On the Feedback Circuit Analysis using Return Ratio

I. Introduction

The precision circuit technique of implementing linear analog signal-processing functions with nonlinear devices has thus far been based upon the stabilized feedback amplifier invented by Harold Black in 1927 [1]. The most significant benefit derived from negative feedback is the desensitization of the closed-loop gain against active device parameter variations. This desensitization also leads to low distortion and noise rejection, which are highly desirable features of precision amplifiers. In addition, the employment of negative feedback allows a designer to modify the input and output impedances and to widen the bandwidth of a circuit, which proves to be useful techniques in making design tradeoffs in practice.

The downside of negative feedback, however, is the potential of instability, which entails a long history of efforts of analyzing the loop transmission and studying the stability of feedback circuits [1]–[27]. The majority of these analyses can be categorized into two approaches—the two-port analysis based on loop gain [22], [24], [25] and the return-ratio analysis [6], [7], [9], [17], [21], [23], although the names loop gain and return ratio have been used interchangeably in the literature.

It was noted in [11] that it appears that return ratio was first introduced by Bode [17]. In contrast to the complex procedures involved in the loop-gain-based two-port analysis, the return-ratio method embodies a somewhat simpler approach[1] in feedback circuit analysis [10]–[12]. Especially, in a bilateral feedback configuration,[2] it requires little or no manipulation of the original circuit topology in analyzing the loop transmission. However, as the original treatments of both return ratio and loop gain were developed for unilateral single-loop feedback, care must be taken in applying the developed formulas and procedures to bilateral and/or multi-loop feedback circuits. Specifically, the return ratio calculated in a bilateral configuration depends on the exact loop breakpoint, as will be shown later in this note. A closer examination of the discrepancies reveals that

[1] However, the “simpler” approach is error-prone as discussed in the following text.
[2] A configuration when the forward gain block, the feedback block, or both are bilateral.

multiple feedback loops exist in a bilateral feedback circuit, violating the assumptions of the return-ratio-based analysis. In addition, this observation offers more insight on the much debated difference between loop gain and return ratio—although the end results of the closed-loop gain are the same, the two approaches may reach at vastly different intermediate results for the loop transmission [10]–[12]. As perhaps the predominant motivation for the derivation of return ratio or loop gain is to study the feedback-loop stability (as well as to give rise to a closed-loop transfer function), some observations from a stability standpoint will also be given in this note.

Section II reviews the basics of the feedback circuit analysis using return ratio. The approach is applied to a bilateral shunt-shunt feedback circuit in Section III, followed by a discussion on the apparently disagreeing return ratios obtained. In Section IV, a theorem is given to relate the various return ratios to the loop gain used in an alternative approach for feedback analysis; and an in-depth examination on the fundamental cause of the discrepancies is covered in Section V. Lastly, the paper is concluded with a brief summary in Section VI.

II. Feedback Analysis using Return Ratio

A. Return Ratio

Consider the feedback circuit shown in Fig. 1, which consists of passive elements and controlled sources, with one controlled source k drawn explicitly. By the linearity of the circuit, the signals S_o and S_{ic} can be expressed as linear combinations of the signals S_i and S_{oc} [24]:

$$\begin{aligned} S_o &= dS_i + b_2 S_{oc}, \\ S_{ic} &= b_1 S_i - H S_{oc}, \end{aligned} \tag{1}$$

where, d, H, b_1, and b_2 have the same definitions as those in [24]. These equations readily solve to the closed-loop gain:

$$A_{cl} = \frac{S_o}{S_i} = \frac{b_1 k b_2}{1 + RR} + d, \tag{2}$$

where, $RR = kH$ is called the return ratio of the controlled source k. An alternative form of (2) is expressed in the form of the asymptotic gain formula [6]:

$$A_{cl} = A_{\infty}\frac{RR}{1+RR} + \frac{A_0}{1+RR}, \tag{3}$$

where, $A_{\infty} = \lim_{RR\to\infty} A_{cl} = \frac{b_1 b_2}{H} + d$ and $A_0 = \lim_{RR\to 0} A_{cl} = d$.

The return ratio RR of the dependent source k can be found with the following procedure:[3]

1) Set all independent sources in the circuit to zero;
2) Disconnect the dependent source from the rest of the circuit, which introduces a breakpoint in the feedback loop;
3) On the side of the break that is not connected to the dependent source, connect an independent test source s_t of the same sign and type as the dependent one;
4) Find the return signal s_r generated by the dependent source;
5) The return ratio of k is given by $RR = -s_r/s_t$.

Although not explicitly stated, the formulation of the feedback equations using return ratio implies a unilateral feedback. This fact can be readily appreciated by examining Fig. 1(b) in [11] (reproduced here in Fig. 2), where the core feedback loop is formed by k and H that only allows the signal to travel in one direction. The direct feed-forward term d is out of the loop, and does not participate in the signal circulation in the loop, i.e., RR is independent of d.[4]

B. Generalized Return Ratios

The previously described procedure to find the return ratio for a controlled source works for hand analysis, where the internal nodes of the small-signal model of an active device are accessible. In SPICE simulations with compact transistor models (and in experiments), an alternative procedure was proposed in [7] and [9], and is termed the generalized return-ratio method: the return ratio of a single-loop feedback circuit can be

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[3] The readers can refer to pp. 600 of [24] for a concrete example of applying the five-step procedure to derive RR.

[4] The same observation can be drawn from the asymptotic gain formula, by examining Fig. 8.42 in [24].

found by breaking the circuit at an *arbitrary* point to find a voltage return ratio RR_V and a current return ratio RR_I. RR_V and RR_I can be found using a similar procedure as previously described, and are related to the total return ratio by

$$\frac{1}{RR} = \frac{1}{RR_V} + \frac{1}{RR_I}. \tag{4}$$

The reason for both voltage and current return ratios to be measured is to handle the loading effect of the finite driving-point impedances at the breakpoint. Therefore, a single test is sufficient when the breakpoint is inside an ideal controlled source.

C. Blackman's Impedance Formula

Feedback modifies the input and output impedances of a circuit. In general, a driving-point impedance can be found using the Blackman's impedance formula [3]:

$$Z_{port} = Z_{port}\Big|_{k=0} \cdot \frac{1 + RR_{sc}}{1 + RR_{oc}}, \tag{5}$$

where, $Z_{port}\Big|_{k=0}$ is the same port impedance with the controlled source k set to zero, RR_{sc} is the return ratio of k with the port shorted, and RR_{oc} is that with the port open.

III. RETURN RATIO IN BILATERAL FEEDBACK

In single-loop unilateral feedback circuits, calculating the return ratio of a controlled source using the procedures described in the preceding section is rather straightforward. When multiple controlled sources exist in the loop, the return ratios of these sources should be identical, and also equal to that calculated with the generalized return ratios, no matter where the breakpoint of the loop is. However, practical feedback circuits consisting of real transistors and passive elements can seldom be cast into such an ideal configuration. Especially, passive elements are bilateral, i.e., the signal transmissions from and to both sides are completely symmetrical. Next, it will be shown that the return ratio calculated in a bilateral setup depends on the exact loop breakpoint.

Consider the simple shunt-shunt feedback circuit shown in Fig. 3(a), where the amplifier A is the forward block, and the feedback network is formed by the impedance

Z_f.[5] In general, signals travel both ways in the A and Z_f blocks. To show explicitly all the controlled sources in the loop, both the forward and feedback paths are modeled by a two-port network as shown in Fig. 3(b). Note that, when the loop is closed, both y_{21a} and y_{21f} are driven by V_i, and both y_{12a} and y_{12f} are driven by V_o.

A. Return Ratio of y_{21a} (forward-gain path)

In Fig. 4, break the loop open at the control terminal of y_{21a}, insert an independent source V_t, and apply KCL at nodes V_i and V_o, we have

$$\begin{aligned} \left(y_{11a}+y_{11f}\right)V_r+y_{12a}V_o+y_{12f}V_o&=0,\\ \left(y_{22a}+y_{22f}\right)V_o+y_{21a}V_t+y_{21f}V_r&=0. \end{aligned} \tag{6}$$

Upon eliminating V_o, the return ratio of y_{21a} can be calculated as follows:

$$RR(y_{21a})=-\frac{V_r}{V_t}=-\frac{y_{21a}\left(y_{12a}+y_{12f}\right)}{\left(y_{11a}+y_{11f}\right)\left(y_{22a}+y_{22f}\right)-y_{21f}\left(y_{12a}+y_{12f}\right)}. \tag{7}$$

Note that, when the loop is opened this way, the controlled sources y_{21a} and y_{21f} are driven by different voltages, V_t and V_r, respectively.

B. Return Ratio of y_{12f} (feedback network)

The return ratio of y_{12f} can be calculated in the same way (Fig. 5). By the symmetry of the circuit, we can write down the result directly by inspection:

$$RR(y_{12f})=-\frac{y_{12f}\left(y_{21a}+y_{21f}\right)}{\left(y_{11a}+y_{11f}\right)\left(y_{22a}+y_{22f}\right)-y_{12a}\left(y_{21a}+y_{21f}\right)}. \tag{8}$$

Note that, when the loop is opened in this case, the controlled sources y_{12f} and y_{12a} are driven by different voltages, V_t and V_r, respectively.

Apparently, $RR(y_{21a})\neq RR(y_{12f})$ holds by comparing (7) with (8), which indicates that this is not a unilateral feedback loop. However, if the controlled sources y_{12a} (the feedback of the forward path) and y_{21f} (the forward gain of the feedback path) are set to zero and the loop becomes truly unilateral, both (7) and (8) reduce to the same result:

[5] Although a shunt-shunt feedback is considered here, similar results can be obtained with either one of the other three feedback configurations, i.e., the shunt-series, series-shunt, or series-series feedback.

$$RR(y_{21a}) = RR(y_{12f}) = -\frac{y_{21a}y_{12f}}{(y_{11a}+y_{11f})(y_{22a}+y_{22f})}. \tag{9}$$

C. Generalized Return Ratios

We first pick the loop breakpoint at the output node V_o, as often done in a SPICE simulation; then separately apply a test voltage and current source, as shown in Fig. 6(a); and the generalized return ratios can be calculated as follows:

$$\begin{aligned} RR_V{}^o &= -\frac{V_r}{V_t} = -\frac{y_{21a}y_{12f}}{(y_{11a}+y_{11f})y_{22a} - y_{21a}y_{12a}}, \\ RR_I{}^o &= -\frac{I_r}{I_t} = -\frac{y_{21a}y_{12f}}{(y_{11a}+y_{11f})y_{22f} - y_{21f}y_{12f}}, \end{aligned} \tag{10}$$

and the total return ratio is given by

$$RR^o = \left(\frac{1}{RR_V{}^o} + \frac{1}{RR_I{}^o}\right)^{-1} = -\frac{y_{21a}y_{12f}}{(y_{11a}+y_{11f})(y_{22a}+y_{22f}) - (y_{21a}y_{12a} + y_{21f}y_{12f})}. \tag{11}$$

Next, as shown in Fig. 6(b), we pick the input node V_i as the breakpoint, a similar procedure results in

$$RR^i = -\frac{y_{21a}y_{12f}}{(y_{11a}+y_{11f})(y_{22a}+y_{22f}) - (y_{21a}y_{12a} + y_{21f}y_{12f})}, \tag{12}$$

which is identical to RR^o. Define $RR^G = RR^o = RR^i$, we note that $RR(y_{21a}) \neq RR(y_{12f}) \neq RR^G$ holds by comparing (7), (8), (11), and (12). However, if the controlled sources y_{12a} and y_{21f} are set to zero and the loop becomes truly unilateral, all these return ratios reduce to the same result as in (9).

D. Discussion

The calculations so far in this section have revealed an uncomfortable fact—the return ratio calculated in a bilateral feedback loop depends on the exact loop breakpoint. A closer examination shows that the original circuit actually contains multiple feedback loops formed by either the forward-gain path or the feedback network alone; they appear in shunt when the loop is closed. In fact, were it not for the subscripts *a* and *f* used in the circuit diagram, there would have been no means to distinguish the forward path from the feedback network—signals actually travel in both directions in these blocks. Consequently, breaking the loop open at a specific point does not entirely eliminate the feedback—it simply forms an open-loop circuit with a local closed-loop feedback embedded in the rest parts. Depending on the nature of the embedded loop, a new circuit is formed each time the loop is opened at a different point, and the return ratio thus calculated would most likely be different.

Take $RR(y_{21a})$ for example. In Fig. 4, when the loop is opened at the control terminal of y_{21a} of the forward path, although the feedback controlled sources y_{12a} and y_{12f} are still driven by the same output signal V_o, y_{21f}, the forward controlled source of the feedback network, is now driven by V_r, which is different from V_t, the one driving y_{21a}. It is y_{21f} that introduces the embedded feedback in this test. With the large gain developed around the loop, $V_r \neq V_t$ holds in general, and the feedback (kickback) from y_{21f} can be significant. This justifies the extra term in the denominator of (7) that relates to y_{21f}. A similar argument applies to $RR(y_{12f})$ in Fig. 5, except that the kickback source now is y_{12a}, which is driven by V_r. This explains the extra term in the denominator of (8).

It now becomes somewhat straightforward to analyze the next case where generalized return ratios are calculated for the loop by opening it at V_o—either y_{12a} or y_{21f} forms the embedded loop when the breakpoint is driven by V_t or I_t, respectively. When this case is contrasted to that of Fig. 4, it is apparent that the kickback effect of y_{21f} is the same with a current drive (I_t); however, now a similar kickback is also produced by y_{12a} with a voltage drive (V_t). This justifies the fact that, when y_{12a} is set to zero, (7) and (11) reduce to the same result. The same argument applied to comparing this case to that of Fig. 5 leads to the observation that the kickback of y_{12a} is the same, while that of y_{21f} is different. Thus, when y_{21f} is set to zero, (8) and (11) reduce to the same result.

We conclude that, although the finite driving-point impedances are accounted for at the loop breakpoint in a generalized return-ratio calculation, the resulting total return ratio is only an approximation of that of the controlled sources in a bilateral feedback loop. Furthermore, it needs to be pointed out that the two-port formulation is not the cause of this discrepancy, as it merely embodies an equivalent network representation of the original circuit. Secondly, the passivity of the feedback network Z_f does not provide a satisfactory explanation either—setting $y_{11f} = y_{22f}$ and $y_{12f} = y_{21f}$ does not eliminate the discrepancies.[6]

E. Return Ratio with Respect to a General Reference Value

The discussion presented in the preceding section reveals that a direct application of the procedures developed to calculate the return ratio in a unilateral feedback loop sometimes leads to capricious results in bilateral and/or multi-loop feedback configurations. It seems that Bode has somewhat realized this potential problem in the development of his return-ratio theory—in Figs. 4.7 and 4.8 of [17] (reproduced here in Fig. 7), he argued that, when the feedback network is bilateral, the forward gain of the feedback network can be either *left out* or *lumped into the forward-gain amplifier*.

In the former case ("*left out*"), however, it is not easy (and perhaps wrong) to simply neglect the forward transmission of the feedback network when it consists of passive (symmetrically bilateral) elements, as this forward path is inherently part of the feedback loop. This is especially the case when the return ratio is calculated without resorting to a two-port representation. Examples of this are in Fig. 4.2 of [17], Fig. 8.39 of [24], and Fig. 4 of [11]. Note that this argument is completely different from separating the direct feed-forward term d of the input signal from the intrinsic feedback loop (formed by k and H) when deriving the closed-loop transfer function (Fig. 2). As a matter of fact, this has always been done since, as the first step in a return-ratio evaluation, any independent source in the circuit (of course including the input source) is set to zero, and the effect of d is eliminated.

In the latter case ("*lumped into the forward-gain amplifier*"), the treatment involves re-computing the forward-path gain with a general reference value [17] [21]:

Definition: The reference value of any element (controlled source) is that value which gives zero transmission through the circuit as a whole when all other elements of the circuit have their normal values.

For example, in the bilateral shunt-shunt feedback circuit shown in Fig. 3, y_{21f} can be

[6] It is conceptually not difficult to conceive a feedback circuit where the feedback network is formed by an active device, and $y_{11f} = y_{22f}$ and $y_{12f} = y_{21f}$ do not hold in general.

treated as the reference value for y_{21a} when $RR(y_{21a})$ is evaluated; conversely, y_{12a} can be considered as the reference value for y_{12f} when $RR(y_{12f})$ is calculated. Now let y_{21a} and y_{21f} be lumped together; define $y_{21} = y_{21a} + y_{21f}$, the return ratio of y_{21} can de calculated as:

$$RR(y_{21}) = -\frac{(y_{21a} + y_{21f})(y_{12a} + y_{12f})}{(y_{11a} + y_{11f})(y_{22a} + y_{22f})}. \tag{13}$$

Since there exists a single breakpoint that opens both loops, the same result can be reached at by virtue of the multi-loop return-ratio formula [21], [12]:

$$RR(y_{21}) = RR(y_{21a})\big|_{y_{21f}=0} + RR(y_{21f})\big|_{y_{21a}=0}. \tag{14}$$

Interestingly, another theorem of Bode related to the return ratio with respect to two elements is [21]

$$\frac{1+RR(x_1)}{1+RR(x_2)} = \frac{1+RR(x_1)\big|_{x_2=0}}{1+RR(x_2)\big|_{x_1=0}}, \tag{15}$$

where, $RR(x_1)$ and $RR(x_2)$ are the return ratios of two controlled sources in the feedback circuit, and $RR(x_1)\big|_{x_2=0}$ and $RR(x_2)\big|_{x_1=0}$ are the same return ratios calculated with the other one set to zero. It is straightforward to verify that the equality in (15) holds for y_{21a} and y_{21f}. Note that the loop becomes unilateral when either one of the controlled sources is set to zero for the return ratio of the other to be evaluated.

IV. FEEDBACK ANALYSIS USING LOOP GAIN

A. *Loop Gain in Bilateral Feedback*

An alternative approach for feedback analysis is the two-port method based on loop gain [22], [24], [25]. In this approach, a manipulation of the original bilateral feedback circuit is performed first to rearrange it into a unilateral configuration. The method is probably motivated by the fact that, for example, in Fig. 3, when the loop is closed, the forward-transmission controlled sources y_{21a} and y_{21f} of the forward amplifier and the feedback network, respectively, enter the loop in shunt. Therefore, they are all inherent constituents of the feedback, and should be treated equally. The same argument holds true for the controlled sources y_{12f} and y_{12a}. Although the exact procedures of the loop-gain analysis for a bilateral feedback circuit are complicated [24], [6], [8], the approach is conceptually simple; and as it always results in a unilateral setup, the calculated loop gain is unique no matter where the breakpoint of the loop is. As shown in the literature [24], [10], [11], this loop gain is

$$T = af = -\frac{(y_{21a} + y_{21f})(y_{12a} + y_{12f})}{(y_{11a} + y_{11f})(y_{22a} + y_{22f})}, \tag{16}$$

where, a is the (total) forward gain and f is the (total) feedback factor.

B. Loop Gain and Return Ratio

Note that (16) is identical to (13), the return ratio calculated following Bode's second suggestion. Relating (16) to (7), it becomes obvious that the only difference involved in the derivations is the reposition of the controlled source y_{21f}. In the return-ratio analysis, the kickback of this element interferes with the measurement of the loop transmission. Therefore, when y_{21f} is set to zero, both (16) and (7) reduce to the same result. In general, to relate the loop gain to the various return ratios in a bilateral feedback loop, we propose the following theorem:[7]

Theorem: Consider the forward transmission (from A to Z_f and back to A) of the bilateral feedback amplifier that can be modeled as the two-port network shown in Fig. 3. The loop gain and various return ratios are all numerically closely related if the following conditions hold:

$$\begin{aligned} &1) && |y_{21a}| >> |y_{21f}|, \\ &2) && |y_{12f}| >> |y_{12a}|, \\ &3) && |y_{21a}y_{12a}| + |y_{12f}y_{21f}| << |(y_{11a} + y_{11f})| \cdot |(y_{22a} + y_{22f})|, \end{aligned} \tag{17}$$

where, y should be replaced by the appropriate two-port parameters, i.e., y, z, g, or h, for different types of feedback under consideration. The conditions 1) and 2) ensure that the signal transmission in the forward direction dominates when the loop is closed; and the condition 3) ensures that the kickbacks of y_{21f} (the forward transmission of the feedback network) and y_{12a} (the feedback of the forward amplifier) are negligible when the loop is

[7] Note that the theorem proposed here is an amended (and more accurate) version of that introduced in [11].

opened. With these assumptions, the loop gain and the return ratios can all be approximated as

$$T \approx RR(y_{21a}) \approx RR(y_{12f}) \approx RR_G \approx -\frac{y_{21a}y_{12f}}{(y_{11a}+y_{11f})(y_{22a}+y_{22f})}. \tag{18}$$

V. RETURN RATIO, LOOP GAIN, AND STABILITY

At this point, a few conceptual problems arise–what to do with the multiple return ratios obtained in a bilateral feedback? Which one of Bode's suggestions should be adopted for return-ratio analysis (although both are somewhat confusing)? Should the forward gain of the feedback network be *left out* or *lumped into the forward-gain amplifier*? Does the loop-gain measurement really capture and characterize the loop transmission? Do the return ratios?

To seek answers to these questions, it seems we need to return to the most fundamental motivation for the return-ratio and loop-gain analyses—to study the stability of the feedback loop. As a bilateral feedback loop (such as the one shown in Fig. 3) is likely to contain multiple feedback loops, the use of return ratio to analyze the stability remains an open research problem. In this note, instead of proposing a general theory on the stability of multi-loop feedback, a few observations drawn from the analysis of the bilateral feedback circuit presented so far are summarized as follows.

First of all, return ratio by definition is with respect to a single controlled source, such as the transconductance (g_m) of the input transistor of an amplifier. When the feedback loop is opened at this controlled source to check the stability of the loop transmission, an implicit assumption is that the rest of the circuit is stable. This is mostly true in practice. Consider the shunt-shunt feedback circuit shown in Fig. 3 again, the passive feedback network (although bilateral) may very well be composed of a few resistors and capacitors, and is inherently stable. Examining the return ratio derived in Section III-A for y_{21a} is a valid means to check the loop stability. Although the kickback from y_{21f} still forms an embedded loop inside, and triggers some unintended confusion, it bears no consequence on the stability study due to its passivity. This can be related to the treatment of the first special case presented in [12] of a multi-loop feedback study—the embedded local loops within a global feedback. The usual gain and phase margins of $RR(y_{21a})$ can be evaluated for the open global loop to check its stability with all embedded loops intact and verified stable *a priori* [28].

In a more general sense, however, it is common for the feedback network to consist of active circuits forming the embedded loops. Evaluating the return ratio of y_{21a} alone will then be insufficient to determine the stability of the circuit. Another typical example is the CMOS active-cascode amplifier, e.g., the one shown in Fig. 3.41 of [24]. Checking the stability of the global feedback loop with the local gain-boosting loop closed is routinely performed in simulation, in tandem with another check of the boosting loop for its own stability.

Now we turn our attention to the loop-gain method. In the context of the shunt-shunt feedback amplifier shown in Fig. 3, lumping y_{21f} into y_{21a} and checking the stability of y_{21} (by evaluating its return ratio) presents an alternative way to examine the stability of the circuit,[8] not because the resulting loop gain coincides with that of the second approach of the return-ratio measurement suggested by Bode, but y_{21f} forms an intrinsic part of the forward loop transmission and is not correct to be left out from the overall feedback standpoint. It is not difficult to conceive a case where y_{21f} stems from an active (or passive) device deliberately designed to stabilize the loop. In such a case, $RR(y_{21a})\big|_{y_{21f}=0}$ will not be stable while $RR(y_{21})$ will be stable. In this sense, the loop-gain measurement in a bilateral feedback configuration seems to focus and check on the *overall* stability of the feedback loop, not a specific controlled source at a time. For example, in the feedback circuit shown in Fig. 8, calculating the return ratio of g_{m1} (of M_1) or g_{m2} (of M_2) alone only reveals partly the overall loop transmission; while lumping the two in parallel results in a single loop-gain calculation that concludes the stability check for the whole circuit.

[8] The return ratio thus obtained may drastically differ from the one with y_{21a} alone, and may not exhibit the "nice" two-pole roll-off such as the example shown in [11]. However, the niceness of the shape is not an *a priori* condition for stability analysis.

VI. CONCLUSION

The return-ratio analysis in bilateral feedback circuits is revisited. Results indicate that, depending on the exact loop breakpoint, the calculated return ratio may vary significantly. The fundamental cause of this discrepancy is studied and linked to the loop-gain measurement using the two-port method. In either case, the bilateralism of the circuits invites a multi-loop feedback configuration, which explains the various disagreeing results obtained in the return-ratio and loop-gain evaluations. A theorem is proposed to point out the equivalence between these quantities and the conditions under which the equivalence holds.

From a stability standpoint, either method is valid in evaluating the loop stability for the bilateral and/or multi-loop feedback circuit considered here. The loop-gain method seems to measure the overall loop stability, while the return-ratio approach checks one controlled source at a time. The efficiency and complexity of either approach may vary case by case, and are up to the designer’s discretion.

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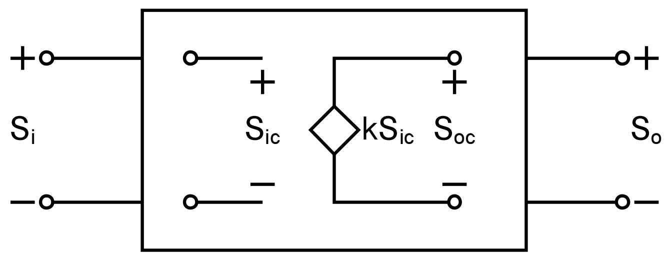


Fig. 1. The general diagram of a feedback circuit with one controlled source *k* shown explicitly.

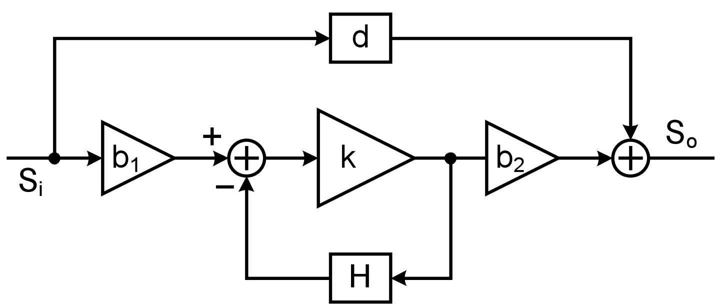


Fig. 2. Fig. 1(b) in [11], an alternative illustration of the feedback circuit in Fig. 1.

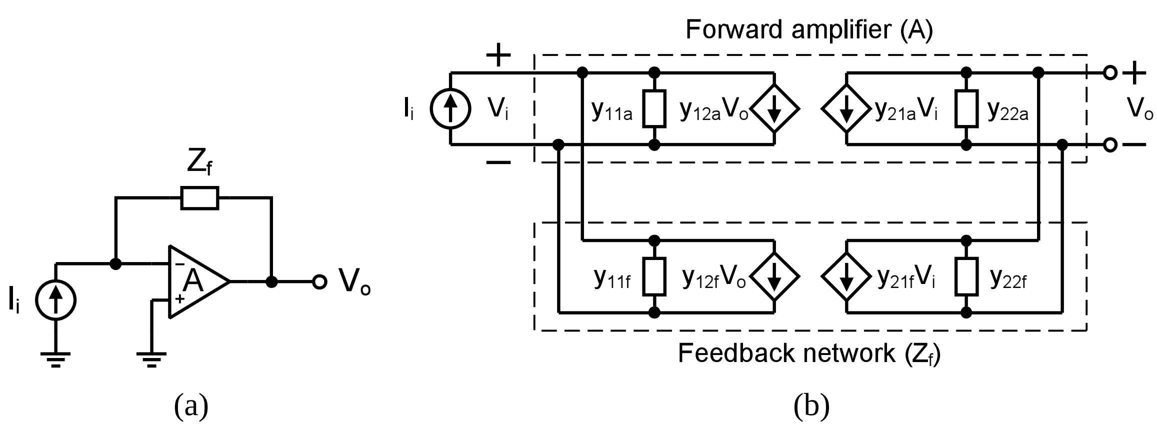


Fig. 3. A simple bilateral shunt-shunt feedback amplifier: (a) circuit schematic, and (b) its two-port model.

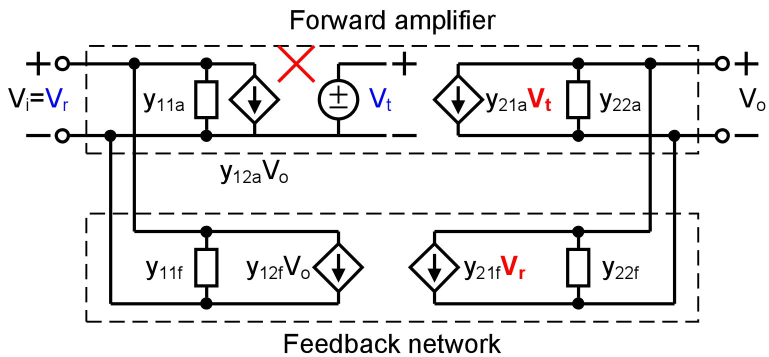


Fig. 4. Return-ratio evaluation of the controlled source y_{21a}.

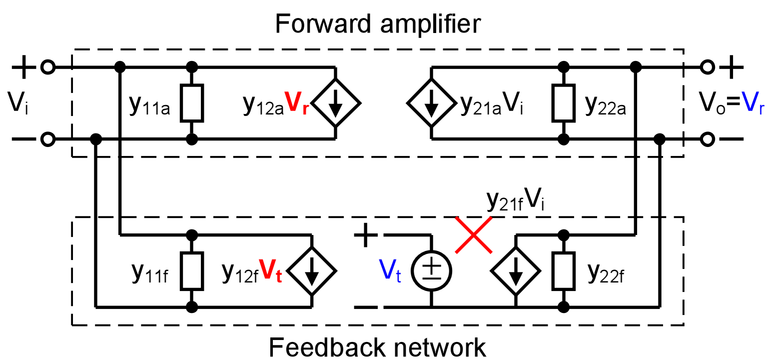
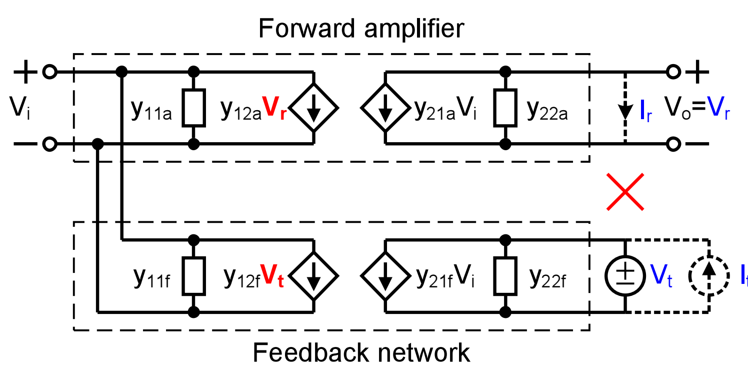
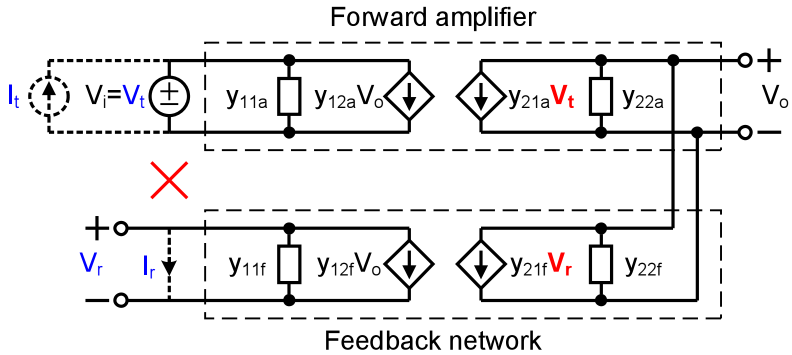


Fig. 5. Return-ratio evaluation of the controlled source y_{12f}.

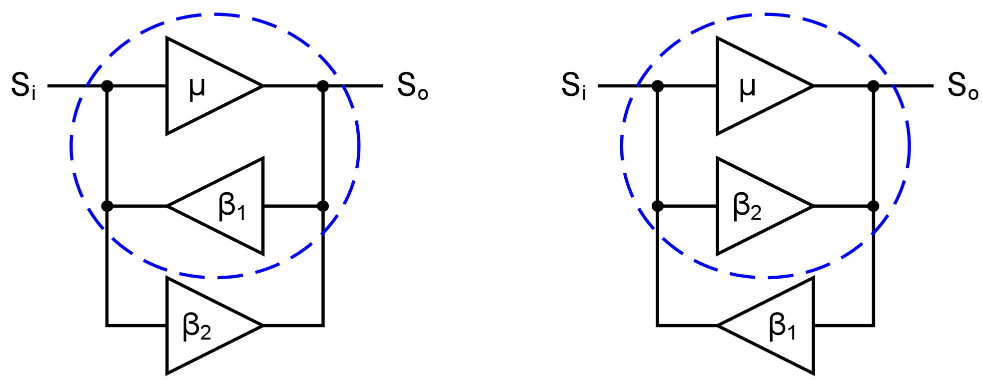


(a)



(b)

Fig. 6. Generalized return-ratio calculation of the circuit in Fig. 3 (a) by opening the loop at the output, and (b) by opening the loop at the input.



(a) (b)

Fig. 7. Figs. 4.7 and 4.8 of [17], return-ratio evaluation in a bilateral feedback configuration (a) by leaving the forward gain of the feedback network (β_2) out, or (b) by lumping it into the forward-gain amplifier.

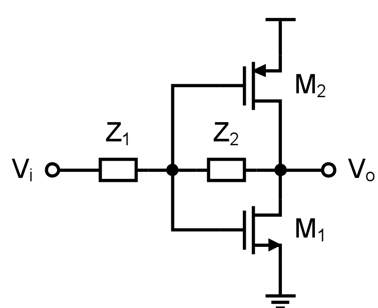


Fig. 8. The circuit diagram of a feedback amplifier consisting of multiple forward-gain active devices.