances and propagates down the pipeline



No resetting
is needed

Kick-back Background Calibration [1, 2]

- ◆ The LMS algorithm is used to estimate the gain/correction coefficient G_{kb} of the dither's kick of the previous sample(s)
- ◆ The correction coefficient G_{kb} is applied to the previous stage-1 flash bits

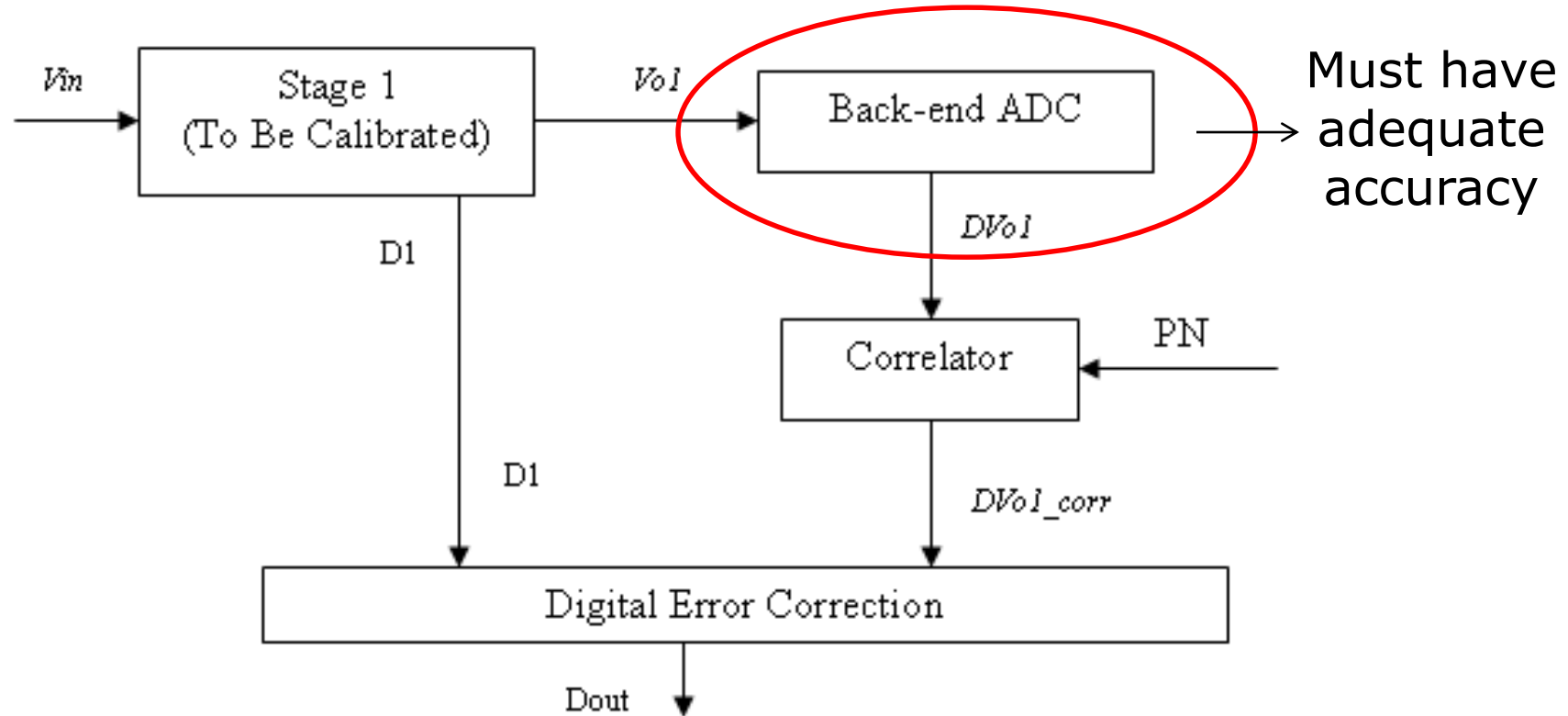
$$G_{kb_{n+1,k}} = G_{kb_{n,k}} - \mu * V_d[n-k] * (V_d[n-k] * G_{kb_{n,k}} - V_{in}[n])$$

$$V_{out_kbc} [n] = V_o[n] + \sum_{i=1}^{M_{kb}} D_1[n-i] \times G_{KB,i}$$

Challenges

- ◆ What is the **accuracy** of this approach?
- ◆ How is the **power** consumption?
- ◆ Is it **robust**?
- ◆ How about the **convergence time**?

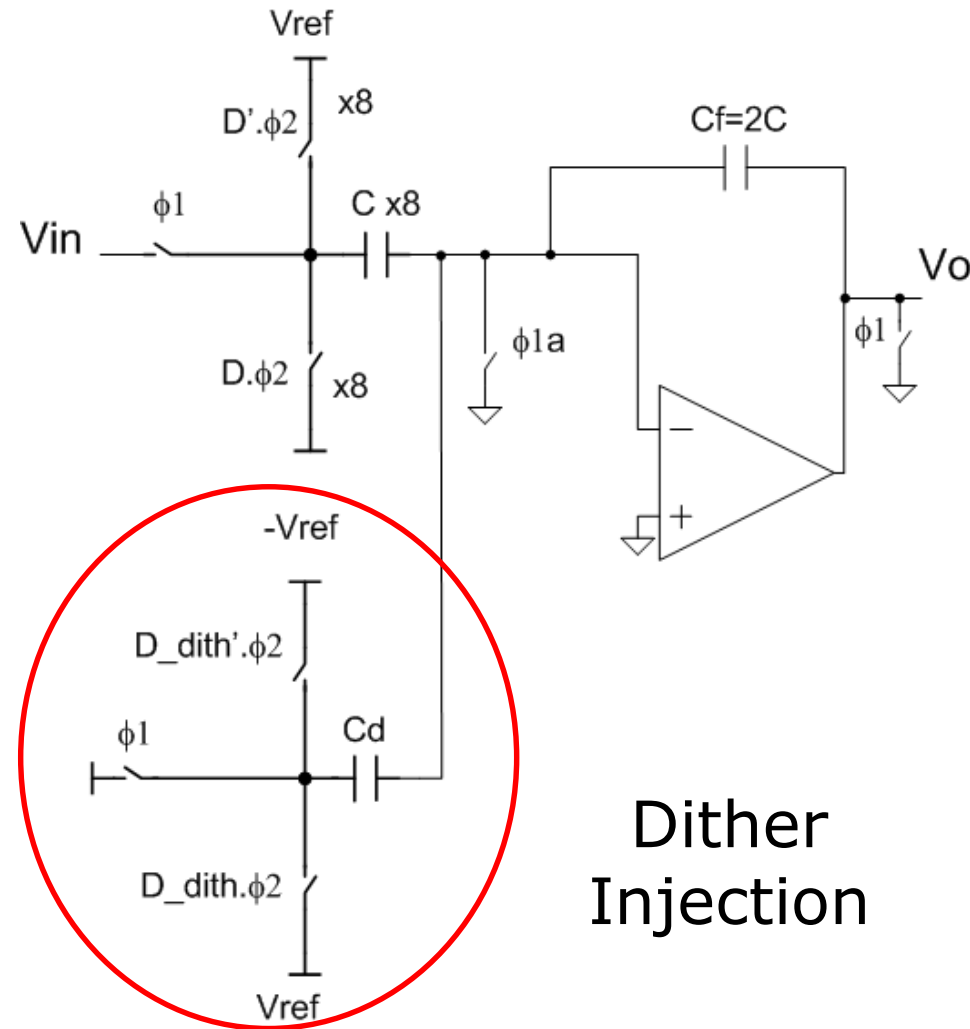
Correlation-Based Calibration



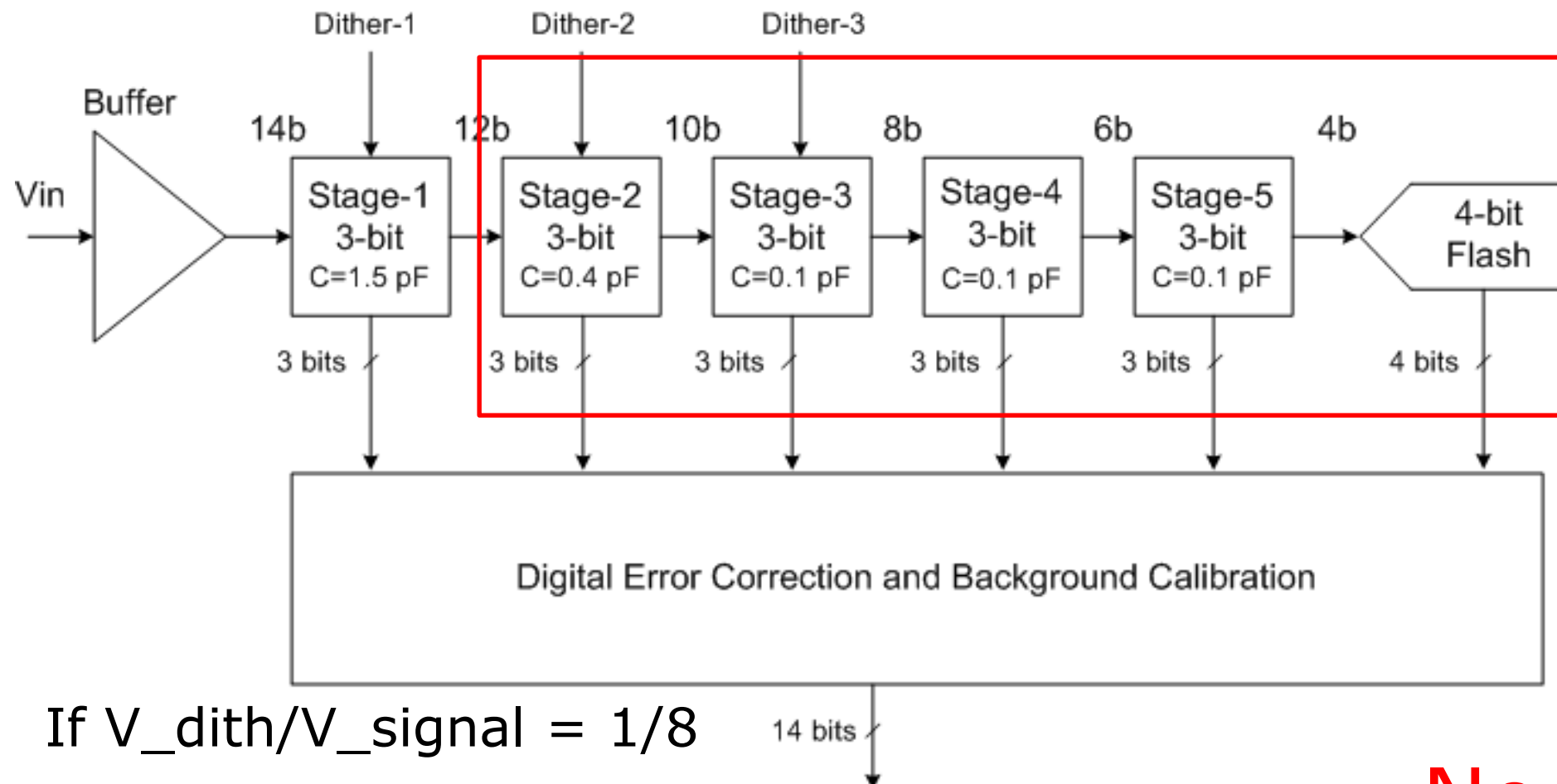
$$\text{LMS Algorithm: } Ge_{n+1} = Ge_n + \mu \cdot V_d[n] \cdot (V_{o1}[n] - V_d[n] \times Ge_n)$$

Correlation-Based Calibration

- ◆ A portion of the correction range is consumed by the calibration random sequence
- ◆ This can cause a significant power overhead
- ◆ It is desirable to minimize the amplitude of the calibration signal (dither): V_{dith}



But ...

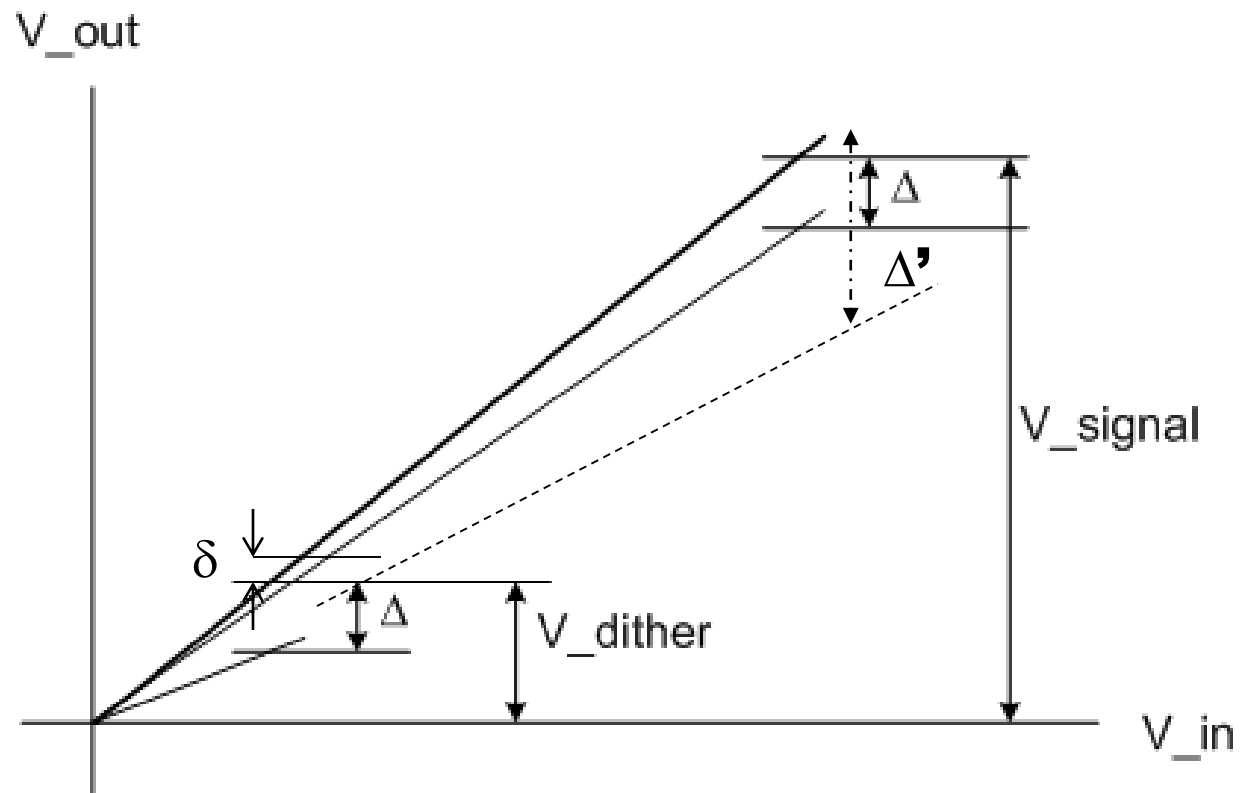


If $V_{dith}/V_{signal} = 1/8$

Is this backend accuracy adequate for the calibration signal? **No**

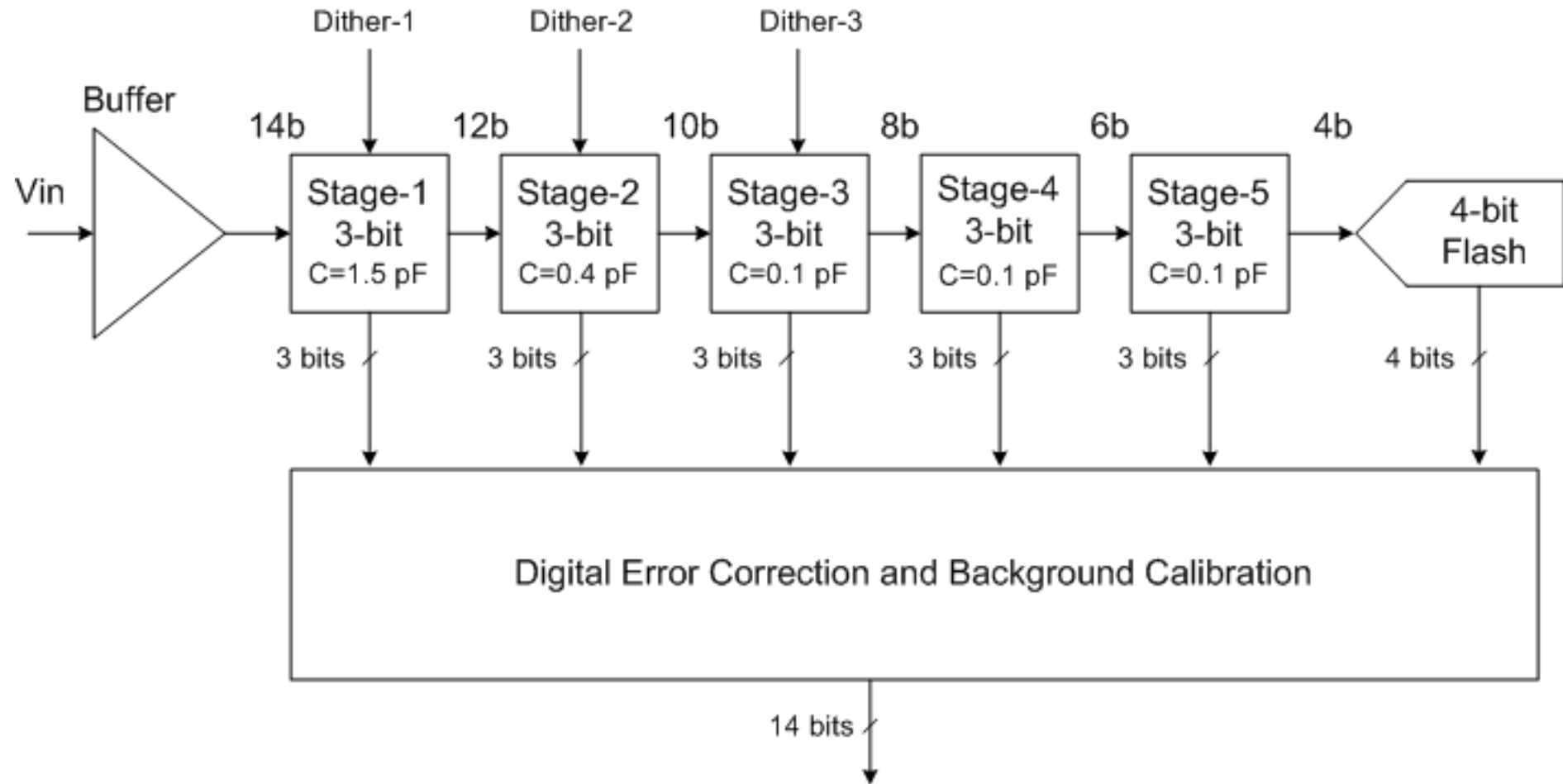
Effect of Using a Small Dither

- The calibration accuracy is degraded by the ratio of:
 $V_{\text{dither}}/V_{\text{signal}}$
- Degrades accuracy when using small dither for calibration
- We need to improve the linearity of the back-end by at least that ratio

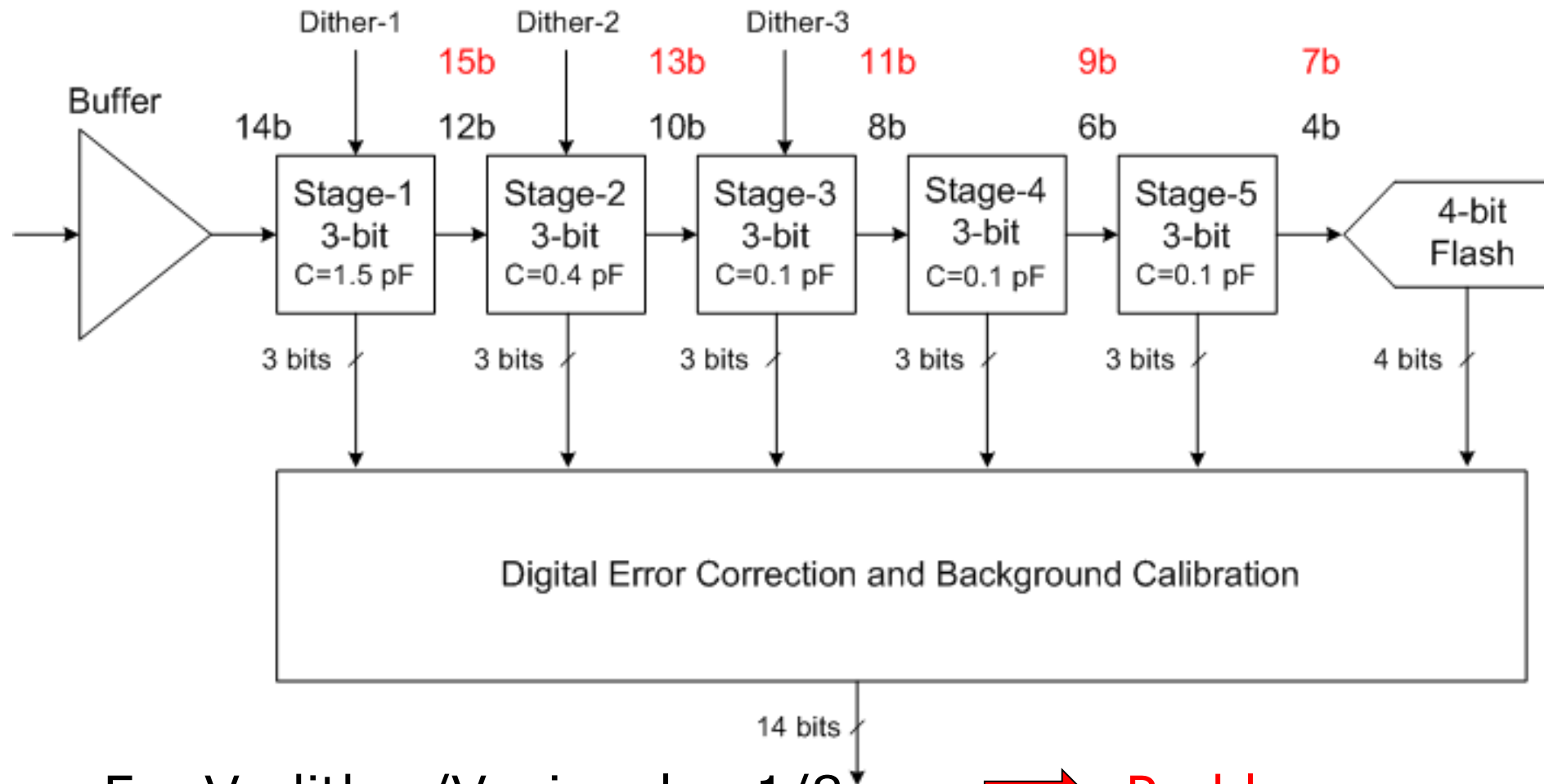


$$\delta/\Delta = \Delta/\Delta' = V_{\text{dither}}/V_{\text{signal}}$$

Accuracy of Pipeline Stages



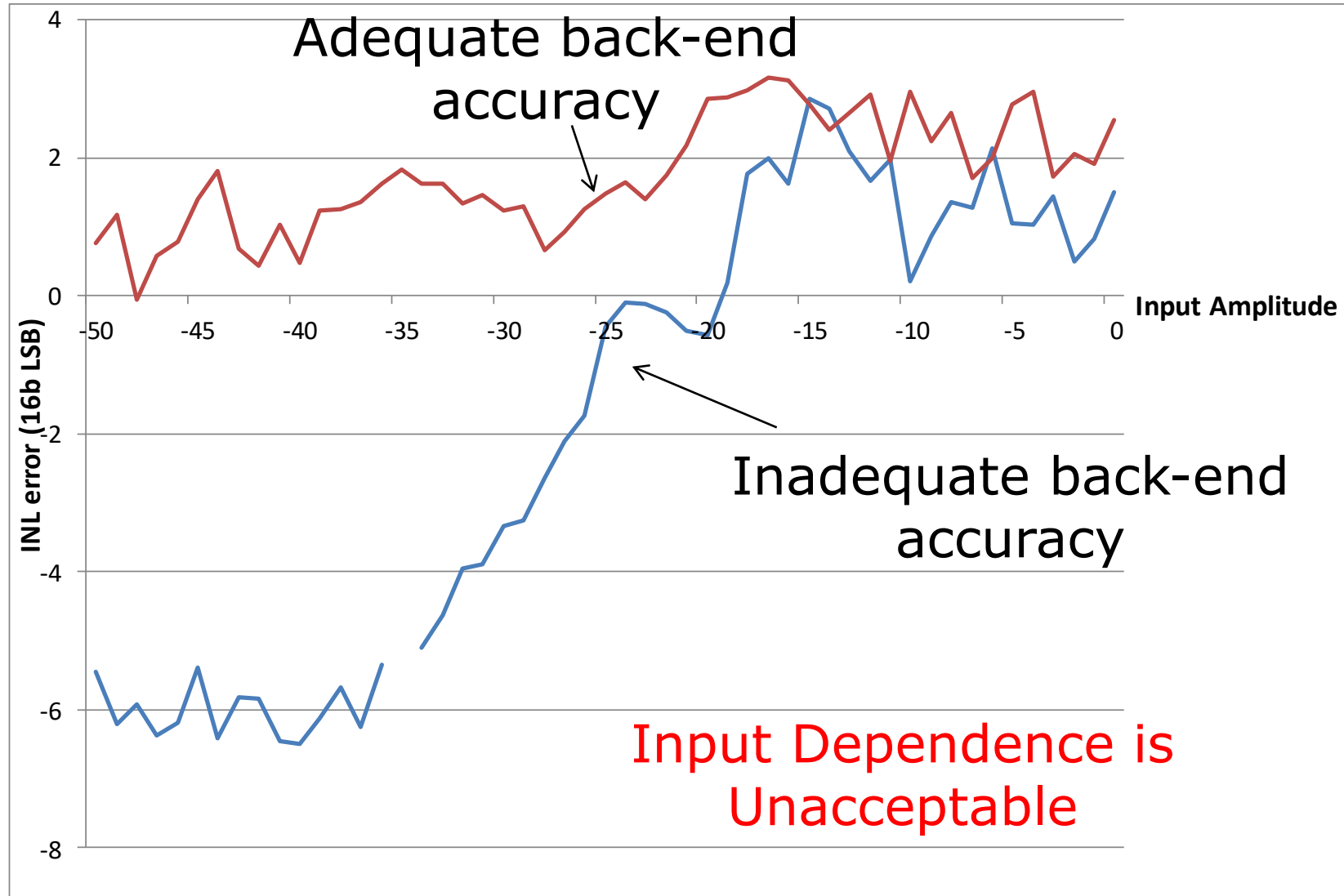
Accuracy of Pipeline Stages



For $V_{\text{dither}}/V_{\text{signal}} = 1/8$

➡ Problem

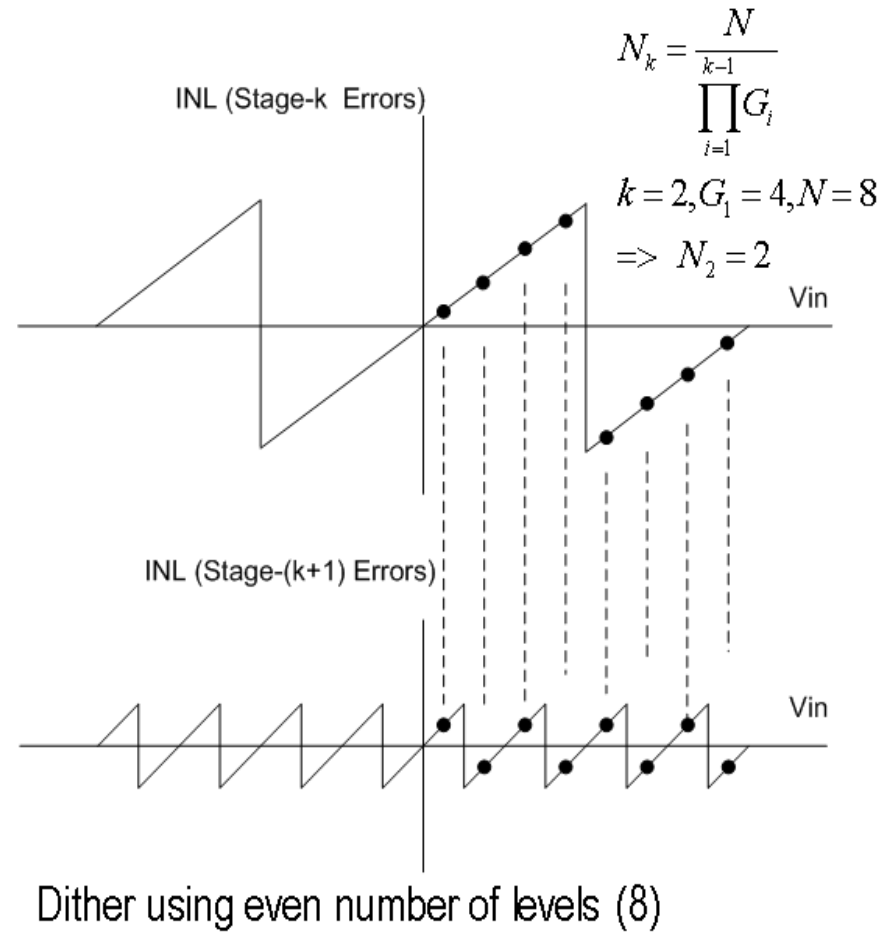
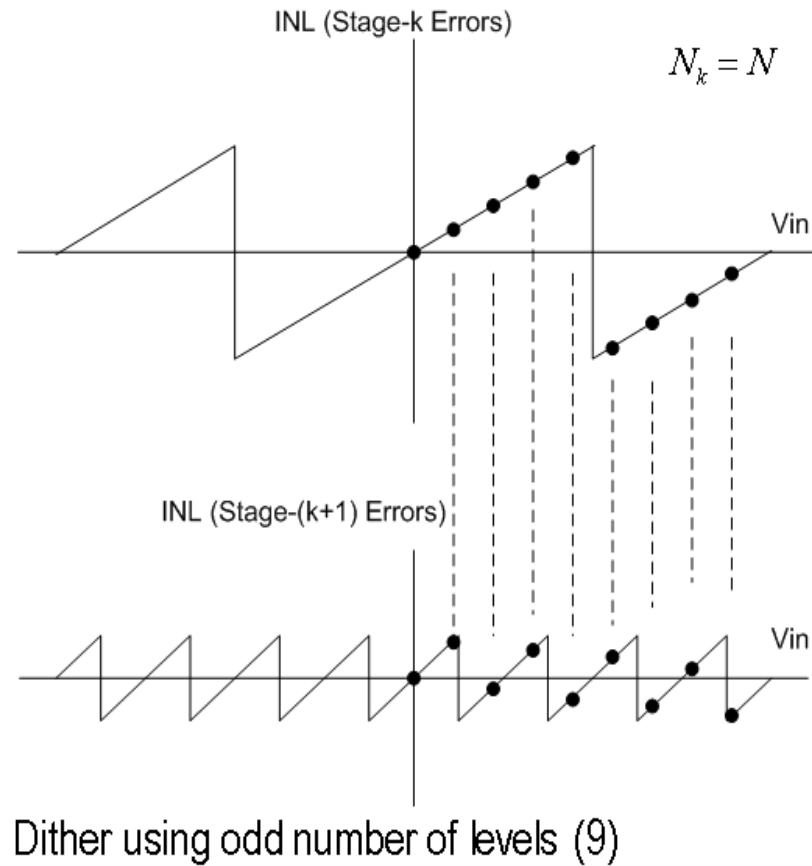
Example of the Problem



Effective Solution

- ◆ Inject a small dither signal in the MDAC
- ◆ Since the dither is small, we only use a small part of the correction range => small power overhead
- ◆ Use multi-level dither (odd number) in every stage to improve accuracy of the back-end (9 levels => 3.2-bits)
- ◆ Dynamic element matching can be utilized
- ◆ Ensure effective dither propagation down the pipeline such that the number of dither levels for each stage is adequate for the required accuracy
- ◆ In addition, large dither injection can be employed

Dither Propagation

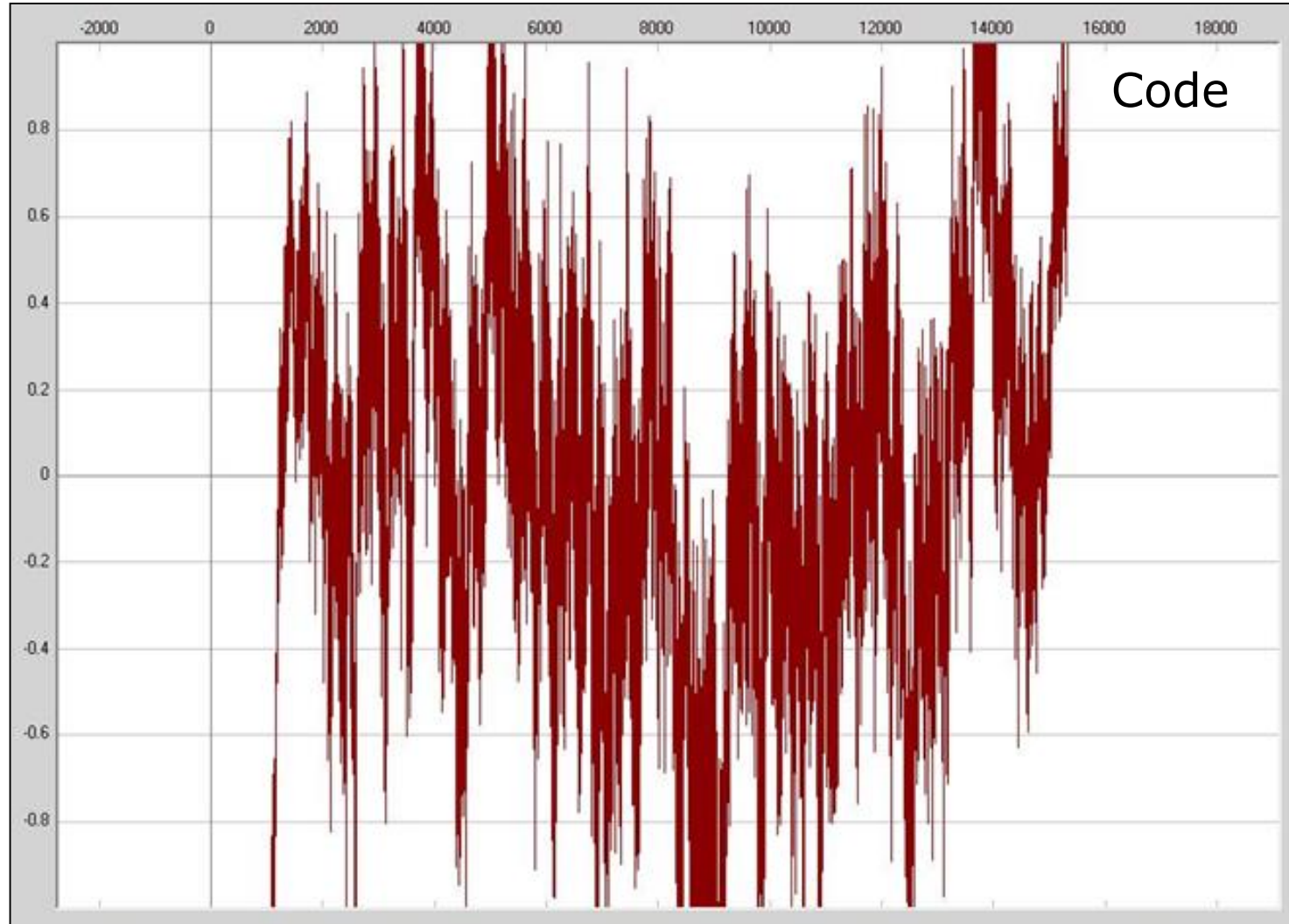


INL at Room Temperature

1 LSB

LSB

-1 LSB

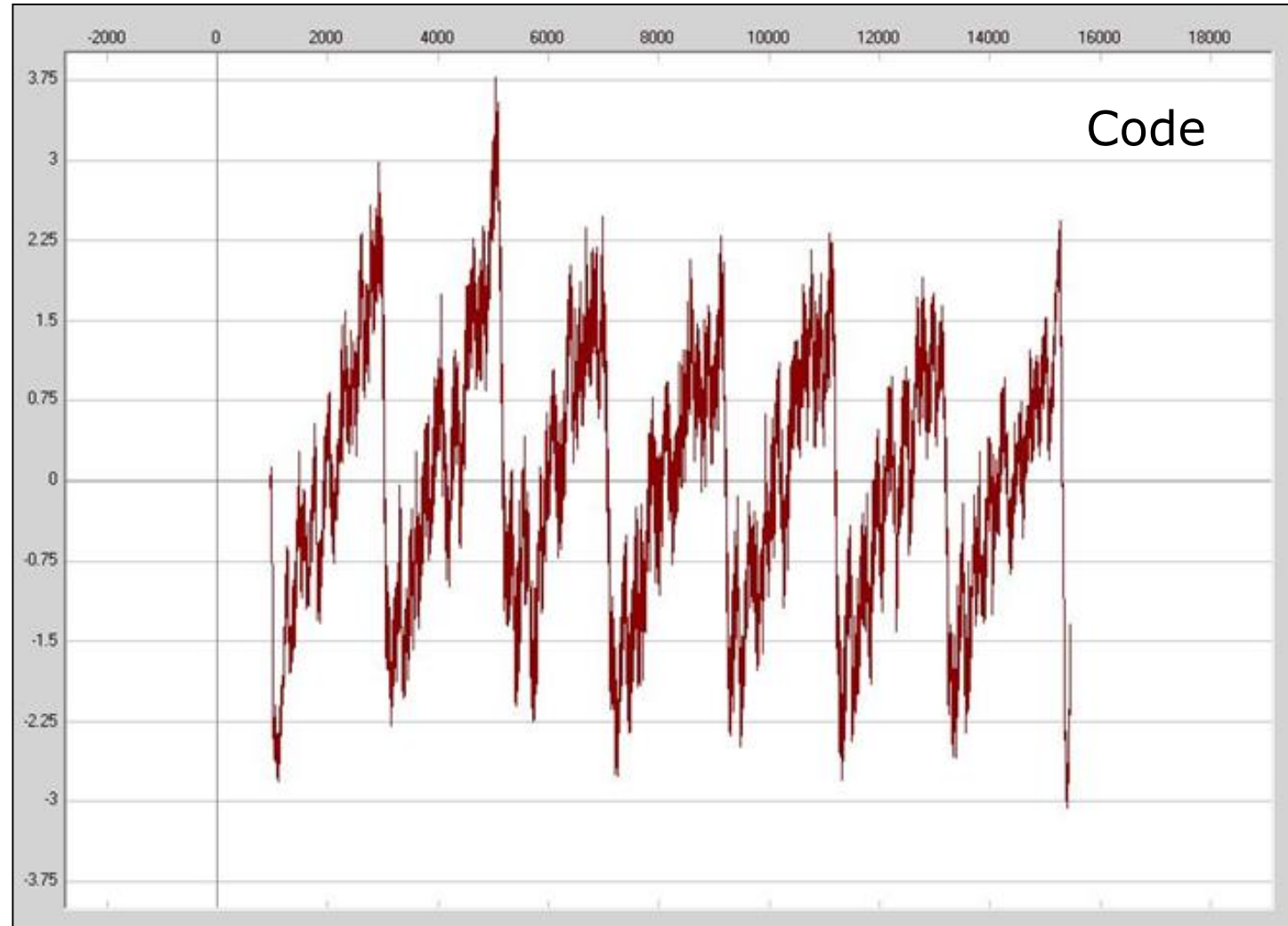


INL at High Temperature Calibration OFF

4 LSB

LSB

-4 LSB

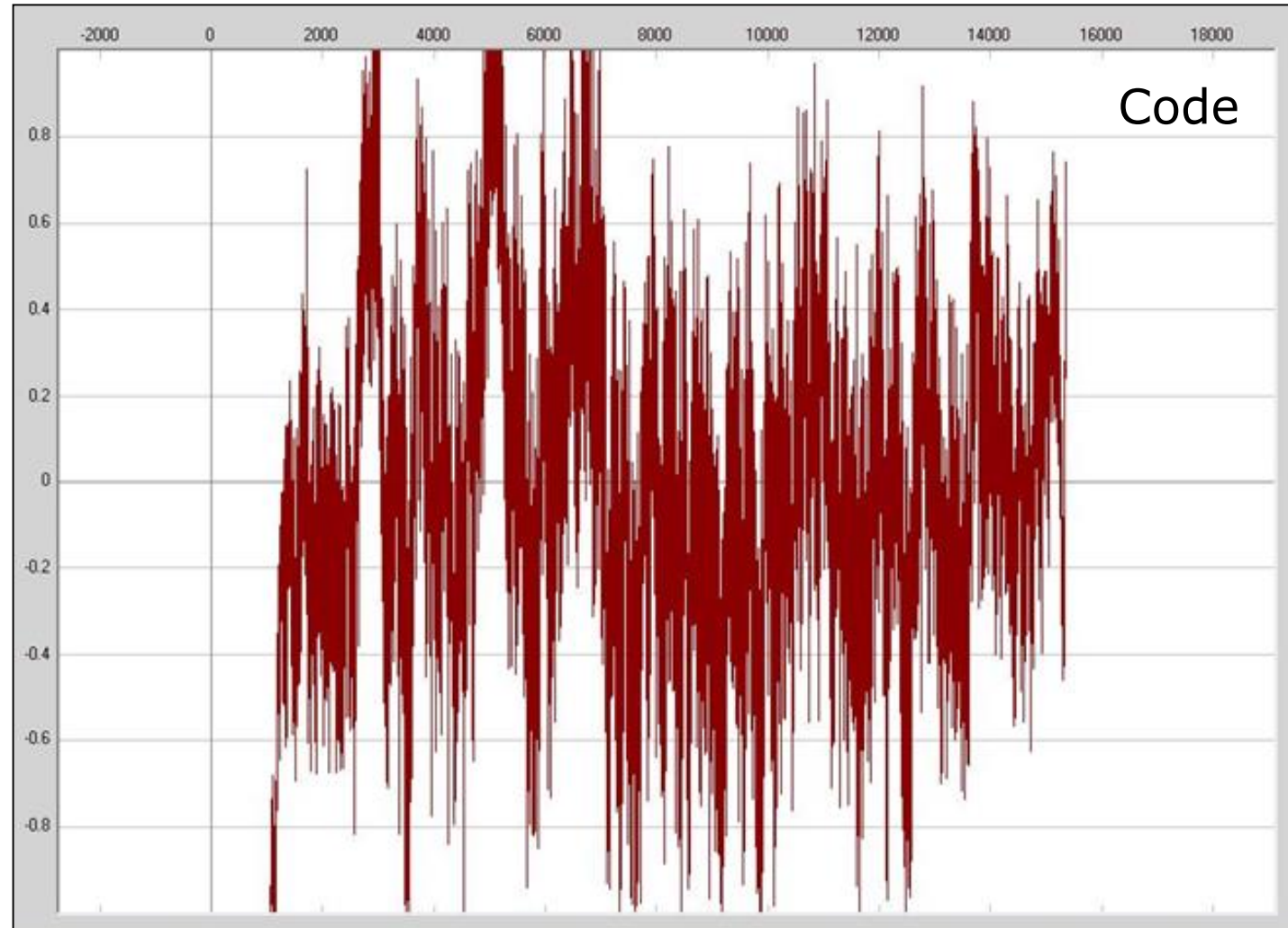


INL at High Temperature Calibration ON

1 LSB

LSB

-1 LSB



INL without Kick-back Calibration

0

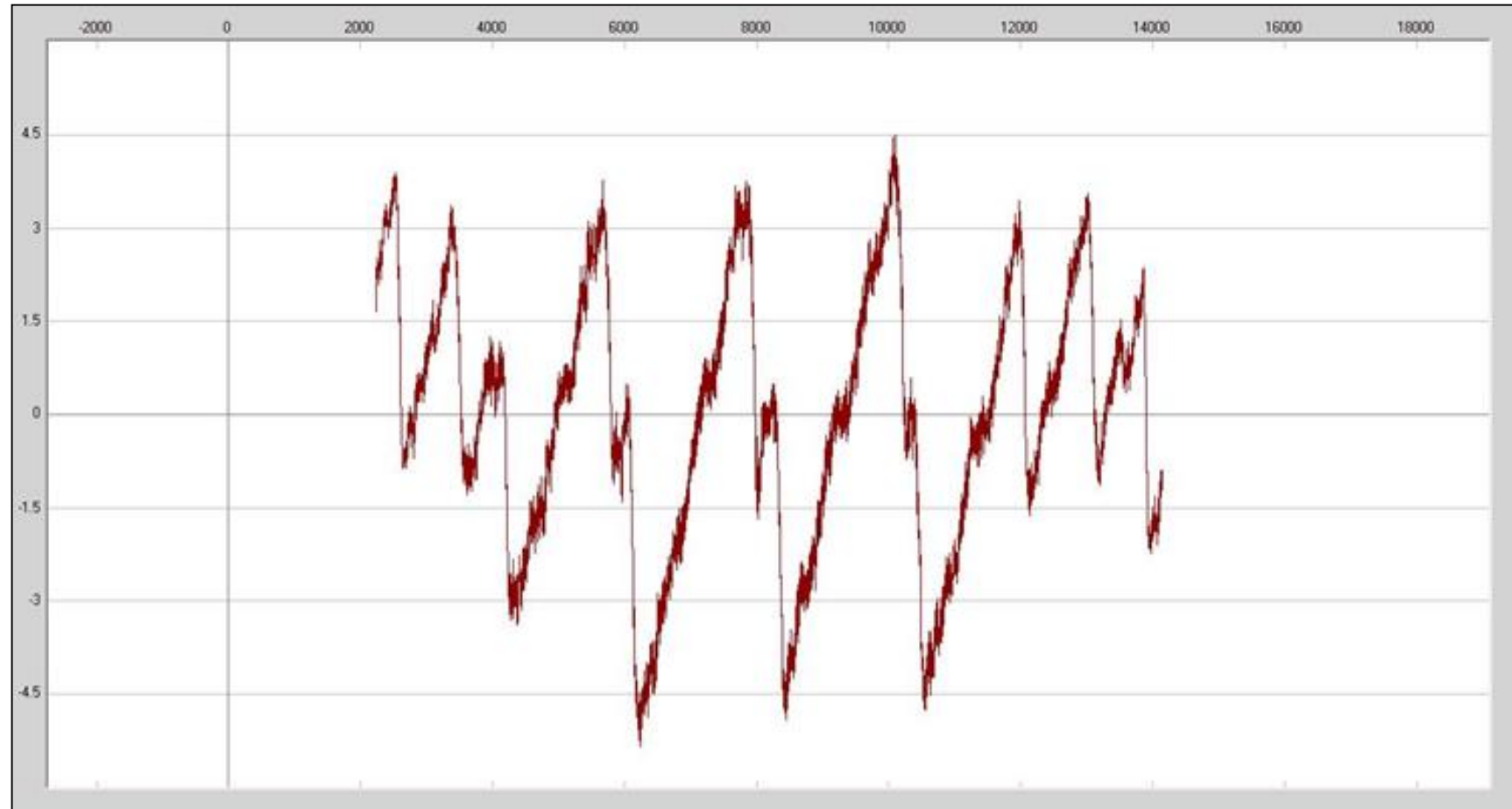
8000

16000 Codes

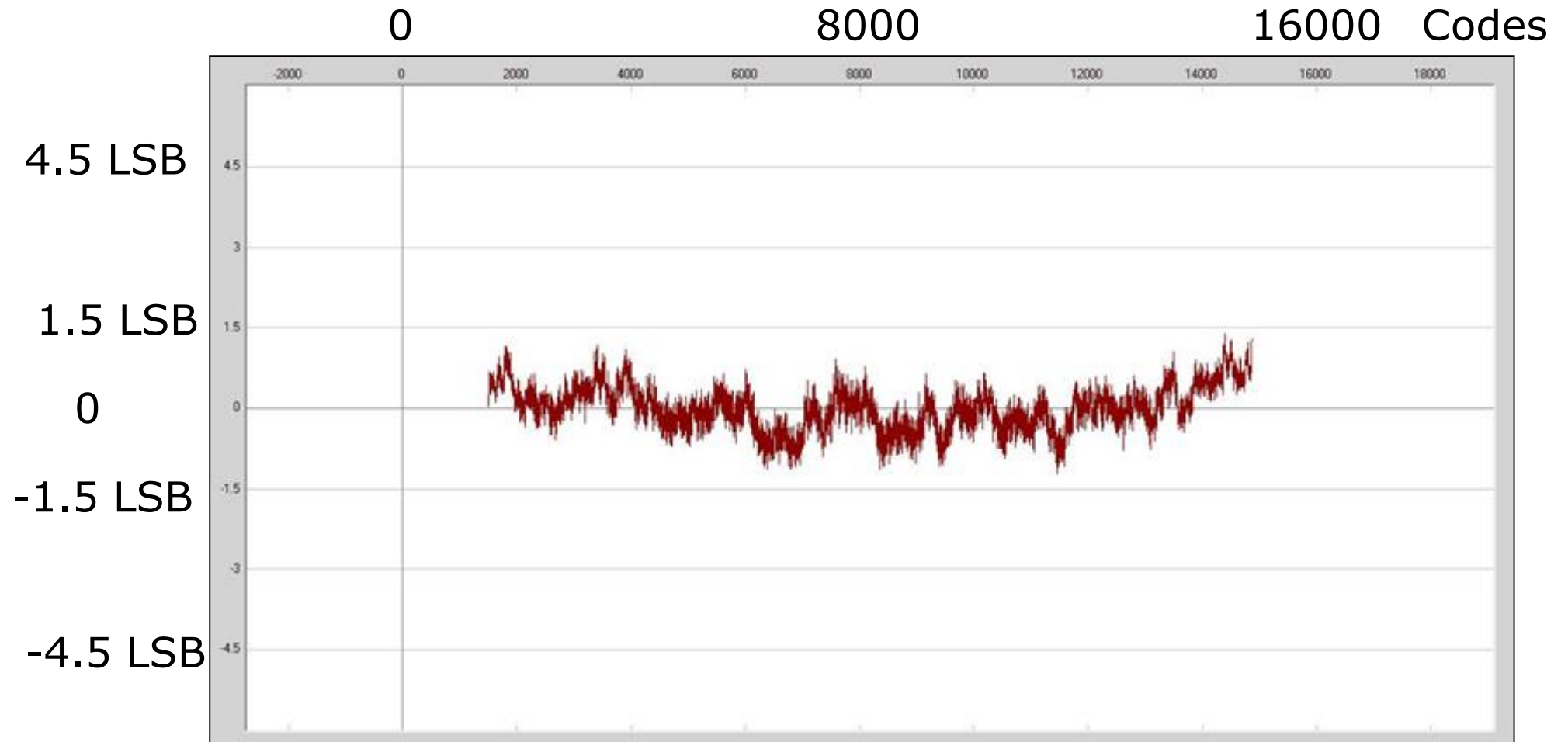
4.5 LSB

0 LSB

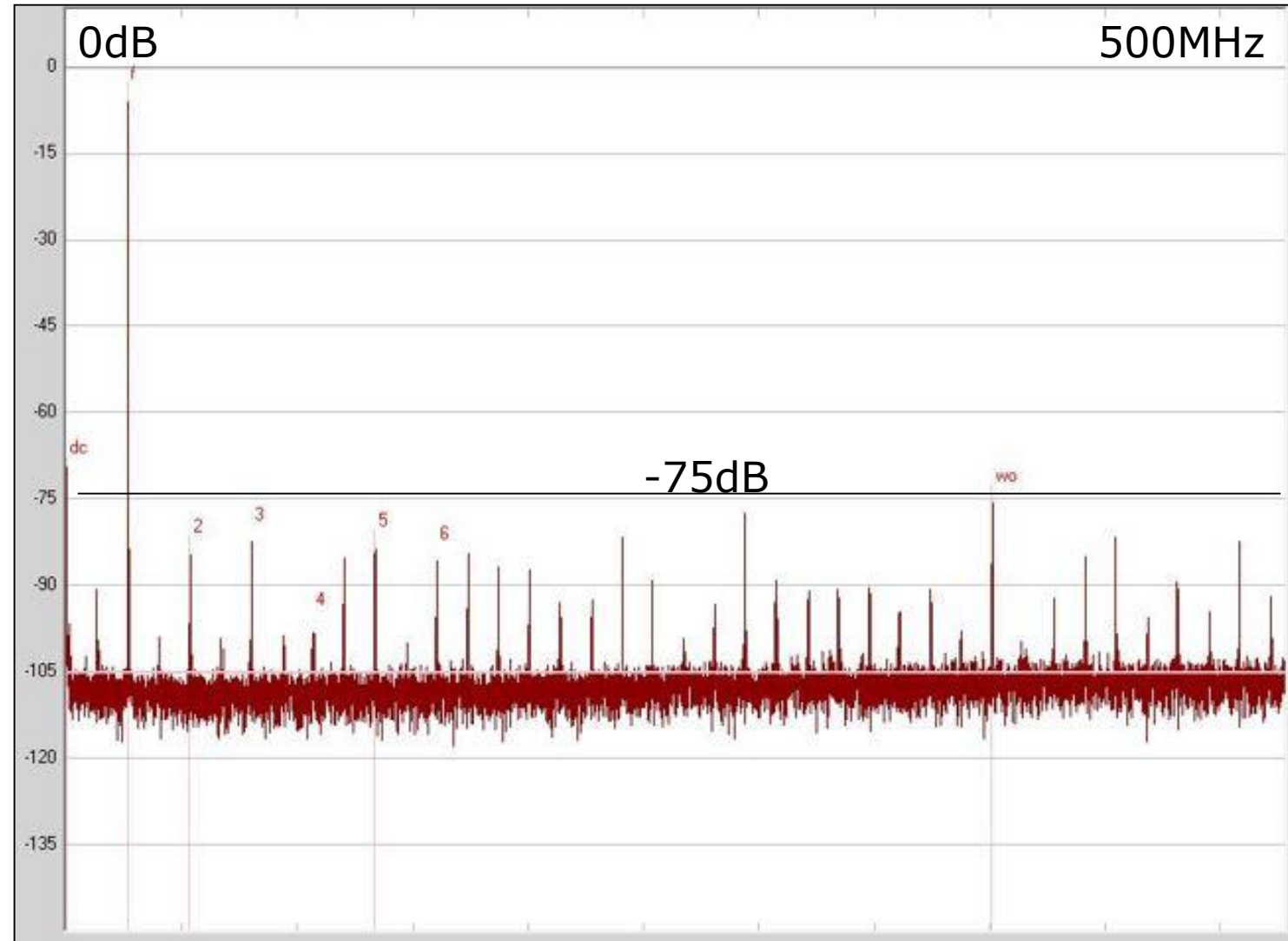
-4.5 LSB



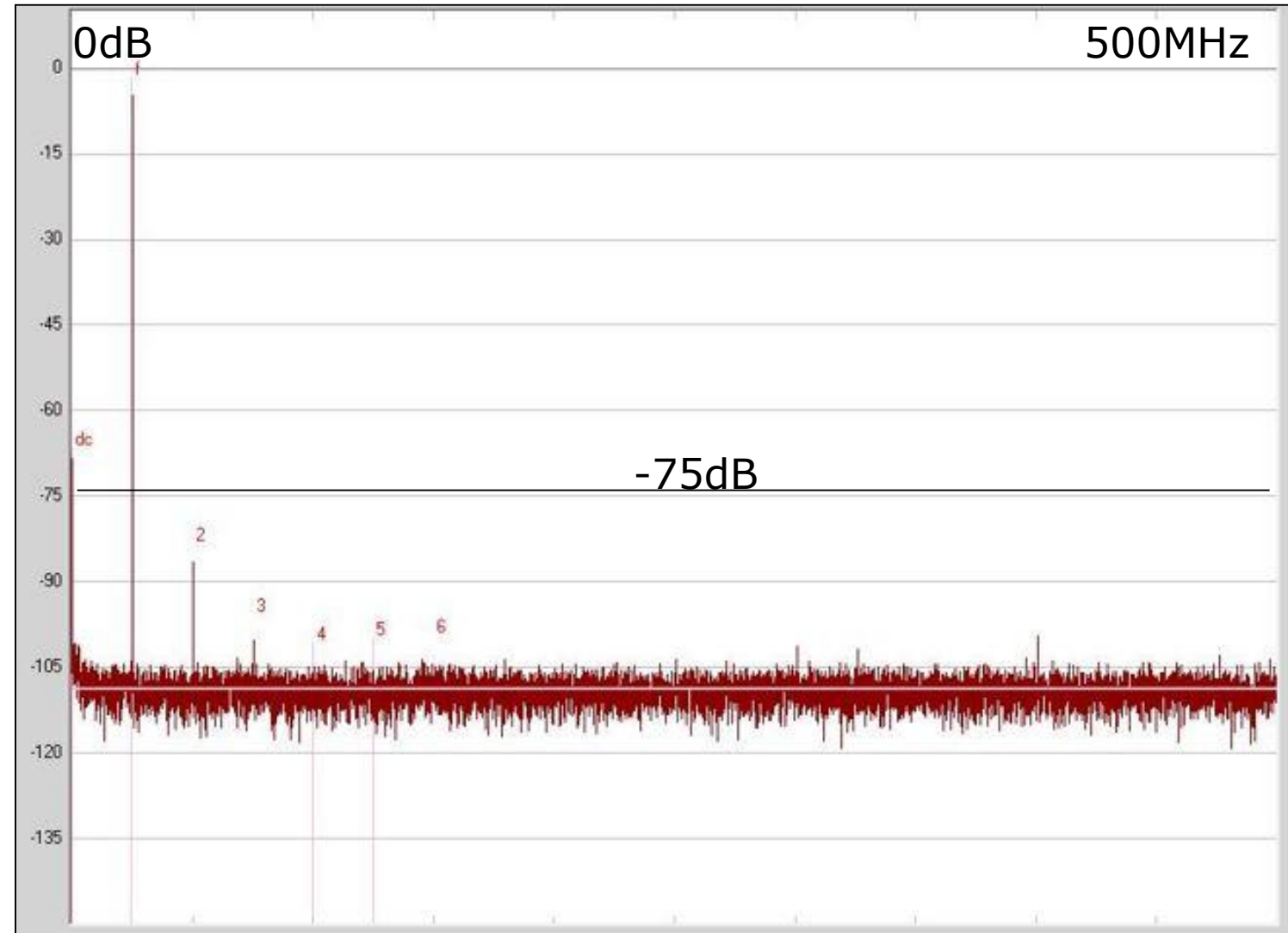
INL with Kick-back Calibration



FFT without kick-back calibration



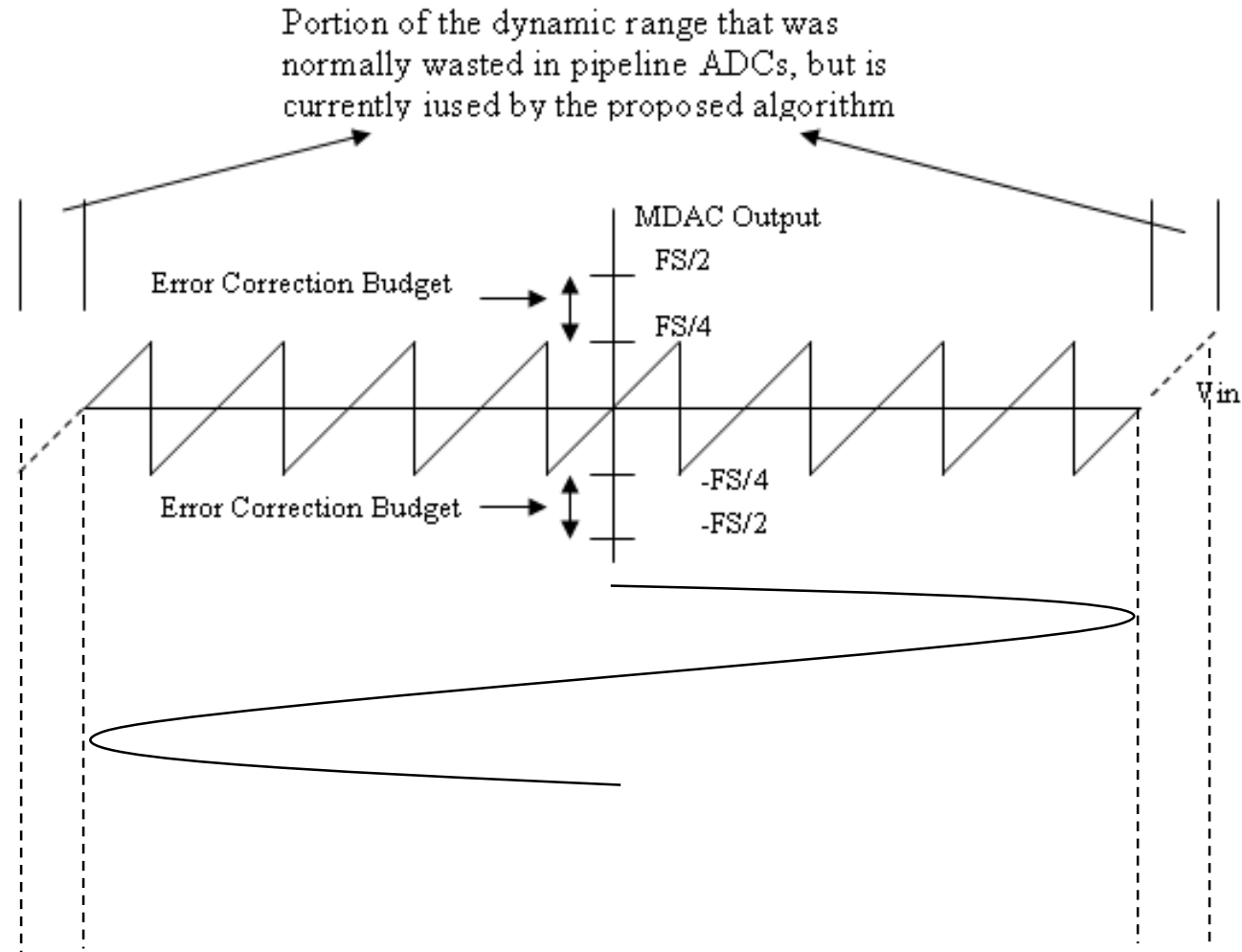
FFT with kick-back calibration



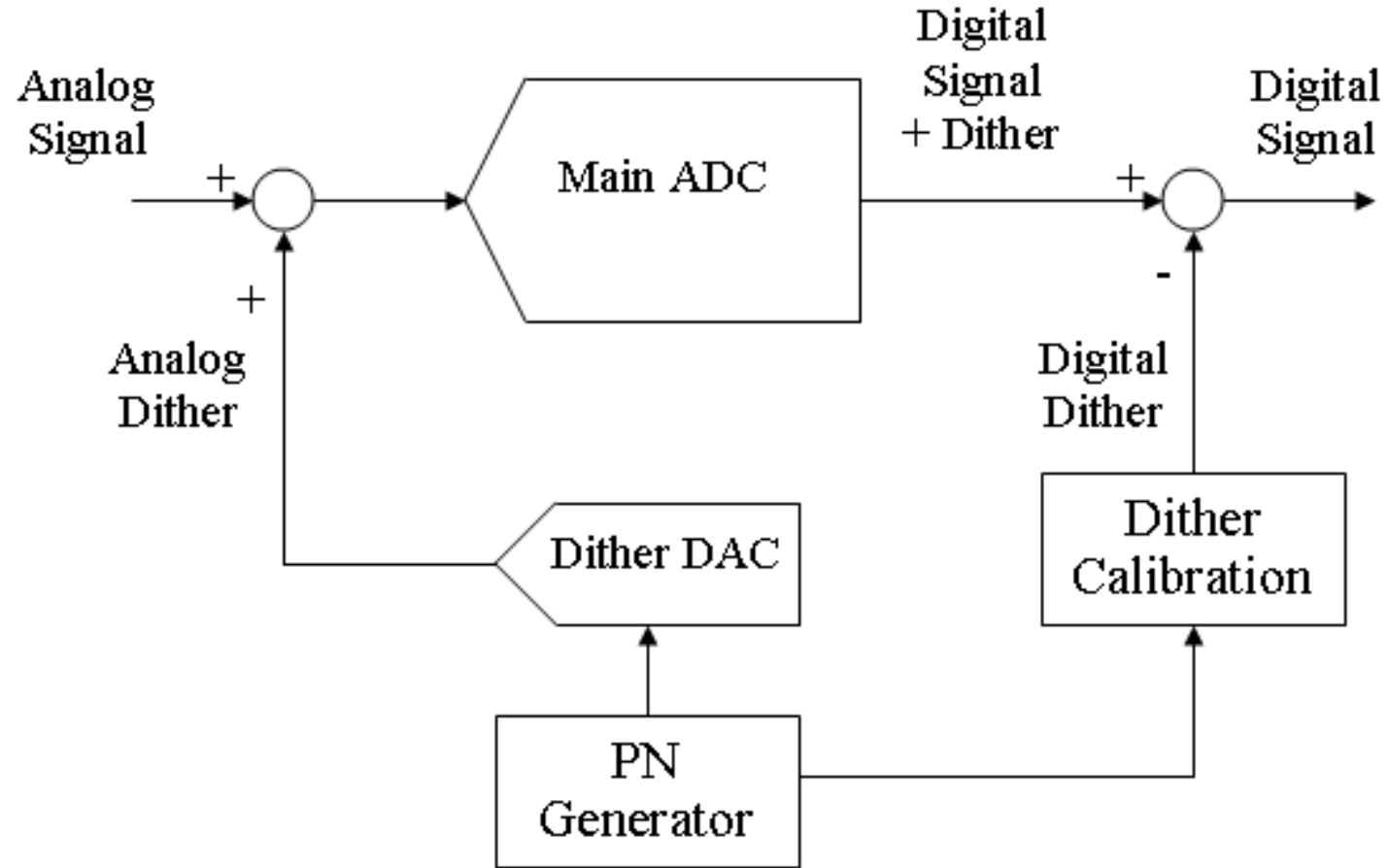
Is this dithering adequate?

- ◆ So far, multi-level dithering was used to:
 - Improve calibration accuracy
 - Dither the backend pipeline effectively
- ◆ How about dithering the front-end stages?
- ◆ Subtractive dither with large amplitude can be useful for dithering the front-end stages
- ◆ This “large” dither should not consume dynamic range

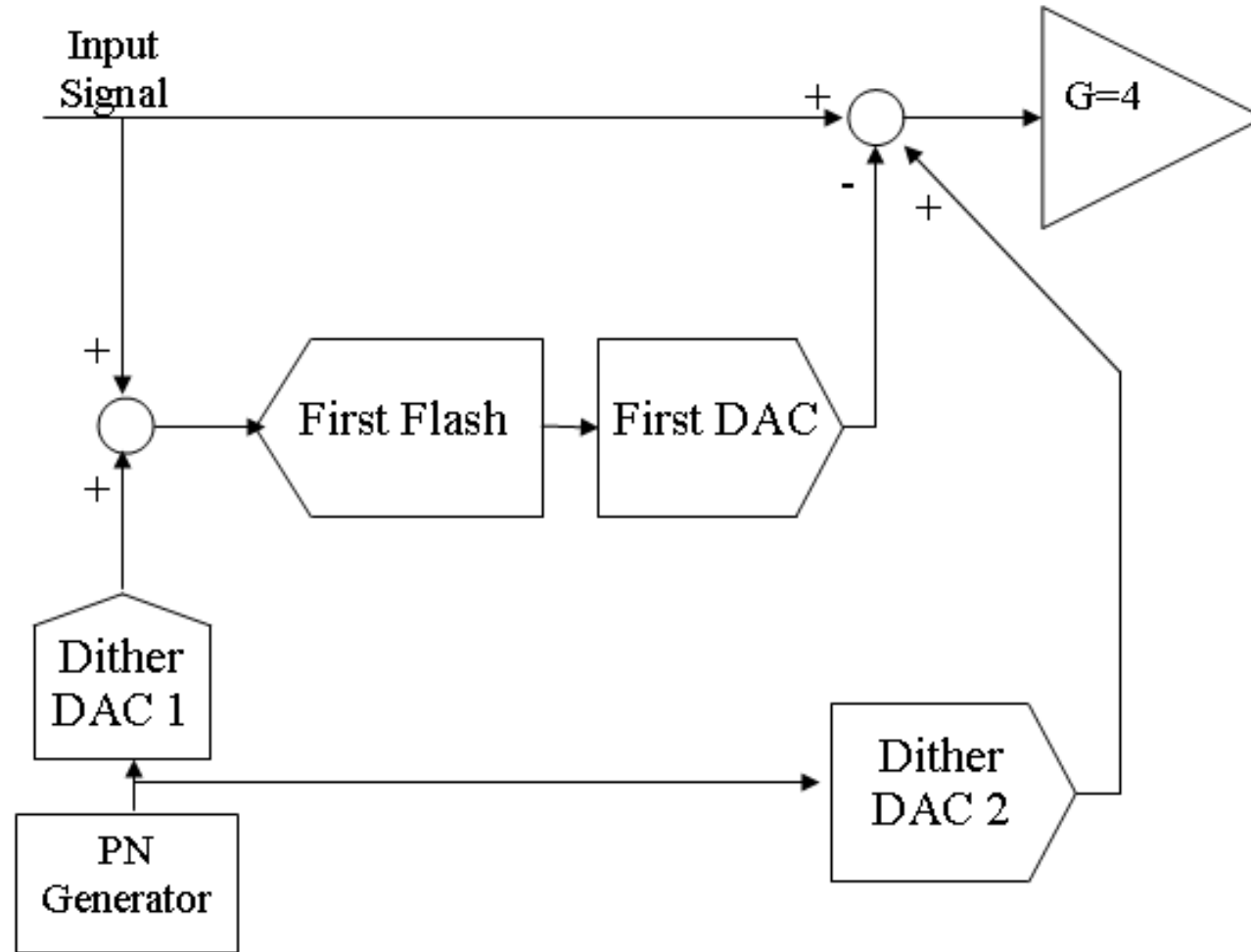
Large Dither Injection



Large Dither Injection and Subtraction

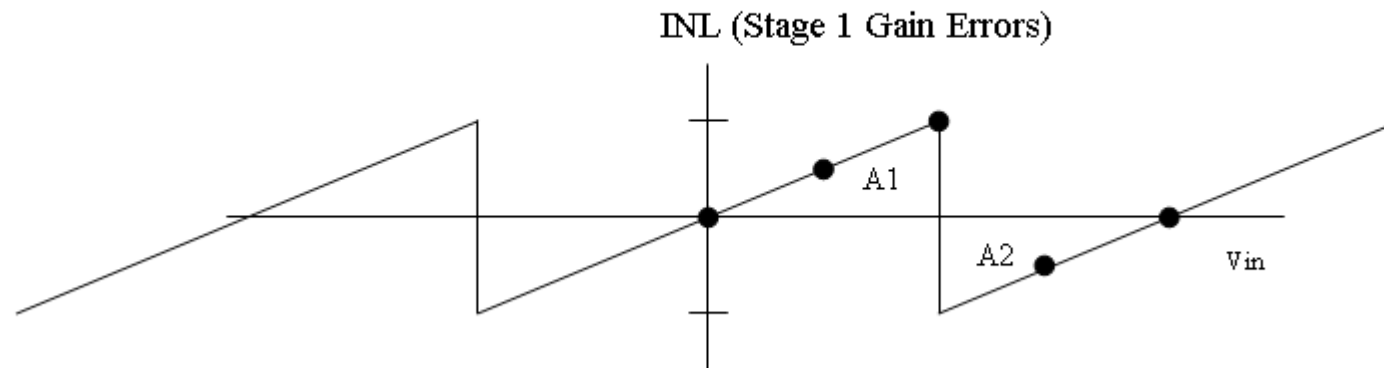


Dither Injection for a SHA-less ADC



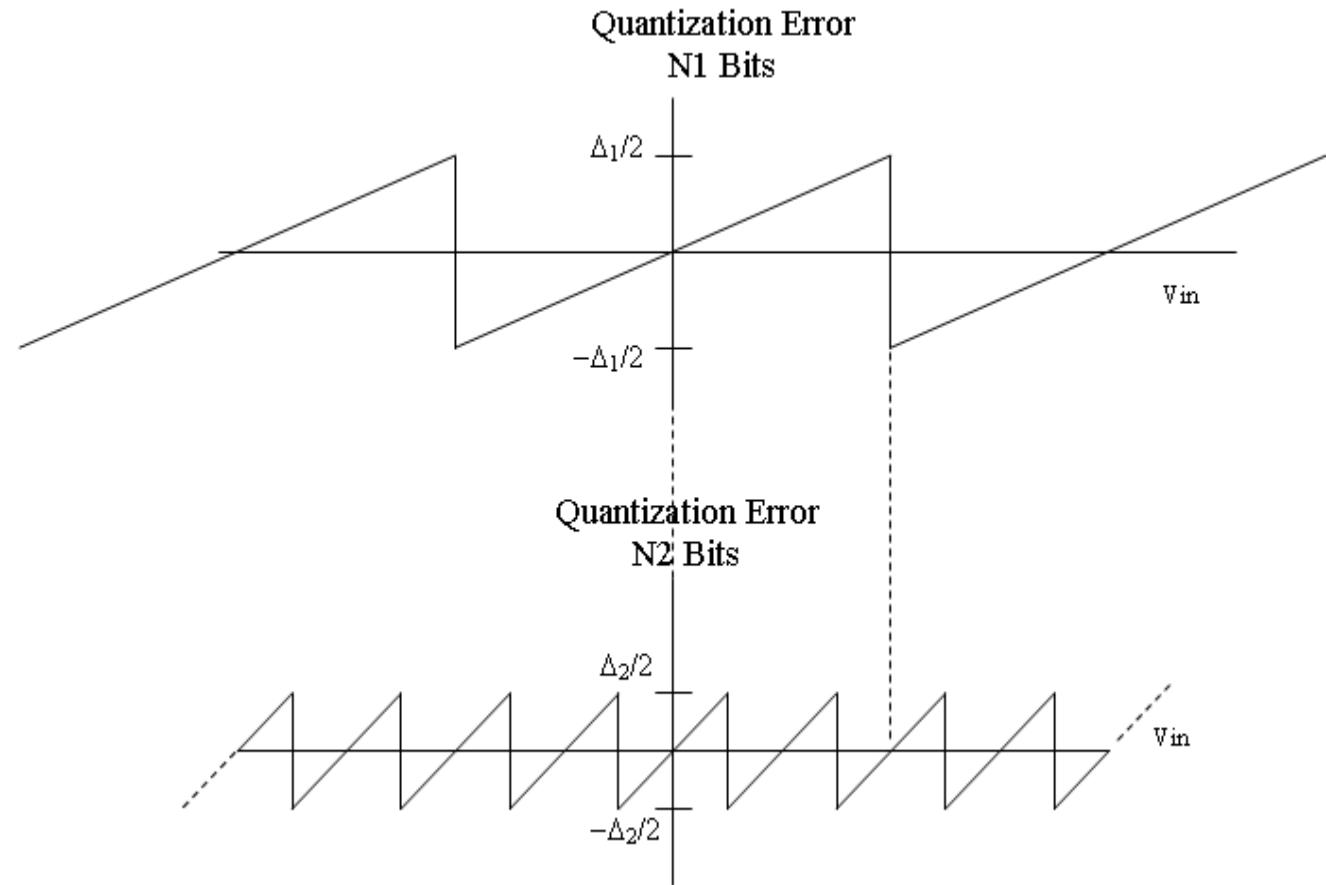
Dither Action with IGE

- ◆ IGE creates a saw-tooth pattern
- ◆ If $A1$ is equal to $A2$, then the dither will randomize (i.e. linearize the IGE)



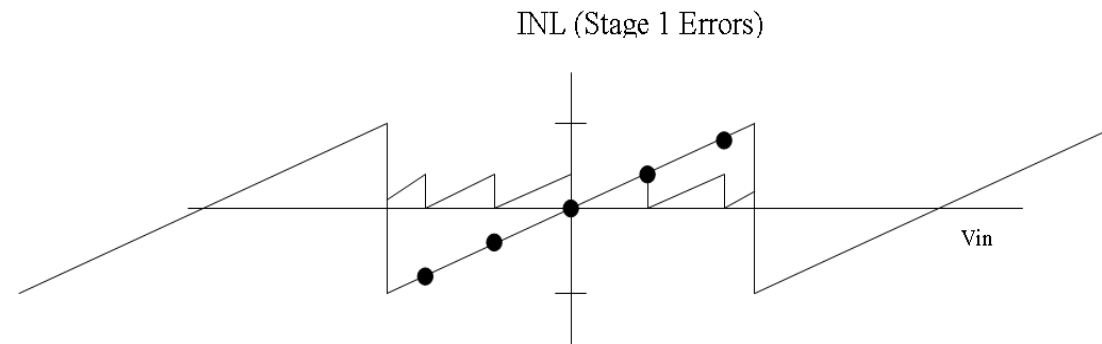
Effect of Saw-tooth Error on SFDR

- ◆ **$\text{SNR} = 6N + K1$**
- ◆ **$\text{SFDR} \approx 9N - K2$**
- ◆ **For every additional bit, the spur energy drops by 6 dB and spreads on double the spurs, which is an additional 3 dB**



Effect of Dither on SFDR

- ◆ SFDR Improves by about $9n = 30 \log N$
- ◆ SFDR of the notch improves by $6n = 20 \log N$
- ◆ The location of the notch moves by $20 \log N$

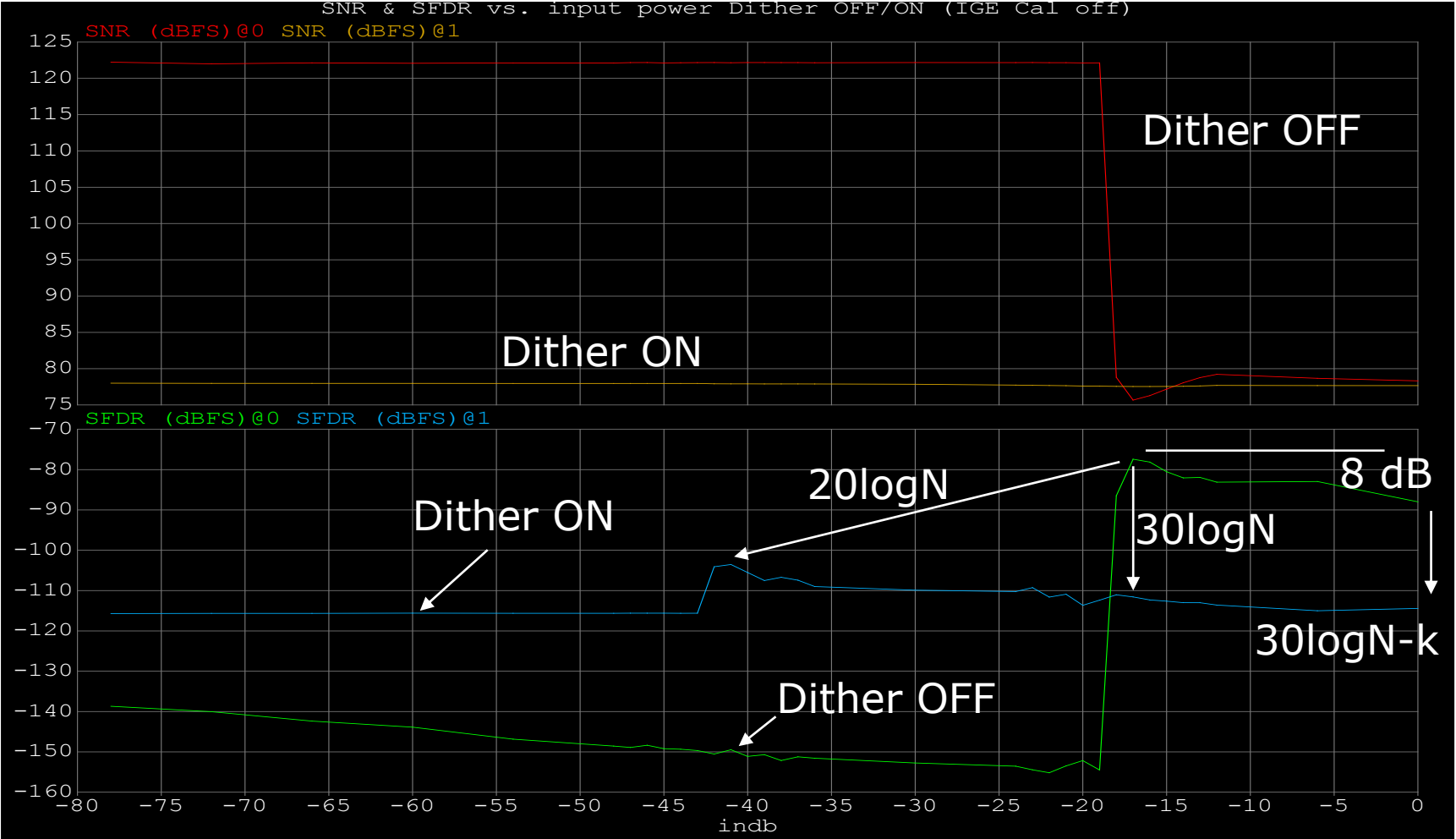


- ◆ Where n is the number of dither bits
- ◆ And N is the number of dither levels
- ◆ For every bit of dither, the SFDR improves by 6 to 9 dB

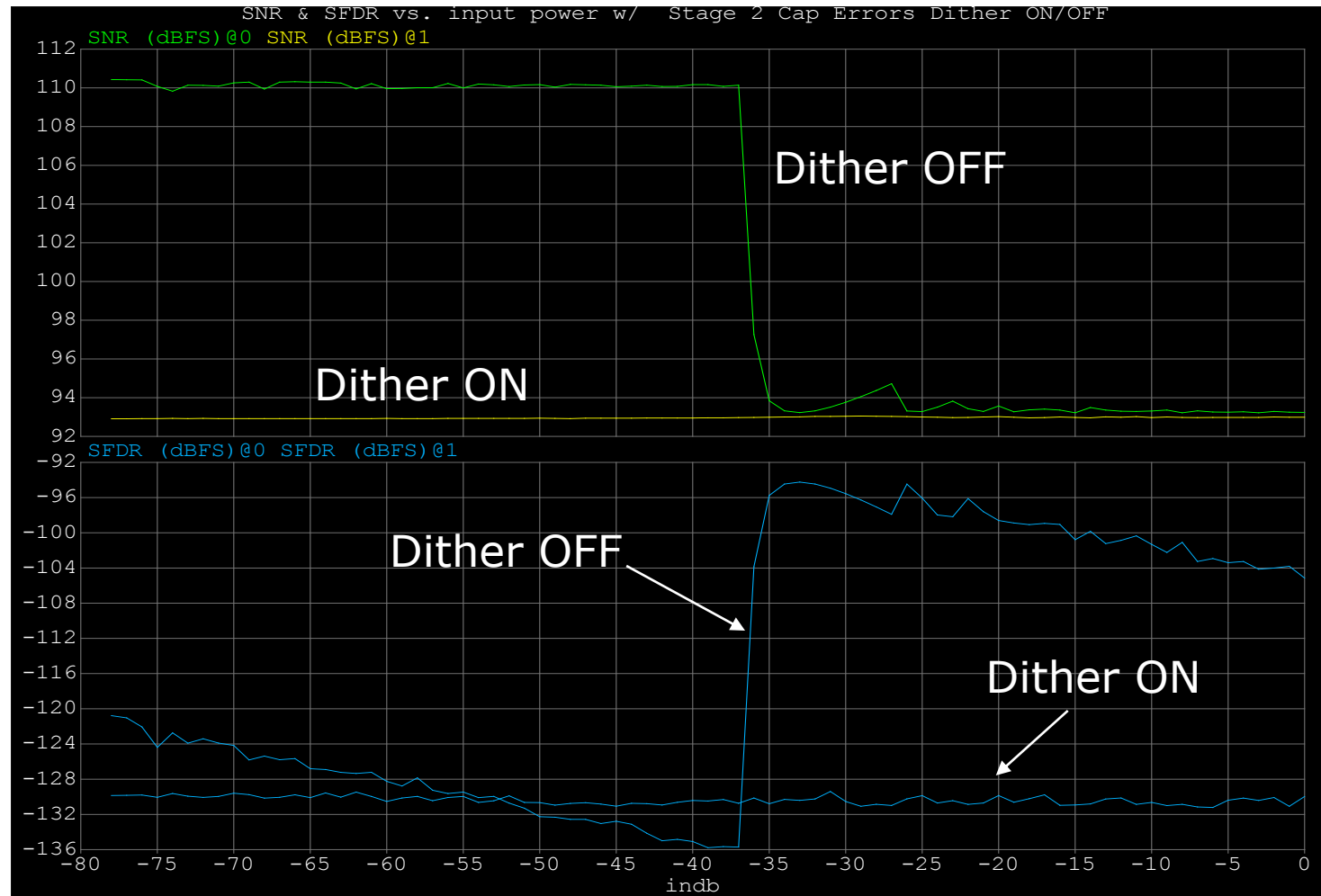
SFDR/SNR Sweep



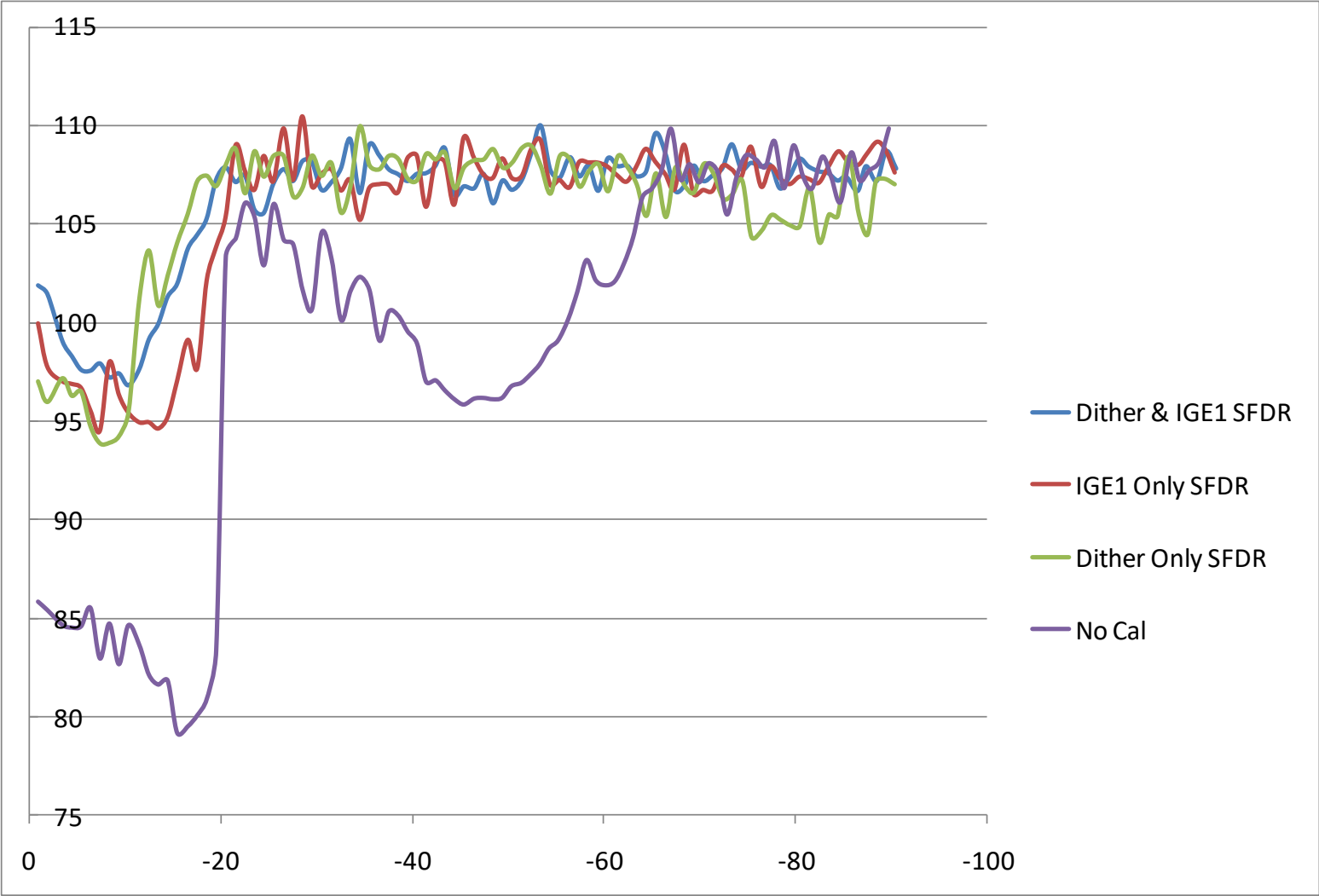
SFDR/SNR Sweep



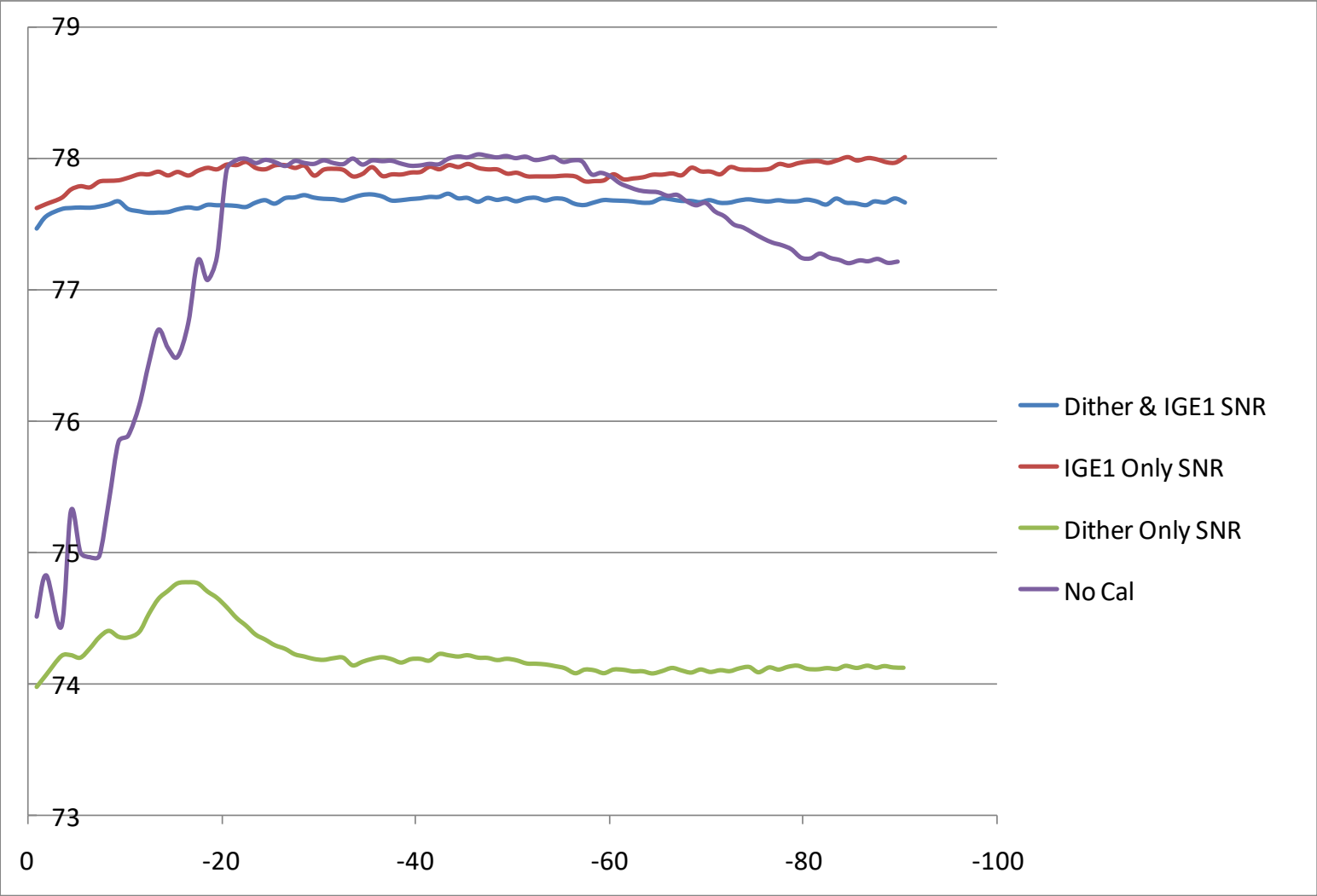
DAC Errors in Stage-2



Input Amplitude Sweep (SFDR)



Input Amplitude Sweep (SNDR)



Challenges (Recap)

- ◆ What is the **accuracy** of this approach?
- ◆ How is the **power** consumption?
- ◆ Is it **robust**?
- ◆ How about the **convergence time**?

Addressing the Challenges

- ◆ Using the above techniques, the required accuracy and robustness for SNDR can be achieved
- ◆ Dithering can be used to improve the linearity (SFDR) further
- ◆ Power overhead is minimized by reducing the dither amplitude
- ◆ Utilizing dither improves robustness and reduces power consumption
- ◆ Convergence time is in the order of seconds for linear correction and can be reduced in special cases [14]
- ◆ Convergence time for the non-linear correction can be in minutes!
 - Deterministic methods can be more attractive for non-linear correction

Conclusion

- ◆ **Digital assistance can be in the form of fixing errors or dithering errors**
- ◆ **Calibration of the IGE, DAC, IME, and kick-back errors are examples of using DSP to improve the analog performance**
- ◆ **Dither can be used to improve the calibration accuracy and improve the linearity further**
- ◆ **Digital assistance enables ADC designers to continue building efficient, accurate and fast ADCs as process geometry shrinks**

References

- ◆ [1] A.M.A. Ali, “High speed data converters”, Institution of Engineering and Technology (IET), London, UK, 2016.
- ◆ [2] A.M.A. Ali, et al., “A 14b 1GS/s RF sampling pipelined ADC with background calibration”, J. Solid-State Circuits, 49, no. 12, pp. 2846-2856, Dec. 2014.
- ◆ [3] A.M.A. Ali, et al., “A 16-bit 250-MS/s IF Sampling Pipelined ADC With Background Calibration”, J. Solid-State Circuits, 45, no. 12, pp. 2602-2612, Dec. 2010.
- ◆ [4] A.M.A. Ali, et al., “A 14-bit 2.5GS/s and 5GS/s RF sampling ADC with background calibration and dither”, IEEE VLSI Symp., 2016.
- ◆ [5] P. Huang et al., “SHA-Less Pipelined ADC With In Situ Background Clock-Skew Calibration”, IEEE J. Solid State Circuits, 46(8), ISSCC Dig. Tech. Papers, pp. 1893-1903, Aug. 2011.
- ◆ [6] M. Brandolini et al., “A 5GS/s 150mW 10b SHA-Less Pipelined/SAR Hybrid ADC in 28nm CMOS”, ISSCC Dig. Tech. Papers, pp. 468-469, Feb. 2015.
- ◆ [7] A. Panigada and I. Galton, “A 130mW 100MS/s pipelined ADC with 69dB SNDR enabled by digital harmonic distortion correction”, ISSCC Dig. Tech. Papers, pp. 162-163, Feb. 2009.
- ◆ [8] N. Rakuljic and I. Galton, “Suppression of Quantization-Induced Convergence Error in Pipelined ADCs with Harmonic Distortion Correction”, IEEE Trans. Circuits and Systems I, 60(3), pp. 593-602, March 2013.
- ◆ [9] A.M.A. Ali, “Methods and structures that reduce memory effects in analog-to-digital converters”, US Patent No 6,861,969. March, 2005.
- ◆ [10] J.P. Keane, “Digital Background Calibration for Memory Effects in Pipelined Analog-to-Digital Converters”, IEEE TCAS-I, March, 2006.

References (cont.)

- ◆ [11] A.M.A. Ali and A. Morgan, “Calibration methods and structures for pipelined converter systems”, US Patent 8,068,045, November, 2011.
- ◆ [12] A.M.A. Ali, et al., “Correlation-based background calibration of pipelined converters with reduced power penalty”, US Patent 7,786,910, August, 2010.
- ◆ [13] A.M.A. Ali, “Pipelined converter systems with enhanced accuracy”, US Patent 7,271,750, September, 2007.
- ◆ [14] A.M.A. Ali, “Method and device for improving convergence time in correlation-based algorithms”, US Patent 8,836,558, September, 2014.
- ◆ [15] E. Siragusa and I. Galton, “A Digitally Enhanced 1.8-V 15-bit 40-MSample/s CMOS Pipelined ADC,” IEEE J. Solid State Circuits, vol. 39, Dec. 2004.
- ◆ [16] S. Sonkusale, et al., “True background calibration technique for pipelined ADC”, Electronic Letters, pp. 786-788, 36(9), 2000.
- ◆ [17] Y. Chiu et al., “Least Mean Square adaptive digital background calibration of pipelined analog-to-digital converters”, IEEE TCAS-I, Jan. 2004.
- ◆ [18] J.A. McNeill et al., “Split ADC” Architecture for Deterministic Digital Background Calibration of a 16-bit 1-MS/s ADC”, IEEE TCAS-I, Dec., 2005.
- ◆ [19] E. Iroaga and B. Murmann, “A 12-Bit 75-MS/s Pipelined ADC Using Incomplete Settling”, J. Solid-State Circuits, Vol. 42, no. 4, April 2007.
- ◆ [20] B. Murmann and B.E. Boser, “A 12 b 75 MS/s pipelined ADC using open-loop residue amplification”, ISSCC Dig. Tech. Papers, pp. 328 – 497, Vol.1, 2003.
- ◆ [21] B. Murmann and B.E. Boser, “Digital Domain Measurement and Cancellation of Residue Amplifier Nonlinearity in Pipelined ADCs”, IEEE Trans. Inst. Meas., Dec. 2007.