The Theory of Bandpass Sampling

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Abstract—**The sampling of bandpass signals is discussed with respect to band position, noise considerations, and parameter sensitivity. For first-order sampling, the acceptable and unacceptable sample rates are presented, with specific discussion of the practical rates which are nonminimum. The minimum sampling rate is pathological in that any imperfection in the implementation will cause aliasing. The susceptibility of the acceptable sampling rates to aliasing decreases with increasing guard-band size from the minimum sampling rate.**

In applying bandpass sampling to relocate signals to a baseband position, the signal-to-noise ratio is not preserved owing to the out-of-band noise being aliased. The degradation in signal-to-noise ratio is quantified in terms of the position of the bandpass signal.

For the construction of a bandpass signal from second-order samples, the cost of implementing the interpolant (dynamic range and length) depends on Kohlenberg's sampling factor k, the relative delay between the uniform sampling streams. An elaboration on the disallowed discrete values of k shows that some allowed values are better than others for implementation. Optimum values of k correspond to the quadrature sampling values with the band being either integer or half-integer positioned.

I. Introduction

The subject of bandpass sampling is an aspect of digital signal processing which has relevance to a variety of disciplines including, for example, optics [1], radar [2], sonar [3], communications [4], biomedical signals [5], power measurement [6], and general instrumentation, such as sampling oscilloscopes.

Historically, Cauchy seems to have been the first to hypothesize the bandpass sampling requirement [7]. Nyquist [8] and Gabor [9] also alluded to the bandpass case. Kohlenberg [10] introduced second-order sampling and provided the basic interpolation formula, which was then reported in the texts by Goldman [11] and Middleton [12].

The bandpass situation is depicted in Fig. 1, where the sampling rate is expressed as f_s Hz and the bandpass signal is located between f_L Hz and f_u Hz. The signal bandwidth B is less than $f_u - f_L$, and the positive frequency band can be expressed as the interval (f_L, f_u).

The *band position* refers to the fractional number of bandwidths from the origin at which the lower band edge resides. A special case is *integer band positioning*, which holds when the band is located at an integral number of bandwidths from the origin, i.e., $f_L = c(f_u - f_L)$, $c = 0, +1, +2, \cdots$; and $c = 0$ is the low-pass case. Fig. 2(a) indicates integer band positioning, for $c = 3$.

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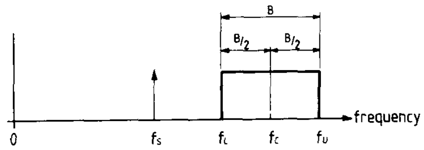

Fig. 1. The bandpass situation as an analog signal spectrum. The sampling rate is expressed as f_s Hz and the band is located at (f_L, f_u). Only the positive frequencies are shown.

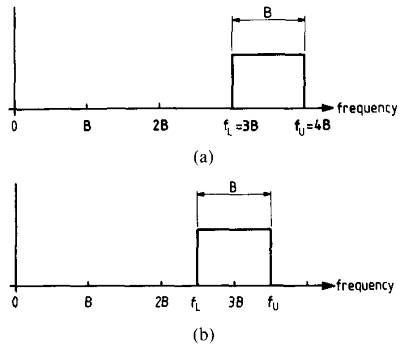

Fig. 2. Examples of (a) integer band positioning for $c = 3$, and (b) half-integer band positioning for $c = 3$.

Another special case is *half-integer positioning*, when $f_L = ((2c + 1)/2)(f_u - f_L)$. Fig. 2(b) shows an example of half-integer positioning for $c = 2$. These cases of band positioning are useful for illustrating aspects of aliasing, and are particularly important in the application of sampling to the effective relocation of signals between bandpass and low-pass positions.

The classical bandpass theorem for uniform sampling states that the signal can be reconstructed if the sampling rate is at least $f_s^{(\min)} = 2f_u/n$, where n is the largest integer within f_u/B, denoted by $n = I_g[f_u/B]$.

This minimum rate for uniform sampling is of interest from a theoretical viewpoint, but for practical applications this rate is often presented in a misleading way. Fig. 3 from Feldman and Bennett [16], also reproduced in several texts (e.g., [13]–[15]), illustrates the minimum uniform sampling rate for bandpass signals. The theoretical minimum rate $f_s = 2B$ is seen to apply only for integer band positioning. The skew lines represent the minimum uniform rate required when the passband is not located in an integer position. The vertical lines, which represent a

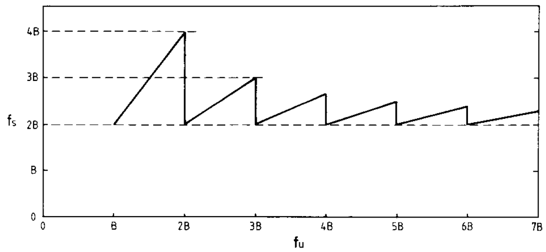

Fig. 3. Minimum sampling frequency for band of width B [16].

discontinuity, traverse *disallowed* values of the sampling frequency, a point elucidated in Section II. Gregg's [17] version of Fig. 3 displays more clearly the discontinuity in the function by using dotted lines for the vertical sections.

Note that $f_s = 2B$ is a valid uniform sampling rate only if there is no signal component at the frequencies f_L or f_u. This is analogous to the anomaly of the low-pass case (cf. Shannon's theorem) where the equality in $f_s \geq 2f_u$ can hold only if there is no signal component at f_u, i.e., the signal is band limited to *less than* B and the signal occupies the interval $(0, f_u)$.[1] Kida and Kuroda [18] discuss the disallowed signal components for higher order sampling.

The theoretical minimum sampling rate is pathological in the sense that any engineering imperfections in an implementation will cause aliasing. This illustrates the need to sample at above the theoretical minimum rate or, alternatively stated, guard-bands need to be included. Gregg [17] states without elaboration that rates between $f_s^{(\min)}$ and $2f_u$ (the low-pass case rate) are not, in general, valid. These rates correspond to the region above the function in Fig. 3. Nevertheless, some authors mistakenly interpret figures such as Fig. 3 to mean that no aliasing will occur as long as $f_s > 4B$. Gaskell [1] gives an expression (see (16)) for the valid rates, but otherwise there is little information regarding bandpass sampling at rates above the theoretical minimum but still well below the frequencies of the signal being sampled. In Section II, this subject is discussed and results are depicted graphically. The existing information is clarified and the cases for bandpass sampling at above the minimum rate are reviewed. The discussion includes sensitivity of sampling rates and offers a practical guide to bandpass uniform sampling.

The theoretical minimum uniform sampling rate $f_s = 2B$ can be applied only when the band is integer positioned. Kohlenberg's [10] second-order sampling theorem allows the minimum rate, in the form of an *average* rate, to be applied independent of the band position. The technique is second order in that two interleaved uniform sample sequences are applied. Each sequence is of rate B (i.e., sampling every $1/B$ s), and the sequence separation is denoted by k s.

Using the samples to construct a signal $f(t)$ at (f_L, f_u), the interpolation is given by

$$f(t) = \sum_p \left[f\left(\frac{p}{B}\right) S\left(t - \frac{p}{B}\right) + f\left(\frac{p}{B} + k\right) S\left(-t + \frac{p}{B} + k\right) \right] \tag{1}$$

in which p is the sample number index and the interpolant is

$$S(t) = S_0(t) + S_1(t) \tag{2}$$

where

$$S_0(t) = \frac{\cos\left[2\pi(rb - f_L)t - r\pi Bk\right] - \cos\left[2\pi f_L t - r\pi Bk\right]}{2\pi Bt \sin r\pi Bk} \tag{3}$$

$$S_1(t) = \frac{\cos\left[2\pi(f_L + B)t - (r+1)\pi Bk\right] - \cos\left[2\pi(rB - f_L)t - (r+1)\pi Bk\right]}{2\pi Bt \sin (r+1)\pi Bk} \tag{4}$$

and r is an integer given by the band position such that

$$\frac{2f_L}{B} \leq r < \frac{2f_L}{B} + 1. \tag{5}$$

The right-hand side of (5) can be expressed in terms of the center frequency f_c,

$$\frac{2f_L}{B} + 1 = \frac{f_L + f_U}{B} = \frac{2f_c}{B}. \tag{6}$$

Geometrically, there are at least $r - 1$ bandwidths between the two-sided spectrum, i.e., between $-f_L$ and $+f_L$. The interpolant has the properties that $S(0) = 1$, $S(p/B) = 0$ for $p \neq 0$, and $S(\,p/B \; + k) = 0$, and in general it has extrema which are larger than unity.

In the general case when the equality of (5) does not hold, the value of k may not take on the discrete values

$$kBr,\ kB(r+1) = 0, 1, 2, \cdots \tag{7}$$

for which the interpolant blows up. k is otherwise unrestricted, but implementation difficulty using finite word-length machines is affected by the choice of k. This sensitivity is discussed in Section III.

When the equality in (5) holds, the first term of the interpolant, $S_0(t)$, is zero, and the restrictions $kBr \neq 0, 1, 2, \cdots$ on k are removed. The situation now corresponds to either integer band positioning (r even) or half-integer band positioning (r odd) and for these conditions there are exactly r bandwidths between $-f_L$ and $+f_L$.[2] Three special cases are noteworthy:

[1]The low-pass case is exceptional in that the component at $f_L = 0$ can be recovered because it contains trivial phase information.

[2]An alternative definition for r in (5) is to place the equality with the right-hand term instead of the left-hand term. This causes a swap in the above meaning of odd and even r, and a swap of the term that drops out of the interpolant when the equality holds. For this alternative definition, r cannot be zero and the low-pass case corresponds to $r = 1$.

i) The low-pass case is $f_L = 0$ and $r = 0$ for which the interpolant reduces to

$$S_{LP}(t) = \frac{\cos(2\pi Bt - \pi Bk) - \cos \pi Bk}{2\pi Bt \sin \pi Bk}. \tag{8}$$

ii) The special case of $k = 1/2B$ corresponds to uniform rate sampling, with its well-known sinc $(2\pi Bt)$ interpolant, which, as mentioned above, requires integer band positioning.

iii) The quadrature sampling case has

$$k = \frac{1}{4f_c} \pm \frac{m}{2f_c}, \qquad m = 0, 1, 2 \cdots \tag{9}$$

and the simplified interpolant

$$S_Q(t) = \frac{\sin \pi Bt}{\pi Bt} \cos 2\pi f_c t. \tag{10}$$

Quadrature sampling is important because, as the name indicates, the in-phase and quadrature components are sampled explicitly from the bandpass signal. This is seen by denoting the bandpass signal in the usual way:

$$f(t) = I(t) \cos \omega_c t - Q(t) \sin \omega_c t \tag{11}$$

where $I(t)$ and $Q(t)$ are quadrature components of the center angular frequency ω_c rad/s.

To extract $I(t)$ directly from $f(t)$, i.e.,

$$f(t_p) = I(t_p) \tag{12}$$

requires sampling at times

$$\omega_c t_p = p\pi, \qquad \text{i.e.,} \quad t_p = \frac{p}{2f_c}, \qquad p = 0, \pm, \pm 2, \cdots. \tag{13}$$

Similarly, to obtain $Q(t)$

$$\omega_c t_p' = (2p + 1)\frac{\pi}{2}, \qquad \text{i.e.,} \quad t_p' = \frac{2p+1}{4f_c} \tag{14}$$

whence

$$f(t_p') = -Q(t_p'). \tag{15}$$

The signals $I(t)$ and $Q(t)$ are band limited to the interval $(0, B/2)$ so that each stream requires sampling at a rate B.

Grace and Pitt [3] drew attention to this special case of Kohlenberg's sampling scheme. Rice and Wu [19] applied the special case of uniform sampling combined with the band being integer positioned, which is a form of quadrature sampling in that $k = 1/2B = 1/4f_c + m/2f_c$ where $m = 2/r(= 2$ for integer band positioning). They used the fact that $I(t)$ and $Q(t)$ are a Hilbert transform pair to derive the required interpolation weights used for time aligning the samples. Their innovation was to "decimate by two" the samples before applying the Hilbert transform interpolator. Waters and Jarrett [4] implemented quadrature sampling using the $m = 0$ case and also used decimation before the interpolation filter. Jackson and Matthewson [2] also discussed implementation of quadrature sampling.

The spectrum of a sampled signal is periodic. Whenever a band-pass signal is reproduced at a baseband position by sampling, the noise from all the aliased bands is combined into the baseband. Even with an ideal antialiasing filter, the signal-to-noise ratio (SNR) is not preserved for bandpass sampling owing to (postfilter) thermal noise contributions from the aliased spectra. This has clear implications for communications and radar receivers where bandpass sampling is applied to relocate the signal effectively to a low(er) pass position. This is in contrast to analog complex mixing (i.e., using an image rejecting mixer), in which the signal-to-noise ratio is ideally preserved. The degradation in signal-to-noise ratio does not seem to have been discussed to date, despite its ubiquitous presence in less critical applications, such as sampling oscilloscopes. Section IV describes and quantifies the signal-to-noise ratio degradation, with a simple experiment to demonstrate the effect. Also, the aliasing noise resulting from using typical bandpass filters is presented in terms of a signal-to-(aliasing) distortion ratio. This estimates the effect of applying "acceptable" sampling rates with practical, instead of ideal, rectangular bandpass filters.

II. Uniform Sampling

The conditions for acceptable uniform sampling rates can be written [1]

$$\frac{2f_u}{n} \le f_s \le \frac{2f_L}{n-1} \tag{16}$$

where n is the integer given by

$$1 \le n \le I_g\left[\frac{f_u}{B}\right]. \tag{17}$$

The equality conditions of (16) hold only if the band is confined to the interval (f_L, f_u): a frequency component at $f = f_L$ or $f = f_u$ will be aliased, as discussed in Section I and by Kida and Kuroda [18].

Equations (16) and (17) are depicted graphically in Fig. 4. The sampling frequency, normalized by B, is on the ordinate, and the abscissa represents the band position f_u/B. Fig. 4 is a corrected and extended version of Fig. 3. The areas inside the wedges are the allowed zones for sampling without aliasing. The shaded area represents uniform sampling rates that result in aliasing. The low-pass case $f_s \ge 2f_u$, given by $n = 1$, corresponds to the large wedge to the left of the figure. Each wedge across the figure corresponds to a successive value of n.

The theoretical minimum sampling rate $f_s = 2B$, corresponding to integer band positioning, occurs at the tips of the wedges, and these rates are clearly dangerous points for implementation, since any engineering imperfection will move the sampling rate into the disallowed area. In a sampling system, the allowed sampling rates are represented by a vertical line within the allowed areas and above the band position given on the abscissa.

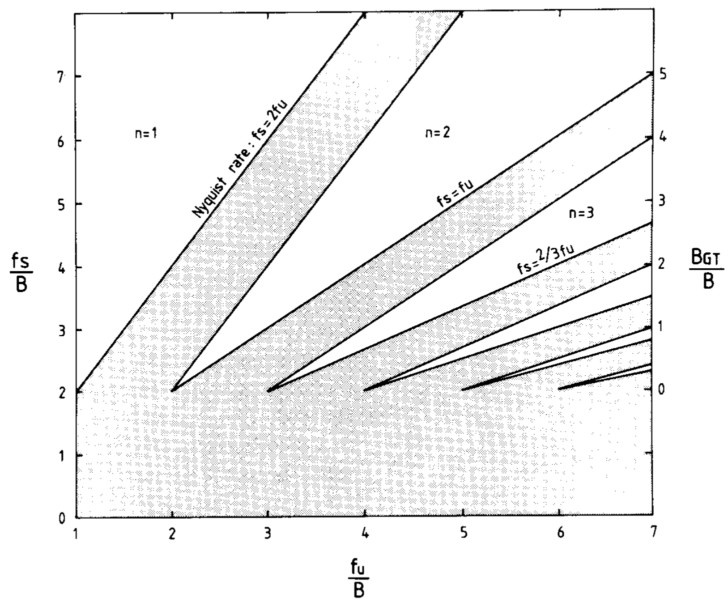

Fig. 4. The allowed and disallowed (shaded areas) uniform sampling rates versus the band position. The figure is an extension of Fig. 3, to show the practical areas of nonminimum sampling rates. The right-hand ordinate is the total guard-band (sum of upper and lower guard-bands). f_s is the sampling rate, B is the bandwidth, and the band is located at (f_L, f_u).

Brown [20] has pointed out that for symmetric double sideband signals, the spectra can "fold over" each other without loss of information, allowing a theoretical minimum sampling rate of $1/B$ for a band-limited signal where the total bandwidth (both sidebands) is B. This case would have an allowed operating "region" in Fig. 4 of straight lines bisecting the disallowed areas between the parallel borders. The case also represents a pathological condition, because any sampling rate variation will not only cause aliasing, but will also cause the signal to become folded incorrectly with a likely loss of otherwise retrievable information.

Sampling at nonminimum rates is equivalent to augmenting the signal band with a guard-band. In fact, any allowed operating point away from the tips of the wedges of Fig. 4 can be interpreted as having a guard-band between positions where the spectrum will be aliased. The total guard-band is given by $B_{GT} = f_s - 2B$ Hz, which is labelled on the right-hand ordinate of Fig. 4.

To find the relation between the guard-bands and the sampling frequency, the order of the wedge n is required. This is found from the overall bandwidth (signal bandwidth plus total guard-band)

$$W = B + B_{GT} \tag{18}$$

and its location (f'_L, f'_u), where

$$f'_L = f_L - B_{GL} \tag{19}$$

$$f'_u = f_u + B_{Gu} \tag{20}$$

and B_{GL} and B_{Gu} are the lower and upper guard-bands, respectively, with $B_{GT} = B_{GL} + B_{Gu}$. The lower order wedge is then given by

$$n' = I_g\left[\frac{f'_u}{W}\right]. \tag{21}$$

Fig. 5 is an expansion of Fig. 4, showing the nth wedge with the guard-bands and the sampling frequency tolerance.

The allowable range of sampling frequencies is

$$\Delta f_s = \frac{2f'_L}{n' - 1} - \frac{2f'_u}{n'} \tag{22}$$

and this range is divided into values above and below a nominal operating point (see Fig. 5)

$$\Delta f_s = \Delta f_{su} + \Delta f_{sL} \tag{23}$$

giving the lower and upper guard-bands as

$$B_{GL} = \Delta f_{su} \frac{n' - 1}{2} \tag{24}$$

$$B_{Gu} = \Delta f_{sL} \frac{n'}{2}. \tag{25}$$

Note that symmetric guard-bands imply asymmetric sampling frequency tolerances. However, the asymmetry decreases with increasing n.

If the operating point is the vertical midpoint of the wedge, then the sampling rate is

$$f_s = \frac{1}{2}\left(\frac{2f'_u}{n'} + \frac{2f'_L}{n' - 1}\right) \tag{26}$$

and the guard-bands become

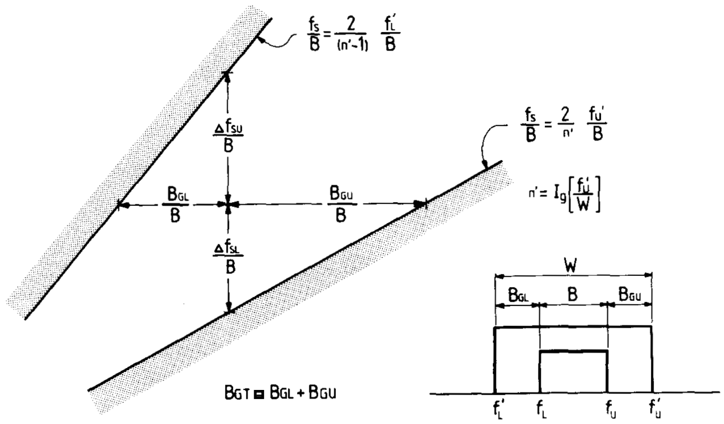

Fig. 5. Parameters in the nth wedge from Fig. 4. B_{GL} and B_{Gu} represent guard-bands against aliasing of the signal occupying bandwidth B, and Δfs_u and Δfs_L represent the allowable upward and downward range of the sampling frequency from its nominal value.

$$B_{\mathrm{GL}} = \Delta f_s \frac{n' - 1}{4} \tag{27}$$

$$B_{\mathrm{Gu}} = \Delta f_s \frac{n'}{4}. \tag{28}$$

An example illustrates the use of the equations. If a 25-kHz signal is allocated symmetric 2.5-kHz guard-bands so that the overall band is (10.7, 10.703 MHz), then $n' = I_g[10703/30] = 356$ and the sampling rate has limits of $2(10703)/356 = 60.130$ kHz to $2(10700)/355 = 60.282$ kHz, i.e., an allowable range of $\Delta f_s = 152$ Hz. The theoretical minimum sampling rate (for the signal without guard-bands), corresponding to a zero allowable sampling rate range, is on the lower edge of the nth wedge, where $n = I_g[10702.5/25] = 428$, and is $f_s^{(\min)} = 2(10702.5)/428 = 50.017$ kHz.

Note, for this example, that in moving from the theoretical minimum rate to a rate which includes the guard-bands, a total of $428 - 356 = 72$ separate ranges of disallowed sampling rates have been traversed. This illustrates the pitfall of arbitrarily increasing the sampling rate from its theoretical minimum value for band-pass sampling.

The relative precision of the minimum sampling rate required to avoid aliasing clearly increases with the increasing separation of the band-pass signal and the origin, i.e., increasing n. If an operating point is within the nth wedge, above (i.e., infinitesimally to the left of) the tip of the $(n + 1)$th wedge, then the difference between the minimum and maximum allowed sampling rates is

$$\Delta f_s = \frac{2(f_u - B)}{n - 1} - \frac{2 f_u}{n} \tag{29}$$

so that the relative precision required of f_s is

$$\frac{\Delta f_s}{2B} = \frac{1}{n(n-1)} \left(\frac{f_u}{B} - n\right) \approx 0\left(\frac{1}{n^2}\right) \tag{30}$$

i.e., related to the inverse square of the separation of the band from the origin. (For the above-mentioned points, $f_u/B - n = 1$.) For the example with $n = 428$, the relative frequency precision required of the sampling rate is $1/(428 * 427) \approx 5$ ppm. This sensitivity corresponds to the vertical locus within the 428th wedge, above the tip of the 429th wedge. Moving to lower order wedges (smaller n), while remaining above the tip of the 428th wedge (same f_u/B) will relax the sampling oscillator precision requirements and increase the guard-band, but of course requires higher sampling rates.

The relative precision required of the sampling oscillator is depicted in Fig. 6 as the line $1/(n(n - 1))$. This line gives the relative precision required of the sampling rate when operating with $f_u/B - n = 1$ as a function of n. The positions of a typical crystal and RC oscillator are indicated to give a feel for practical requirements. With the inclusion of larger guard-bands by operating in the various nth wedges, the line interpolating the various $\Delta f_s/2B = 1/n'(n' - 1)(f_u/B - n')$ runs above the $f_u/B - n = 1$ line. For example, for $n' = n + 10$, the line would be an order of magnitude in $\Delta f_s/2B$ higher than and parallel to the $f_u B - n = 1$ line in Fig. 6. Varying an operating point within a given wedge corresponds to a positive slope locus in Fig. 6 which steepens as f_u/B decreases. After intersecting the $f_u/B - n = 1$ line, the locus approaches a vertical line at $f_u/B = n$ as Δf_s approaches zero at the tip of the wedge.

The $f_u/B - n = 1$ line represents operating with the minimum allowed set of sampling rates and the maximum available guard-band. The guard-band size decreases with increasing n, and the line is $\sim 0(1/n^2)$. If the relative guard-band is now held at a constant size, then the relative precision of the sampling rate becomes, from (24) and (25), $\Delta f_s/2B \approx B_{\mathrm{GT}}/B \cdot 1/n'$. This demonstrates the tradeoff between the precision required of the sampling rate and the guard-band size. For practical situations, a modest increase in the sampling rate permits a

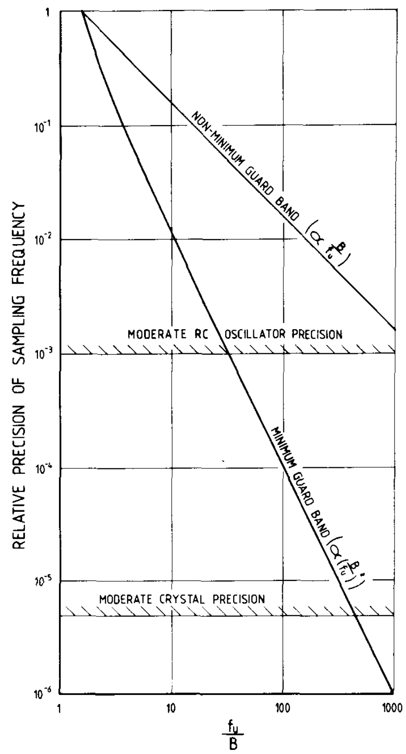

Fig. 6. The relative precision of the set of minimum allowed uniform sampling rates. The line is an interpolation of the operating points corresponding to $f_u/B - n = 1$. In terms of Fig. 4, these operating points are within the nth wedge at a position above the tip of the $(n + 1)$th wedge.

realizable precision requirement, as shown in the example above.

In addressing sampling rate variation, it must be borne in mind that so far the discussion has been limited to avoiding aliasing in the sampling process. Exact (re)construction of the bandpass analog waveform requires exact implementation of the sampling parameters used in the interpolant computation, a point addressed within the following section.

III. Interpolation in Second-Order Sampling

Kohlenberg's second-order sampling result gives a theoretical minimum *average* sampling rate which would correspond to the straight line $f_s = 2B$ on Fig. 4. The interest in second-order sampling has been not only from the theoretical viewpoint of maintaining a constant minimum average sampling rate independent of the band position, but also from its practical application in the simplified form of quadrature sampling a bandpass signal.

This section concerns aspects of bandpass signal construction using various k; specifically, the required range (number of bits in the coefficients) and length (number of coefficients) for realizing the interpolant as a finite impulse response filter, and the sampling rate sensitivity. The application is the generation of a band-pass signal from second-order samples.

For second-order sampling, the spectra resulting from each of the two interleaved sampling sequences applied to the bandpass signal are [21]

$$F_A(f) = F(f) * \sum_p B\,\delta(f - pB) \tag{31}$$

$$F_B(f) = F(f) * \sum_p Be^{-j2\pi Bkp}\,\delta(f - pB) \tag{32}$$

where $F(f)$ is the analog spectrum, $*$ denotes convolution, and p is a spectral position index. From here on, the explicit frequency dependence of F_A and F_B is dropped. Equations (31) and (32) show that the sampled spectrum is a series of analog spectra located B Hz apart and scaled by B. For F_B, there is a progressive phase change of $2\pi Bk$ imposed on $F(f)$ at successive spectral locations, caused by the sampling stream being delayed by k seconds.

Fig. 7, which is an elaboration of Linden's [21] Fig. 3, illustrates the geometric aliasing relations. There are two regions in the band in which the aliasing arises from different aliases of the negative frequencies from the analog spectrum. These regions correspond to where the $S_0(t)$ and $S_1(t)$ terms of the interpolant (see (2)) are applied, and are

$$R_0: \quad f_L \le f \le rB - f_L \tag{33}$$

and

$$R_1: \quad rB - f_L \le f \le f_L + B. \tag{34}$$

Note that from the choice of definition for r, R_0 cannot encompass the band whereas R_1 can encompass the band, whence $S(t) = S_1(t)$, i.e., integer or half-integer band positioning.

The spectra resulting from the two sampling sequences are

$$R_0: \quad F_A = F(f) + F(-f_0') \tag{35}$$

$$F_B = F(f) + e^{j2\theta_0}F(-f_0') \tag{36}$$

and

$$R_1: \quad F_A = F(f) + F(-f_1') \tag{37}$$

$$F_B = F(f) + e^{-j2\theta_1}F(-f_1') \tag{38}$$

where

$$R_0: \quad \theta_0 = \pi Bkr \tag{39}$$

$$R_1: \quad \theta_1 = \pi Bk(r + 1) \tag{40}$$

and

$$R_0: \quad f_0' = f + 2f_L - rB \tag{41}$$

$$R_1: \quad f_1' = f + 2f_L - (r + 1)B. \tag{42}$$

The scaling factor B in (31) and (32) is omitted in (35)–(38) and hereon.

The aliased contribution (the term containing $F(-f')$) is the same for F_A and F_B, except for a phase shift which is constant throughout each region. The phase shifts are, however, different for each of the R_0 and each of the R_1 in the aliased spectra.

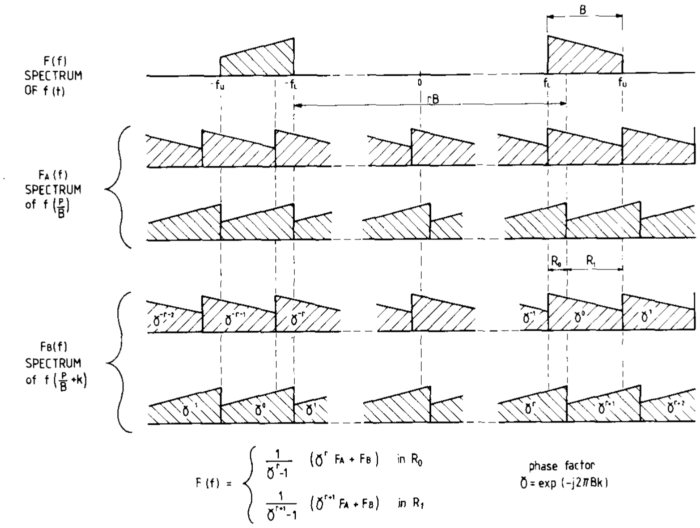

Fig. 7. The geometric relations for aliasing in second-order bandpass sampling and general band position. $F(f)$ is the analog spectrum, and $F_A(f)$ and $F_B(f)$ are the spectra from each of the two interleaved (by k s) uniform sample sequences. The aliased bands are progressively phase shifted relative to the analog band as indicated by the notation $\gamma^r = e^{-j2\theta_0}$. For integer and half-integer band positioning, the region R_0 becomes zero.

The bandpass signal construction is (cf. (1))

$$R_0: \quad F(f) = \frac{1}{e^{-j2\theta_0} - 1} (e^{-j2\theta_0} F_A - F_B) \tag{43}$$

$$R_1: \quad F(f) = \frac{1}{e^{-j2\theta_1} - 1} (e^{j2\theta_1} F_A - F_B) \tag{44}$$

so the disallowed values for k are

$$R_0: \quad k = \frac{m}{rB}; \quad m = 0, \pm 1, \pm 2, \cdots \tag{45}$$

$$R_1: \quad k = \frac{m}{(r+1)B}; \quad m = 0, \pm 1, \pm 2, \cdots \tag{46}$$

which are unique except when $k = 0, 1/B, 2/B, \cdots$; whence the sampling sequences coincide.

In Fig. 7, the phase factor notation

$$\gamma^r = e^{-j2\pi Bkr} = e^{-j2\theta_0} \tag{47}$$

is used for brevity. Considerable insight into the interpolant can be gained by expressing it in terms of the frequencies in Fig. 7. Using the angular frequencies $\omega_u = 2\pi f_u$, $\omega_L = 2\pi f_L$, $\omega_B = 2\pi B$ and $\omega_i = 2\pi(rB - f_L)$ which is the common border of R_0 and R_1 in the analog spectrum, and noting that R_0: $\arg(\gamma^r) = -2\theta_0$, R_1: $\arg(\gamma^{r+1}) = -2\theta_1$:

$$S_0(t) = \frac{\cos(\omega_i t - \theta_0) - \cos(\omega_L t - \theta_0)}{\omega_B t \sin\theta_0} \tag{48}$$

$$S_1(t) = \frac{-\cos(\omega_i t - \theta_1) + \cos(\omega_u t - \theta_1)}{\omega_B t \sin\theta_1} \tag{49}$$

which shows the composition of the interpolant in the same form as the low-pass case of (8).

As already noted for integer and half-integer band positioning, $2f_L/B = r$ and R_0 becomes zero. Further simplification in the signal construction occurs if $e^{-j2\theta_1} = -1$, i.e., for quadrature sampling, in which case

$$F(f) = \tfrac{1}{2}(F_A + F_B). \tag{50}$$

This occurs when

$$j2\pi kB(r+1) = \pm j(2m+1)\pi \tag{51}$$

i.e.,

$$k = \frac{2m+1}{2B(r+1)} \tag{52}$$

$$= \frac{1}{4f_c} + \frac{m}{2f_c} \tag{53}$$

since $f_c = (r + 1/2)B$, so that these values of k indeed correspond to quadrature samples.

As k approaches its disallowed values, construction of the bandpass signal becomes more difficult to implement since the interpolant becomes larger. Both the dynamic range (number of bits required for the filter coefficients) and the length (number of filter taps) required for the interpolant contribute to the cost of an implementation. The general interpolant form (2) predominantly resembles a modulated (by f_c) sinc function, so the dynamic range and the length are related, although the relation is not simple. The dynamic range is a minimum for quadrature sampling when the interpolant assumes its maximum value at $t = 0$, as evident in (10). In this case, the filter length is also clearly defined for a given coefficient size as long as $f_c >> B$, i.e., for the narrow-band situation. When k deviates from the values corresponding to quadrature sampling, both the dynamic range and the length of the interpolant increase.

The low-pass case illustrates the increase in interpolant length with a poor choice of k. Assuming integer band positioning, the low-pass interpolant can be written, from (8), in the form

$$S_{LP}(t) = -\cot \pi Bk \frac{\sin^2 \pi Bt}{\pi Bt} + \frac{\sin 2\pi Bt}{2\pi Bt}. \quad (54)$$

The first term decays as $1/\pi Bt$ whereas the second term decays as $1/2\pi Bt$. For a given dynamic range and coefficient word size, the magnitude of the first term should be minimized, or at least kept small with respect to the second term, for minimum filter length. Consequently, uniform sampling where $k = 1/2B$ (corresponding to quadrature sampling since $f_c = B/2$), is optimum in this case. For baseband interpolation of a sampled bandpass signal, configuring the system such that the signal is integer positioned and applying uniform sampling is the simplest arrangement. In an implementation, it should be borne in mind that half-integer band positions alias to be spectrally inverted at the low-pass position.

In a nonuniformly sampled low-pass system, the alternative to applying the interpolant of (54) directly is to generate uniform samples and interpolate with sinc $2\pi Bt$. The required resampling is a shift of one of the sample sequences using the sinc $2\pi Bt$ interpolant on alternate samples.

Further expense in the implementation of bandpass signal construction may arise because of the need to include "guard bits" in the sampled signal and the interpolant coefficients. This is because errors in the measurement or generation process of F_A and F_B, such as those due to quantization, may in turn create errors in the reconstructed signal. The effect of the choice of k on the resulting uncertainty in the constructed signal is of interest, and is estimated here using a sensitivity analysis in the frequency domain. The specific aim is to get an idea of the cost of a poor choice of k.

Progress can be made by assuming that the variations of corresponding (same analog frequency) components in F_A and F_B are uncorrelated. The variance of a component in F can then be written in terms of its magnitude:

$$\sigma_F^2 = \left|\frac{\partial F}{\partial F_A}\right|^2 \sigma_{F_A}^2 + \left|\frac{\partial F}{\partial F_B}\right|^2 \sigma_{F_B}^2 \quad (55)$$

where $\sigma_{F_A}^2$ and $\sigma_{F_B}^2$ are the variances of the components of F_A and F_B. For correlated variations, caused for example by k becoming very small, σ_F^2 will increase, so (55) represents a best case. The units of σ_F^2 are watts/hertz and the noise power in the constructed signal is found by integrating over the band of F.

Letting $\sigma^2 = \sigma_{F_A}^2 = \sigma_{F_B}^2$, and using (43) and (44) in (55) and integrating, the noise power in F is

$$\sigma_F^2 2B = \frac{\sigma^2 B}{2}\left(\frac{\Delta r}{\sin^2 \theta_0} + \frac{1 - \Delta r}{\sin^2 \theta_1}\right) \quad (56)$$

where

$$\Delta r = r - \frac{2f_L}{B} \quad (57)$$

denotes the extent of the R_0 portion of the band and $0 \leq \Delta r < 1$. The minimum noise power is $\sigma^2 B/2$, and from the choice of definition of r, occurs for $\Delta r = 0$ and $\theta_1 = \pi/4 + m\pi$, $m = 0, \pm 1, \pm 2, \cdots$; i.e., for integer or half-integer band positioning and quadrature sampling.

For general values of Δr and θ_0, θ_1; the noise power relative to its minimum value can be expressed as the number of bits of resolution irrecoverably lost in calculating F; viz.,

$$N_b = \log_2 \left(\frac{\sigma_F^2 2B}{\sigma^2 B/2}\right)^{1/2} \quad (58)$$

$$= \log_2 \left(\frac{\Delta r}{\sin^2 \theta_0} + \frac{1 - \Delta r}{\sin^2 \theta_1}\right)^{1/2}. \quad (59)$$

In terms of an implementation, (59) indicates for poor choices of k, how the errors in the sampled data magnify during the construction of an analog bandpass signal. N_b is plotted against k normalized by B in Fig. 8(a) for the example where $r = 9$ and $\Delta r = 0$ (half-integer band positioning); and in Fig. 8(b) for $r = 10$ and $\Delta r = 0.5$. The functions are symmetric about $k = 0$ and $k = 1/2B$. The asymptotes correspond to the disallowed values if k; viz., $k = m/10B$, $m = 0$ to 10, for Fig. 8(a); and both $k = m/10B$ and $k = m/11B$ for Fig. 8(b). For the half-integer band positioning case, the quadrature sampling values

$$k = \frac{1}{4\left(\frac{9+1}{2}\right)B} + \frac{m}{2\left(\frac{9+1}{2}\right)B} = \frac{1}{20B}, \frac{3}{20B} \cdots;$$

are located at the function minima of Fig. 8(a). The uniform sampling case is given by $k = 1/2B$ and, as already noted, it is only possible to apply this in the special case of integer band positioning (r even and $\Delta r = 0$) and is clearly not viable (i.e., $k = 1/2B$ is a disallowed value) in Fig. 8(a) and (b). The interpolant is simplified and is the same for each of the quadrature sampling values (*NB*: (10) is independent of k). This is evident in Fig. 8(a) where any choice of quadrature sampling gives the optimum k with respect to sensitivity of the signal construction to uncertainty in the sampled data. Since the quadrature sampling locations are at the minima, small perturbations (relative to $1/r$) of k from the quadrature values do not result in a large penalty in the uncertainty caused by constructing an analog bandpass signal. This is particularly clear for the low-pass case, where there is only one quadrature sampling location which is at $k = 1/2B$.

In Fig. 8(b), the quadrature sampling points are at $k = 1/21B, 3/21B, \cdots$; and these do not occur at the function minima. For constructing a bandpass signal whose band is not integer or half-integer positioned, it is unwise to use an arbitrary choice of quadrature sampling since the construction process could well give rise to an error

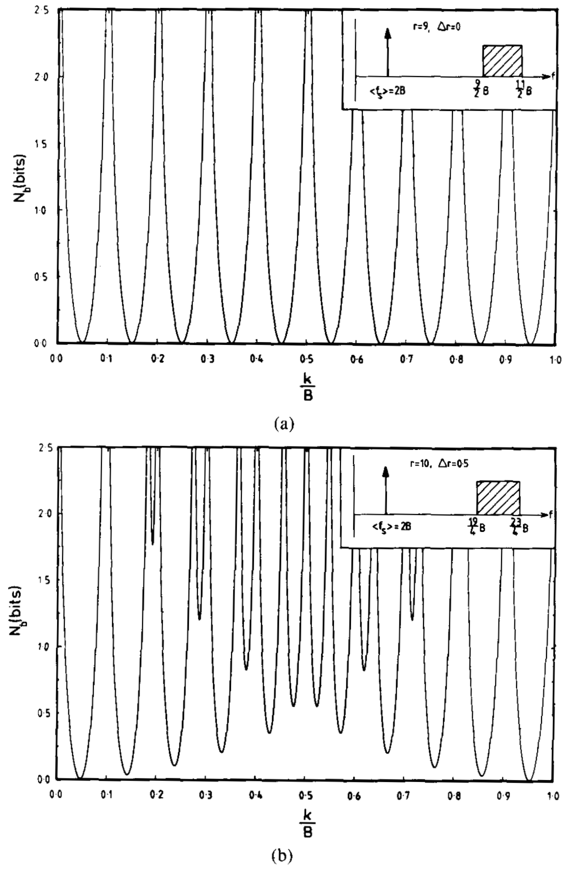

Fig. 8. An estimate of the equivalent number of bits of resolution irrecoverably lost in interpolating a bandpass signal using uncertain baseband rate samples, as a function of k. The graphs represent a cost of a poor choice of k in the second-order bandpass sampling process. The average sample rate is $\langle f_s \rangle = 2B$. (a) $r = 9$, $\Delta r = 0$ (half-integer band positioning). The minima correspond to cases of quadrature sampling. (b) $r = 10$, $\Delta r = 0.5$ (neither integer nor half-integer band positioning). The minima do not, in general, correspond to quadrature sampling.

in the signal. However, the first quadrature sampling value is close to the first minimum of the function, which is the global minimum. In this case of general band positioning, optimum quadrature sampling requires the minimum k, i.e., $k = 1/4f_c$. The best general value of k occurs at the first function minimum (or the last minimum, which amounts to the same sampling sequence interleaving), and so the configuration of the optimum general sampling system requires finding this minimum. This disadvantage, as well as the need here for a complicated interpolant, make it worthwhile arranging the band to be integer or half-integer positioned.

The accurate construction of the bandpass signal from its samples also depends on the accuracy of the sampling rate and k. If the sample sequences are interleaved by k_a and the interpolant implementation features $k = k_a + \Delta k$, then the constructed spectrum of interest $\hat{F}(f)$ and its alias $\hat{F}(-f')$ in terms of the wanted spectrum is, expressed here for R_1 only,

$$\begin{bmatrix} \hat{F}(f) \\ \hat{F}(-f_1') \end{bmatrix} = \frac{1}{e^{-j2\theta_1} - 1} \begin{bmatrix} e^{-j2\theta_1} & -1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & e^{-j2\theta_{1a}} \end{bmatrix} \begin{bmatrix} F(f) \\ F(-f_i') \end{bmatrix} \tag{60}$$

where $\theta_{1a} = -\pi k_a(r + 1)B$. For small Δk

$$\hat{F}(f) \approx F(f) + \frac{\pi(r + 1)B\Delta k}{e^{j\theta_1} \sin 2\theta_1} F(-f_1') \tag{61}$$

which approximates how the degree to which the alias is suppressed depends on the difference of k and k_a. The relative difference between the constructed spectrum and the wanted spectrum can be expressed as

$$\Delta_F = \left| \frac{\hat{F}(f) - F(f)}{F(f)} \right| \approx \pi B(r + 1)\Delta k \tag{62}$$

a result which is also readily obtained from time domain considerations.

As r increases, the timing error Δk becomes increasingly critical. For large values of r pertinent to communications or radar, etc., the demands on Δk become considerable. For the previous example (in Section II, using guard-bands) with $r = 356$ and $B = 30$ kHz, and $f_L = 10.7$ MHz, and an alias suppression of 40 dB, the required accuracy of the sample sequence interleaving is

$$\Delta k \approx \frac{1}{357} \cdot \frac{10^{-2}}{\pi 30(10)^3}$$

i.e., $\Delta k \sim 0.3$ ns. For lower intermediate frequencies in communications, such as 455 kHz, the requirement is correspondingly simpler, viz., $\Delta k \sim 7$ ns ($r = 15$).

IV. Noise in Bandpass Sampled Signals

Noise considerations are important not only where the signal-to-noise ratio is important, such as in communication receivers, but also in the measurement of noise itself, such as in noise power measurements. In a sampling system, the periodicity of the spectrum means that wideband noise, such as the thermal noise introduced by the relevant hardware, is all combined into each of the $fs/2$ bands.

In applying bandpass sampling to relocate a bandpass signal to a low-pass position, the resulting signal-to-noise ratio will be poorer than that from an equivalent analog system (i.e., an ideal image-rejecting mixer), in which the signal-to-noise ratio is preserved.

Consider a system with a bandpass signal of spectral power density S, in-band noise power density of N_p, and out-of-band noise power density N_o. The analog signal-to-noise ratio is thus S/N_p. The signal-to-noise ratio for the sampled signal becomes degraded by at least the noise aliased from the bands between dc and the passband, and is thus

$$\text{SNR}_s \approx \frac{S}{N_p + (n - 1)N_o}. \tag{63}$$

Often, $N_p >> N_o$ and the signal-to-noise ratio is established before sampling. However, if $N_p = N_o$, and assuming $n >> 1$, then the degradation of the signal-to-noise ratio in decibels is simply

$$D_{\mathrm{SNR}} \approx 10 \log n. \tag{64}$$

Equivalently, the effective noise temperature is increased by at least n. Note that this analysis assumes a flat analog noise spectrum, ideal filters, and an infinitesimally small sampling aperture.

For the example with $B = 30$ kHz located at the integer position $n = r/2 = 30(f_u \sim 455$ kHz), or $n = 356(f_u \sim 10.7$ MHz), then the increase in the noise power is at least 15 and 25 dB, respectively.

In the case where the noise spectrum is not uniform, a conservative estimate of the noise power density in each of the bands in the sampled spectrum can be obtained by equating the noise power before and after sampling. Denoting N_{EA} as the equivalent noise spectral density (ENSD) and B_{EA} as the equivalent noise bandwidth (ENBW) of the analog signal; and N_{ES} as the ENSD of the sampled signal with $2B$ the sampling rate (it is known [22] that the ENBW of a sampled system cannot exceed B), then

$$N_{\mathrm{EA}} B_{\mathrm{EA}} = N_{\mathrm{ES}} B. \tag{65}$$

The degradation in a signal-to-noise ratio is $N_{\mathrm{ES}}/N_{\mathrm{EA}}$ which, in decibels, is then at least

$$D_{\mathrm{SNR}} \approx 10 \log \left(\frac{B_{\mathrm{EA}}}{B}\right). \tag{66}$$

A simple experiment using a noise source and a sampling oscilloscope demonstrates the effect. The test signal was a 10.95-kHz carrier amplitude-modulated by a 200-Hz sine wave. A Bruel and Kjaer 0–20 kHz noise source signal was added to the test signal. The resultant signal was sampled at 125 kHz, windowed using a minimum 3-term Blackman–Harris function, and the spectra calculated by FFT on a LeCroy oscilloscope. The average of 10 power spectra is shown in Fig. 9(a). The signal-to-noise ratio is about 38 dB. The signal was then sampled at 1250 Hz giving a Nyquist bandwidth of 625 Hz, and $n = 20\,000/625 \approx 32$. The average spectra is shown in Fig. 9(b), with the signal-to-noise ratio now being ~ 21 dB. The observed degradation is thus about 17 dB.

The minimum degradation given by (64) is 10 log 32, i.e., about 15 dB. Recall that this minimum is given by considering only the aliased noise from the noise source, and assumes that B_{EA} is 20 kHz, while in practice it is a little greater.

In general, the potential for signal-to-noise degradation in bandpass sampling systems is considerable, even when the analog spectrum contains only thermal noise. In most communications systems, for example, the out-of-band power (adjacent band signals, etc.) has similar, or even higher levels, than that in the band of interest, so an antialiasing filter is necessary.

To get a feel for the filter requirements, the signal distortion ratio (SDR) [24] is a useful metric. It is the ratio of the mean-square aliasing error and the total signal power passed through the filter and can be interpreted as a signal-to-noise ratio. It is here defined for the bandpass case [23] as

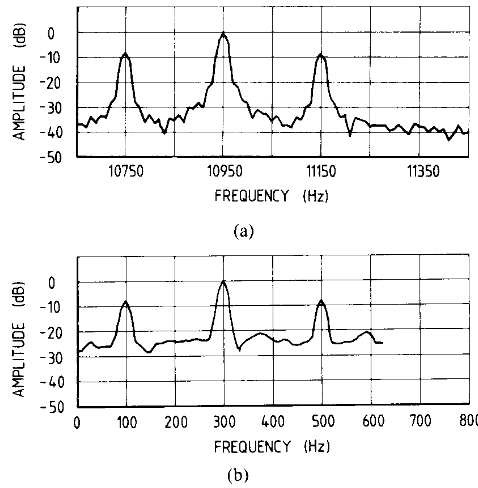

Fig. 9. Averaged sampled spectra of a 10.9-kHz carrier amplitude modulated by a 200-Hz sinusoid in the presence of white noise. The sampling rate is (a) 125 kHz, giving a sampled spectra SNR of about 38 dB, and (b) 1250 Hz, giving a sampled spectra SNR of about 21 dB.

$$\mathrm{SDR} = \frac{\int_0^{\infty} |H(\omega)|^2 \, d\omega}{\int_0^{2\pi f_c - (\pi/2) f_s} |H(\omega)|^2 \, d\omega + \int_{2\pi f_c + (\pi/2) f_s}^{\infty} |H(\omega)|^2 \, d\omega} \tag{67}$$

where $H(\omega)$ is the filter transfer function and f_c is the arithmetic center frequency of the upper and lower 3-dB points. The limits are chosen so that the wanted signal band occupies a Nyquist bandwidth. In practice, this requires selecting f_s such that the band is integer or half-integer positioned.

This signal distortion ratio offers only a rough guide for trading off the filter requirements against sampling rate and is only one of many factors contributing to the effective signal-to-noise ratio [23]. This is because of the assumption in (67) that the signal power at the filter input is constant across the spectrum. In practice, the presence of guard-bands will normally ensure this will not be the case at the edges of the filter. On the other hand, adjacent channel power levels may be tens of decibels above the power level in the channel of interest. In radio communications cases, the filter $H(\omega)$ not only represents the antialiasing filter, but also the receiver front end, including the antenna. Nevertheless, the antialiasing filter will normally be the dominant effect on the signal distortion ratio.

Fig. 10 [23] shows the signal distortion ratio for two filter types against the sampling rate normalized to the

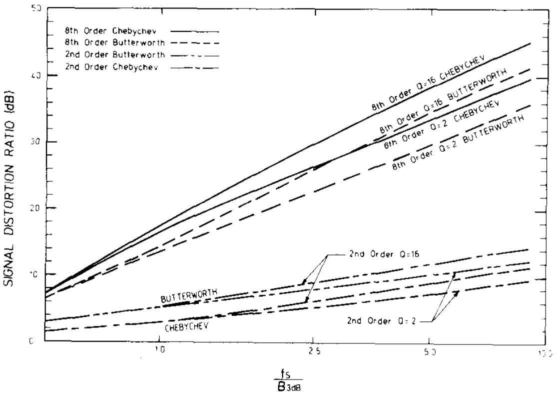

Fig. 10. The signal distortion ratio (SDR) as defined for the bandpass case for second-order and eighth-order Butterworth and Chebychev filters. The envelope edges for each curve represent $Q = 1$ and $Q = 200$ [23].

3-dB bandwidth of the filter $B_{3\,\mathrm{dB}}$. Because of the selection of f_s mentioned above, there are only discrete points on the curves which are valid. The envelopes enclose the curves for filters with Q values from 1 to 200. The parameter $Q = f_c / B_{3\,\mathrm{dB}}$ is related to the band position parameter $r \approx 2f_L/B$ (for integer band positioning) by $Q = \frac{1}{2}(r + 1)$. In the example with a 25-kHz band at 455 kHz, i.e., $r \approx 36$, a signal distortion ratio of 37 dB requires a sampling rate of $f_s/B_{3\,\mathrm{dB}} \approx 2$ (i.e., $f_s \approx 50$ ksamples/s) with a filter equivalent to an eighth-order Chebychev with $Q = 16$.

The definition of the signal distortion ratio in (67) can perhaps be more easily related to a signal power to "distortion power" ratio if the integration in the numerator is over the 3-dB bandwidth only, i.e., the denominator corresponds to the signal power only rather than total power. For such a definition, however, the curves of Fig. 10 change by less than a couple of decibels. Strictly speaking, only interleaved portions of the filter "stop band" contribute to the aliasing distortion, but again, this definition does not appreciably shift the curves of Fig. 10. This matter is discussed in [23]. In applying Fig. 10, it should be borne in mind that the signal distortion ratio is only an approximation to the effective signal-to-noise ratio caused by the aliasing.

V. Conclusion

For practical bandpass sampling, there is a need to sample at above the theoretical minimum rate. Care must be taken when increasing the sampling rate from its minimum value because there are bands of rates for which aliasing will occur. Operating at a nonminimum sampling rate is equivalent to the introduction of guard-bands or a corresponding tolerance to variations in the sampling rate. The relations between the sampling rate tolerance, the guard-band size, and the various allowed nonminimum sampling rates give practical design choices.

For sampling an analog bandpass signal in order to process digitally the signal, the simplest system is to configure the band to be integer positioned and apply uniform (which is also quadrature) sampling.

In the sampling process, the signal becomes degraded owing to aliasing of the noise (introduced after any bandpass antialiasing filter) between dc and at least the original bandpass spectral position. This degradation is unavoidable and a simple estimate for the minimum degradation is available in terms of the band position.

For constructing an analog bandpass signal from its second-order samples, it is wise to configure the system such that the band is integer or half-integer positioned. The advantages are a simplified interpolant and the potential to apply quadrature sampling with the flexibility to use several values for the sequence spacing. Quadrature sampling represents the optimum sample sequence interleaving in the sense that it requires the shortest filter length (least number of taps) for interpolant implementation using a given coefficient word size, and displays the least sensitivity to error propagation in the construction of a bandpass signal from its samples.

For cases where the band is not integer of half-integer positioned, the optimum sampling sequence interleaving for bandpass signal construction is not trivial to find. Here, quadrature sampling spacing should not be applied in general, owing to possible error propagation from imperfect samples, although the minimum quadrature spacing $k = 1/4f_c$ is usually close to the optimum.

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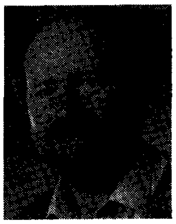

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