ISSCC 2012 Tutorial Transcription
Jitter: Basic and Advanced Concepts, Statistics, and Applications
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1. Introduction

Good morning everybody. This morning I'm going to talk to you about jitter and about some basics and advanced steps, and some applications. So first of all I would like to start with some motivation. Why are we looking about jitter?

2. Motivation: A Complex System

So here what you see in the slide here is a complex system. It contains some ADC on the left hand side here which are sampling some analog data using some clock. This digital data might be processed them by some digital core which is clocked by another clock. And maybe you want this data to be shipped to an external to an off-chip to another processing unit via serializer and a serial transmitter. The data then are sent through some channel which could be a back plain could be a chip-to-chip connection or a long cable, and they are received on the other chip through the receiver. There is a clock in data recovery system which recovers the data starting from the clock starting from the data and samples of flip-flop and transfers the data into the second chip. You can see here that there are several different sub-systems, and every sub-system is more or less clocked by a different clock. And of course since we are talking about ADC, we are talking about digital of course, we are talking about serial interfaces where each of this block has different jitter requirements. We are going to talk today a little bit about jitter requirements of each of these block.

3. Motivation: Jitter in Serial High Speed TX

The second slide I show you from motivation is the following: when you have for instance a serializer, a transmitter, and a channel, jitter shows different phases. You can look here for instance at the output of the transmitter when there is no disturbances applied, you can see an eye diagram which is very fine. The transitions are very sharp and the jitter is really limited. But once you serve for instance on the supply of the serializer of the transmitter inject some disturbances, then you will see that the transitions get blurred - they get expanded. And if you look after the channel when after a long channel then you will see that the eye is completely disturbed and the transition show very complex behavior. We have here for instance some discrete lines you can see here, merged together with some random behavior. So from this simple example you can see already that jitter showed different phases. So and how can we define and quantify this jitter? This will be the topic of today's talk.

4. Overview

Okay this is a small overview of what we are going to talk about. We will start with a definition of absolute jitter. We will talk about the measurements of jitter both in time and frequency domain. We will see the relationship between phase noise and jitter. We will have a look at some typical sub-system electronics and the jitter which is relevant for them. We will then go through another topic which is the so called self-referenced jitter, and then we will talk about jitter statistics for both Gaussian and non-Gaussian distributions. At the end we will have a look at bathtub curves, and we apply them to examples from the high-speed serial data transmission. And as a last topic we will show you something about jitter and spectral spurious tones. So I hope you will enjoy this talk today.

5. Absolute Jitter: Definition

So we'll start with definition of absolute jitter. This is a definition which is very well known and actually accepted in the community. So you have on top your ideal clock which is a clock with a given frequency but without jitter, where each period is exactly the same duration. Of course this ideal clock is not existing. What you have the reality is some real clock where the edges are displaced from the ideal positions. And actually what we will be talking today is about clock and data, serial data. So also regarding data you can do the same definition. You will have the edge of the data is not positioned where you would like it to be but it does some displacement with respect to this ideal clock. So the absolute jitter is nothing else then the difference between the real and the ideal edges of the clocks. So if it is depicted here described by the blue box here it is just this difference.

6. Absolute Jitter: Time Domain Meas

So how can we measure this absolute jitter? Actually we will see there are two ways to measure this absolute jitter: one is the time domain and the second is in the frequency domain. The first we will now deal with a time domain measurement. In time domain measurements you have a device-under-test which is generating your jitter this could be a PLL but it could also be a data transmitter. Well the clock generated by the device-under-test is jitter so you have blurred edges here. So what you do is you take this data or this clock and you feed it to some clock recovery and filter system. This block here has the function of locking to the incoming clock and data and recovering the clock out of it and generating a clock here, which has the same frequency as the original clock but with a very, very low noise content. So the jitter here is very, very low. So what you can do now is to take an oscilloscope and feed your device-under-test clock on the data input and the recovered clock, the very clean recovered clock, on the trigger input. And then what you do with the scope is measuring actually the time difference between the data and the trigger and this is exactly the absolute jitter, we will call it the J_{abs}. The result of this measurement is typically shown in form of histograms. What you can plot with the scope is a histogram of this absolute jitter here.

7. Estimation of RMS from Histogram (1/3)

Now what can we do with this histogram? The first thing we can do with a histogram is try to calculate the RMS value of the jitter. So I plotted here a histogram just for as an example. The RMS, or the 1-sigma value can be calculated by using this formula which is basics formula from statistics. You just take the jitter samples you square them you make the sum divided by the number of samples, take the square root and you get the absolute the RMS value of the absolute jitter. As you can see from this formula you have a parameter here which is N which is the number of hits in your histogram. So it's easy to show that the larger the N, the more accurate is the estimation. Here I show you on the right hand side some simulations which are run based on this distribution and this simulations I use this formula to estimate the RMS value of this distribution here. The distribution here as an RMS value of one, so the real results should be here. On the X axis you see I use here different values of N, there is ten, one hundred, one thousand, ten thousand N's. If you use a very low value of N, a very low value of hits to estimate your RMS you have a very wide spread of your results of your estimated value you can go from 0.4 up to 1.8, whereas the real value is one. The more you increase N, the lesser it gets the spread of these estimation and for, in this case, for N equals to ten thousand you get in any case, a very, very accurate estimation of the RMS. Now the question that we ask our self is how is the relationship between this spread of estimation and the values of N here? Or in other words how many samples do you have to take in order to have a very good accurate estimation of the RMS?

8. Estimation of RMS from Histogram (2/3)

To explain this I will make a short deviation from the topic of jitter and we go into, I will try to explain to you in a few words what are the confidence level and confidence interval in statistics. Let’s mention that you are hired by a coffee company and your first task is to estimate the average size of coffee beans from a farm, okay. The bad thing is that you will have to pay a fine if you are wrong in your estimation. So what you can do here is you take N number of coffee beans, start measuring, averaging, and then at the end let's say you come up with a number which is eleven millimeters okay so you can tell now your company okay now the average value of my coffee beans is eleven millimeter, but since you have to pay a fine and you don't want to pay this fine so you want to research some margin for error. So you say okay I am not really completely sure..it's eleven millimeter plus minus one millimeter, right? So this plus one minus one is an interval in your estimation and this is called in statistics confidence interval. It's a margin for error. Well now since you have some time left you repeat these measurements on N number of coffee beans many times, and you find out that actually in 90% of the cases indeed the average you find is within your eleven millimeter plus/minus one millimeter. So this number 90% of the cases is called confidence level. So your answer to your company should be okay I am 90% confident that the coffee beans average size is 11mm plus/minus one millimeter. It is a very complicated answer but I think it is the only one which allows you not to pay the fine. And now the question is if you increase the number of coffee beans you measure, you can either tighten the confidence interval or for the same confidence interval you can improve the confidence level. Now we are not measuring coffee beans we are measuring jitter but actually it is the same thing.

9. Estimation of RMS from Histogram (3/3)

So our goal is to estimate the RMS with an error margin epsilon. This is our confidence interval and with a given confidence level okay starting from the histogram. Now the question is how large should N be? How many samples should I take? Actually it turns out that the answer to this question is a very, very easy formula, astonishing actually. And this formula is written here so the number of samples you have to take is greater, larger than one half Q over epsilon squared, where epsilon is our error margin and the Q is a factor which determines your confidence level. You can see from the table below if you want to have a confidence level of 99.7% you take Q equals to 3 if you want 95% you can take Q equals 2 and for Q equals to one you have the confidence level of 68%. So based on this formula you can determine how many samples you want you have to take in order to be able to estimate your RMS in a meaningful way. So this axis this plot shows actually this formula, plotted for the three different confidence levels here. So as an example assume you want to have an error margin of 1% - this is this point here. So if you want then the confidence level of 95% you just go to the green line here and you see that the number of samples you have to take is about greater than 20,000. If you want higher confidence level let's say 99.7%, you have to take a number of samples which is about 50,000. So this is the order of magnitude of number of samples that you should take when you have to estimate the RMS of a Gaussian distribution.

10. Absolute Jitter: Freq Domain Meas

So now we analyze up to now how to measure the absolute jitter in the time domain. Now we look at another kind of measure which is in the frequency domain. Okay in the frequency domain you use to measure your jitter, you use a spectrum analyzer. You have your device-under- test which is generating the clock, and the spectrum analyzer perform the following action: it takes actually your incoming clock and mixes it with a clock which is generated by a local PLL here. This is a very simplified version of the spectrum analyzer. The main thing is here, the PLL is generating internally a very clean clock because it is a very low noise generator, local oscillator. And what you are doing here is mixing your device-under-test clock with a local oscillator clock and analyzing the spectrum, the power of the signal which is coming out here. If you compare this picture to the picture that I showed you some slides back

11. Absolute Jitter: Freq Domain Meas

this one, slide number six, you would see that the main structure is actually the same. This one slide number six you will see that the main structure is actually the same.

12. Absolute Jitter: Freq Domain Meas

Also here what the spectrum analyzer is doing is comparing somehow the device-under-test clock with a very clean local generated clock. So also in this case you get information about the absolute jitter - Deviation of the clock, of the real clock, with respect to an ideal one. The result is normally shown in form of a spectrum but the spectrum is the spectrum of the voltage signal and not of the jitter that you would like to have. The spectrum shows looks more or less like this: you have your carrier at f zero, you can have some harmonics at $2f_0$, $3f_0$ and so on, and then all around these harmonics and the basic harmonic you have skirts which are due to the noise on the clock, which are due to the jitter.

13. Voltage Spectrum to Phase PSD

Okay so we now zoom into the first harmonic of this voltage spectrum and we ask our self: okay now I have this voltage spectrum, how can I derive here information about phase? about jitter? So now we assume that the voltage signal is made of a sinusoidal having a constant amplitude A. And it has an ideal let's say nominal frequency let's say omega zero, and it has some noise super imposed to this frequency which I indicate with phi, phi of T. So this omega zero T, this term is responsible for the carrier here and this phi of T is responsible for the noise around the carrier. So now how can we go from the spectrum of this signal to the spectrum of our disturbance of Phi? Okay it turns out that it is very easy that if you take the energy or the power in a one hertz band-width at the frequency F from the carrier take the power in one hertz band-width here and you divide this by the power of the carrier, it turns out that this is equal to the power spectral density of phi which we will indicate as the big S of phi. So this operation is a very simple operation here and allows you to go from the spectrum of the voltage to the spectrum of your phase signal. And it is a very easy operation. This kind of relationship is valued under two assumptions: The first is that we have no amplitude modulation. So all the noise in the skirts is due to phase modulation. And that we have a small angle modulation, this means that the power of the noise is small compared to the power of the carrier. The second thing is that we will always consider the PSD the power spectral density as a single side band. Okay so now with this way we can go from measurements with a spectral analyzer, we can derive the phase, the PSD of the phase.

14. Phase Noise

This PSD of the phase is what is normally called phase noise and is indicated by a big L of F. Normally it's computed automatically by most spectrum analyzer, you have only to have one with a phase noise option, and they aren't shown in a local log scale like you see here where on the X axis you have the off-set from the care frequency and on the Y axis you have the value of the phase noise. Okay now we have the power spectral density of the phase noise how can we go to jitter? which is the topic of today's talk.

15. Phase Noise to Absolute Jitter

Well it is not that difficult. If you there are just a few passages. If you now take this signal here it will be our starting point, and you put out the omega zero from the as a factor here you will see that you get omega zero times T plus phi over omega zero, and you will recognize that this T is your ideal time and this factor, phi over omega zero is actually the deviation of the real clock from the ideal time. So this is what we have been calling up to now the absolute jitter. So you get this very easy relationship between absolute jitter and phase deviation. So now if we take the PSD of both members here, you get on the left hand side the PSD of the jitter is equal to the PSD of the phase error which is what we have called phase noise, divided by the square value of omega zero squared because we are taking the power, right? And now we go through well known theorem in probability theory which is called Weiner-Khinchin Theorem, and it states actually that if you want to have the variance of a given random variable, all the thing you have to do is to integrate the PSD of this variable from minus infinity to plus infinity. So we apply this theorem here now and so that we will find that the variance of the absolute jitter is equal to the integral from minus infinite to plus infinite of the power spectral density of the absolute jitter, and by replacing this term here with the L over omega zero squared, you will find this very easy and very important formula. This tells you that to calculate the variance of absolute jitter, the variance is the RMS value squared. You have just to take the integral from zero to plus infinite of the phase noise multiply it by two. The multiplication by two comes from the fact that we are just integrating now from zero to plus infinite instead of from minus infinite to plus infinite. And then divide what you obtain by omega zero squared. Okay, this a very easy formula. It is very powerful one but it has some problems and the problems are actually to be found in these two integration limits, zero and plus infinity. To understand why this is a problem

16. Phase Noise Integration Limits

We have to look at the origin of the phase noise. The phase noise is calculated taking the spectrum of the signal dividing some energy in the skirts by the carrier and so on. But if you look at the normal spectrum, you will see that there are first of all there are some repetitions of the spectrum due to the harmonics. So if you would integrate from zero off-set from the carrier, to plus infinity, you would take the power also of the next harmonics. So but you are not interested in this power and also you are not interested in the power of the noise in this region here because this belongs to the second harmonic. So the first thing you have to do is to limit the integration of the phase noise to this limit here which is actually a limitation to f zero over two. The second problem is that if you try to integrate from zero off the off-set carrier you will have a problem due to the fact that close to the carrier the noise is normally dominated by components which goes like one over F squared or one F cubed, like flicker noise or similar things. And you know from mathematics that if you try to integrate this kind of function close to zero you get plus infinity, so you would diverge. So one thing you can do here is actually to decide okay I don't start integrating from zero off-set from the carrier, but I start integrating from some minimum frequency okay. This minimum frequency of course has to be decided based on the system which is using this clock. But a better way to do this to overcome these problems is to understand what is the jitter transfer function of the system under investigation. So this clock is going into some system and this system has some sensitivity to clock jitter frequencies. Hopefully your system will not be sensitive to jitter frequencies which are very low. So close to the carrier. So if you are able to find the jitter transfer function of your system, the best thing you can do then is to multiply the phase noise by the square of the transfer function of the system and then integrate from zero to plus infinite. I will show now how to apply this concept here to an application taken from

17. Jitter Filtering in CDR Systems (1/3)

wire line, and this is an easy CDR clock and data recovery systems. You have data coming in here, you have a CDR a clock data recovery which receivers the clock from the data and you use a flip-flop here clocked by this clock to latch the data in right. So this is more or less the picture that you have in front of you. You have the data where the edge some jitter here this is the recovered clock and you sample with this edge this data theoretically in the middle of the eye right. So what happens if the jitter here is moving in this direction, closer and closer to the edge then you will get errors right. So the same thing is valid if the clock is moving in this direction. So actually your system which is determined is sensitive to so the metrics of the system is the bit error rate that you get out of this sampling process, is sensitive to the difference between the two phases: the phase of the clock and the phase of the data. Now you can imagine now to have a very slow jitter modulation on the data, the CDR is normally a low pass filter. It has a low pass characteristic so it will follow this small variation so the phase of the clock will follow the phase of the data and the flip-flop will not see any difference. But if the data has a very high jitter component, very high frequency/jitter component, this frequency will not be passed through the CDR so the phase of the clock will almost stay constant, and you will get the whole jitter here on the data input and this will impact your bit error rate. So you can see from this simple example that such a system is sensitive to high frequency jitter - this hurts your bit-error rate but low frequency jitter doesn't hurt.

18. Jitter Filtering in CDR Systems (2/3)

So to get more quantities on this analysis we can do the following: we can model the CDR as a linear system which has a given transfer function, H CDR so that the phase of the output clock is H CDR times the phase of the data. Now the phase of the error is the difference between the phase of the data and the phase of the clock. So plug this into the phase of the clock you see that you have one minus H CDR. So your system that the transfer function of your system is the transfer function from the phase of the data to the phase of the error, and it is one minus the transfer function of the CDR itself. So now normally the CDR is low pass system, you see here the bode diagram of this CDR with the black solid line. And you see that one minus HCDR is a transfer function which is actually a high pass, so this means that your low frequency jitter components are attenuated are not hurting your system.

19. Jitter Filtering in CDR systems (3/3)

So now if you now apply in all this to what we have seen before the thing we have to do is follow the blue line describes your phase noise on the data. I assumed here for very low frequency off-sets that the phase noise is increasing with one over F squared right. Now what you do is you multiply this, you filter this phase noise with your transfer function, which is the red light here, and the result you get is the black line. So the black line is the phase noise which your system sees actually. And now you see that the increasing energy at low frequency is damped by the system itself so that you will not have any issue in integrating the black line, the resulting line from zero let's say, zero carrier to plus infinite. Because in this case the integrate will be final, will have a finite value. So this in my opinion is the best way you can characterize jitter for a system. Understand what is the transfer function of your system, multiplied with a phase noise and integrate, and then you will get the real value of the jitter which your system sees.

20. Summary So Far

Okay we’ll make a summary of what we have seen so far. What we have looked at is definition of absolute jitter we have looked at measurements in time domain. We analyze how to estimate the RMS sigma from the histogram. We looked also measurements in frequency domain, we had to look what is phase noise and what is the relationship between the phase noise and the absolute jitter right. And we have seen okay we have a problem with the integral of the phase noise close to zero and to infinite so we have to do some work around here. So the next step is to analyze some typical applications in electronics, and to find out which type of jitter are they sensitive to. And we will see that for some of them we will have to introduce new types of jitter.

21. Application Scope of Absolute Jitter

Okay what is the application scope of the absolute jitter? We have seen already from the example before that for instance for analyzing receivers in high speed interfaces absolute jitter is the right thing to do we have seen in the example before. We will shortly see that also absolute jitter is the relevant type of jitter you have to use when analyzing analog data converters. Analog-to-digital data converters but also digital to analog-to-data converters. But we will see that absolute jitter is not the right metrics to use when you are analyzing digital clocking. Before going to digital clocking I will spend two slides showing you absolute jitter in ADC's.

22. Sampling of Analog Signals (1/2)

Okay in an ADC you have an analog signal which is this solid black line here, and ideally you would like to sample this signal at the given time and instance which are ideal, so very noise free time instance. And then you get some samples here and these samples are converted into digital values and then they are transferred to the digital domain, and in the digital domain you just find a series of numbers there. They might also be stored in a FIFO. So the main thing is that from going from the left hand side to the right hand side, you lose any information about time. There is no time information associated with any of these sample values. So the thing that our eight digital colleagues assume is that these numbers are taken at the ideal time value. But of course you don't have an ideal clock when for clock ADC you have a real one. So actually you will see now that what counts now is actually the difference between the ideal clock which is assumed here and the real one which are using for sampling the analog domain. So as in this case, what is the relevant jitter is the absolute jitter.

23. Sampling of Analog Signals (2/2)

In this slide I will show you in a very simplified manner what is an ADC sensitive to actually. So this is let's say the first stage of an ADC sampling stage. You have a sampling capacitor which is charged by your incoming signal and at the rising edge of the clock you can assume that this switch here is open so that the voltage of the incoming signal is stored on the capacitor and then used for the A-to-D conversion. Now what happens if you have jitter? So if you have jitter so let us assume this now, this black line here is your analog signal coming in you would like to sample your analog signal here at this time point and get this value out. But in reality you have a band here due to the jitter of the clock. Due to this uncertainty and due to the fact that the signal here is some slope, you have an uncertainty also on the sampled value so you have a sampling error. And you will see here that the higher is the slope of the signal here, the higher would be the sampling error at a constant jitter value. So the very few words we can try to calculate out what is the relationship between the SNR and the absolute jitter. So the SNR is actually the power of the signal divided by the power of the error in this case the power of the error is due to the power of the jitter times the slope of the signal squared, so due to this effect here. But the power of the jitter we have seen is the actually the RMS value of the absolute jitter squared. So by putting all of this together you will find that the SNR of the ADC is proportional to one over the RMS value of the absolute jitter squared. If you analyze this in more detail, it doesn't take long to show that for a sinusoidal signal the exact formula is this one. So the SNR is 20 times log ten of one over two pi F_{ANA}, where this is the frequency of the input signal times the RMS value of the absolute jitter.

24. Digital Clocking (1/2)

So what happens now in digital clocking? In digital clocking the situation is different. So this is a typical digital system you have one flip-flop you have a combinatorial logic and you have a second flip-flop here after the combinatorial logic. So both flip-flops are clocked by the same clock it is the same synchronous design. So now in order for this stuff to work well, you have to be sure that when the clock is coming here on the second flip-flop, the data on the Z are already there. This is the whole magic in digital clocking. So let's have a look at what happens here. So when the first rising edge of the clock comes the data are transferred from the X input of the first flip-flop to the Y output. And here you have some delay. This delay is the so-called clock-to-Q delay of the first flip-flop. Now this signal Y is going through the combinational logic. And after a given time, which depends on how complex this logic is the data will appear at the output of the combinational logic which is the input of the second flip-flop. This B delay here is the delay of the logic, right, and in order for the system to work, you have to make sure that the next rising edge of the clock is happening after that the data is present here. So what happens if the period of the clock here has a suddenly is a shorter one? So this means that this edge is moving on the left and if it's moving too much on the left then it will clock the Z output of the logic when the data is not ready. So you see from this simple example that the constraint in this case is the constraint of the minimal attenuation for the clock period from this edge to this edge. We don't have any ideal clock we are just concerned with one clock and the duration of the period of one clock.

25. Digital Clocking (2/2)

So now what happens when we have a configuration like this one? We have the same two flip-flops like before with the logic in between but now the clock is derived from a high frequency clock through a division by N. So in this case you will see that is the same story. This is the highest frequency clock and this is the clock N, and you will see that this digital system is sensitive to the edge - to the duration of the clock period from this edge to this edge. But these edges are derived by division of the high frequency clock. So if you convert this constraint into this constraint for the high frequency clock you will see that what is important here, is the duration, or let's say the jitter of this edge of the Nth edge compared to the jitter of this first edge here. In between you have exactly N edges which is given by the divider factor here. So also in this case, the constraint is set on the minimum duration of N consecutive clock period. From one edge to the next Nth edge. Also in this case we have no ideal clock in between. We are not, they don't care about ideal clock here, the digital guys.

26. Accumulated Jitter (Jacc)

So this brings us to the second definition that we will see today and the definition is the accumulated jitter. The accumulated jitter is the jitter of one edge relative to the Nth previous edge of the same clock. We will call it Jacc of N. So it is a function of the number of edges of cycles you consider. As a special case if you takes only one cycle, so the edge of this for instance the jitter of this edge compared to the jitter of this edge, this is the accumulated jitter for N equals to one and it deserves also special name, this is what it is normally called the period jitter. Some other guys called it also edge-to-edge jitter.

27. Jacc: Time Domain Measurement (1/2)

So how can we measure accumulated jitter? It is very easy what you have to do here is take your device-under-test feed the clock on both the data and the trigger of an oscilloscope. Then what you have to do is trigger the oscilloscope on one edge of the incoming clock and look at the jitters on the following edges. You can then derive an histogram for each of the following edges and you can apply here for instance all the things we have seen before regarding RMS estimation from the histogram. As you can see here we don't have any ideal clock here in the picture and this is why this is called also a self-reference measurement. And accumulated jitter is also sometimes called self-referenced jitter. So what you can see here is that accumulated jitter is a function of the number of cycles you consider.

28. Jacc: Time Domain Measurement (2/2)

So then what you can do is plot the accumulated jitter for instance the RMS value that you have derived from the previous measurements on a graph, where on the X axis you have the value of cycles. So the accumulated jitter is then a number, is dependent on a function of the number of cycles. And you will see that in real physical systems actually the accumulated jitter for low values of N increases. It could be then that it saturates somehow but it generally for low values of N it gets increased.

29. Jacc in Freerunning Oscillators (1/3)

Okay so now we will analyze shortly how the accumulated jitter looks like in two kinds of systems. The first one is we will look at accumulating jitter in free-running oscillators. So depicted here is an example of a ring oscillator system with three stages. But actually it doesn't matter what I say would be valid in any kind of oscillator whether you have a ring oscillator or an LC oscillator or a crystal oscillator or the relaxation oscillator. But it is easier to show what I want to show you now using a ring oscillator. So now I imagine the ring oscillator is oscillating okay. At one point one edge is coming out here and fed back to the first stage of the ring oscillator. Each of these stage is noisy because the devices are noisy. So once the edge goes through here it will come out here with some uncertainty - this is shown here by these three colours here. Normally you would like have written in the red position, but due to jitter you have some disturbance here. So this second edge now goes back and starts again from the beginning, goes through the same three stages as before, and gets additional jitter on top of the jitter that it had before. So you go from this kind of picture, if you have this kind of edge, this blue edge going back to the beginning of the stage here. And at the end you will have the same sign of uncertainty here on top of the uncertainty that you have on the originating edge. And so on and so on so. So we will see here that the jitter on the edge number N is actually equal to the jitter that you had on the previous edge plus the previous jitter. So this means that actually the jitter is accumulating over cycles and there is no mechanism which can counteract this accumulation. So the jitter will increase and increase and increase.

30. Jacc in Freerunning Oscillators (2/3)

So how can we quantify this from a mathematical point of view? We can say that they accumulated jitter over N cycles is then equal to the sum of the jitter of the period Jitter of each of these N cycles. Now if we assume that the jitter in different cycles are uncorrelated because the noise of your system is a very high frequency noise, so it is not correlated from one period to the next one. You can find that the variance of the accumulated jitter is just the sum of the variances of the period jitters. And now we make the second assumption that is the jitter is time-independent, so each of these variances has exactly the same value. If you put all together and take the square root you will find that the RMS of the accumulated jitter over N cycles is actually equal to the period jitter of one cycle multiplied by the square root of N. So if you plot this formula on a log-log scale you will see that the accumulated jitter rises steadily with a slope of one half. And there is no mechanism in a free-running oscillator which can limit this grow. The starting point here for N equals two one is the period jitter - this value here.

31. Jacc in Freerunning Oscillators (3/3)

So now I plotted here some example which I found for measurements of different VCOs. I wanted to compare here how the jitter, the accumulate jitter looks like for these different oscillators. We have a ring oscillator for SATA2 transceiver. We have an LC oscillator for a GSM we have MEMS oscillator and we have a crystal oscillator. As you can see here based on this phase noise that these here you can calculate the period jitter - we will show you this in a moment. And I just brought it here what are the values that you obtain using this for typical values that you can obtain for this kind of oscillators so you see that the ring one is the very lousy one is the worst on, the LC oscillator for the GSM is a pretty good one and the MEMS oscillator, micro electrical machine oscillator, is also very good. Of course the best thing you can do is to use a crystal oscillator. But apart from the fact that the period jitter of four kinds of oscillators are different and the same behaviors is valid. So versus the number of N, the jitter will increase steadily. And there is the slope of one half for all of these oscillators.

32. Jacc in PLLs

Now what happens in PLLs? In PLLs the situation is a little bit different. The PLL is a closed system for a moment. Imagine that your VCO is free-running so you have open the loop in front of the VCO so then the VCO is just a normal free-running oscillator and it will start oscillating and accumulate in the jitter like we have seen in the previous slides. So at the beginning you will see that the jitter is increasing but then at some point since the clock is fed back and compared to a clean reference clock, the PLL will realize that there is too much jitter between the two clocks here and try to compensate for this. So then the dynamic of the PLL will kick in and try to control the VCO here in such a way that the jitter at this point is limited. This cannot grow indefinitely because you are comparing the divided clock with a clean reference clock here. So this kind of comparison will limit the jitter accumulation. Indeed you will see that when you are measuring accumulating jitter for PLLs you will find the shape, of such a shape, at the beginning it is increasing then at some point which is related to the timing constant of the loop, then the jitter will saturate and will not increase any longer - it will stay flat.

33. Accumulated vs. Absolute Jitter

Okay now we have seen absolute jitter and accumulated jitter. What is the relationship between the two? The relationship is very easy. As I told you before the absolute jitter is the difference between the ideal clock and the real clock edges, while the accumulated jitter is the jitter of 1 edge, for instance this one, compared to some Nth previous edge. If you look at this picture you will easily find out that the relationship is the following one: the accumulated jitter over N cycles is the difference between the absolute jitter on the Nth cycle minus the absolute jitter on the Nth previous cycle. It is just a difference of the absolute jitter at the two edges that you are considering. Of course if you put any positive to one you get the period jitter which is then the difference between the absolute jitter of two consecutive edges. So now why is this important? This is important because we know a way how to go from phase noise to absolute jitter. And now we know relationship between absolute jitter and accumulated jitter.

34. Phase Noise to Jacc

So we could find a way to go from the phase noise that we measured via spectrum analyzer, to the accumulated jitter. Okay we will start from the formula that we have seen in the slide before. So the first thing we can observe here is that if capital N here is very large, you are subtracting the jitter on one edge to the jitter of the edge which is very far away in the future. So if the two edges are uncorrelated, which is reasonable because if the system has no memory then it should be like this, so the jitter on the two edges are uncorrelated, then you can see that the variance of the accumulated jitter is actually the sum of the two variances here - but they are the same because the system in time invariant so you will find this formula that the variance of the accumulated jitter for very big values of N is actually two times the variance of the absolute jitter. Okay the second way we can go is starting from this formula, calculate the PSD - the power spectral density. So it turns out since we have here to do with some delays there is an exponential functioning between. But this is basic mathematics. So you will find that the PSD of the accumulated jitter is the PSD of the absolute jitter times this function, which actually can be expressed in terms of a normal sine function. It is a sine squared of pi F N divided by F zero where N is the number of cycles we consider F zero as the carrier. And F is the off-set frequency. So this sort of function sine of S squared function is what I will call a sort of filter function. Now we use again the Weiner-Khichin theorem that we have seen before to go to derive the variance of the accumulated jitter by applying this theorem to this formula you will find out that the accumulated jitter is given by this one but this formula here where you have the phase noise multiplied by this filter function and then integrated between F min o and F zero over two. Here I just used the hard way of defining the limit but you can also imagine to have your transfer system function like before and multiply this here. And then integrate. okay you have this kind of function here so let's make some example.

35. Phase Noise to Jacc: Example

How does this look like in reality? Here I showed you some phase noise that we measured for a PLL with having a reference frequency of 36 megahertz an output frequency of about two dot seven gigahertz and the division factor was an integer of seventy six. Each of these pictures show you in the black line is the phase noise that we have measured and the blue dotted line is the filter function that we use to calculate the accumulated jitter. The three pictures deferred in that I use in the following thing: So in the first picture we are calculating the period jitter, in the second picture we are trying to calculate the accumulated jitter over thirty-two, and in the third picture we are calculating the accumulated jitter over seventy six periods. As you can see here, the filter function changes the shape and it has also some zeros here as you can see, and the most of its energy is shifting to lower frequency the higher the N. Now let's go back to the first picture. What you do now to calculate the period jitter in this case is to multiply the phase noise by the filter, and as a result you get this blue line here and then you can integrate this. You will find a value an RMS value of your period jitter. In this case you find the RMS value of the accumulated jitter over thirty two samples and so on. You can see here that in some case it could be that one for the zeros of the filter function is actually falling exactly on top of one spurious tone that you have measured, so that this zero is canceling the spurious tone from your result - from the result you have to integrate . So sometimes it could be that for some values of N that accumulated jitter is masking the jitter coming from the spurious tone of the spectrum. So you have to take care of this.

36. Phase Noise to Jacc

So now we have this formula here which allows us to calculate the accumulated jitter starting from the phase noise of our system under test. We will apply now this formula to two cases, the first case is a free running oscillator and then to PLL to see what comes out.

37. Phase Noise to Jacc:Freeruning Oscillator

Okay for a freerunning oscillator we will assume that the phase noise is of the form of one over F squared - so just pure thermal random noise. You can express this function here using this formula, and if you plug this formula into the formula we have seen before, you will find that the accumulated jitter, the variance, is given by this term here. If you take the square root you will see that the RMS jitter accumulates increases like the square root of N. That's what we intuitively have seen before by discussing how the jitter accumulates in a ring oscillator. Now we have an analytical way that allows us to calculate the accumulated jitter starting from the phase noise and we obtain of course exactly the same result. If you take N equals to one, you will see that the period jitter then is equal to the square root of L one, F one square divided by the carrier of frequency at the power of three. This is a very easy formula but a very useful one. It allows you to calculate the RMS period jitter of your oscillator starting from your phase noise plot.

38. Phase Noise to Jacc:PLL

Now for a PLL we do the same thing, the PLL the shape is a little bit different right, so the phase noise has an inband plateau and then we assume it goes down with one over F squared. So this kind of shape can be expressed by this formula here. And if you plug this formula into the formula for calculating the RMS value of the accumulated jitter it takes some time but at the end it comes out that the accumulated jitter is given by this expression which is a little bit more complicated in the case of a freerunning oscillator. Alright we have some exponential factors here we have some factors that we already know from the formula before. But let's try to analyze now how this formula looks when you plot the accumulated jitter versus N.

39. Phase Noise to Jacc: PLL

Since it has exponential inside we can make two assumptions. First look at what happens for a small value of N. For a small value of N you will find by approximating the exponential function in Taylor with a Taylor function you will find that accumulated jitter is just given by this expression. And this is exactly the same expression that we have seen for a freerunning oscillator, so this is this part here. For a very big values of N the exponential in this formula

40. Phase Noise to Jacc: PLL

the exponential goes to zero. So you are just left with the multiplication with two factors which don't contain N any longer.

41. Phase Noise to Jacc: PLL

So you will see that you have really a asymptotical behavior, flat asymptotical behavior of the accumulated jitter for large values of N. And you can also find that the crossing point between the first part of the curve and the second part of the curve is given by F zero over 2 pi F bandwidth, where F bandwidth is roughly the F bandwidth of your PLL. Okay so actually we have seen that we are able to go from phase noise, we have analytical ways to go from phase noise, to accumulated jitter.

42. Summary So Far

Okay so what we have seen so far is the following we have self referenced jitter. We have applied to digital clocking. We have measurements, we have look at measurements in the time domain. We have looked at accumulated jitter for freerunning oscillators and PLLs we have looked at the relationship of absolute jitter and also how to compute absolute accumulate jitter starting from phase noise. So the next step we will look at is to analyze the statistical properties of jitter.

43. Synoptics

So up to now we have looked at this kind of different kind of measurement procedures: time domain versus idea clock, frequency domain versus idea clock, and time domain self-reference measurement depending on the procedure we used we defined we have found different types of jitter: the absolute jitter, the phase noise, the accumulated jitter and the period jitter. And we have found relationship between all of this but actually at the end up to now we are just looking at the RMS values of each of these jitters. So the question is now how far can we go with the RMS value?

44. Digital Time Constraining

Well let's go back to our example of digital timing constraint. So we have seen that digital need to satisfy a setup time for the digital block so we have seen that what we are sensitive to is the duration of the period. So actually what we are looking then for digital application is the peak value of the jitter and not the RMS value. So in this case for digital time application, RMS value has no meaning. All what is interested is the peak value of the jitter of the second edge compared to the first one.

45. High Speed Serial Data Sampling (1/2)

For high speed serial data sampling the situation is similar. Here I have plotted some RX data, the black line is without jitter, and the red line is with some introduced jitter on data. Now we use some clock for sampling data and for instance with this edge we tried to sample this data in the middle of the eye here. But you can see here that if the first transition of the data as a very big jitter is moving past the rising edge here. Instead of sampling a one, you sample a zero. So you get a bit error here. So you could say okay this is a very big jitter, I am not interested in such big jitters, they have very low probability. But please don't forget the high speed serial data sampling the bit error rate probabilities are of the order of ten to the minus twelve, or ten to the minus fifteen. So if jitter on data edges is larger than 0.5 UI an error will occur. A UI is another way of measuring jitter, just one UI is the duration of one bit of data which is actually one clock period.

46. High Speed Serial Data Sampling (2/2)

Okay if you zoom now in more detail into the high speed zero data sampling example, assume we are sampling the middle here, right, a sampling clock, but each of the transitions has its own distribution. So there is a given probability by this plot here that the transition is moving past the sampling point so the area of this part here is then proportional to the probability of the wrong decision. So you can see from this example the bit error rate of a sampling system is determined by the behavior of the tails of the jitter distribution. You don't care about how the jitter looks like here. You are interested on the jitter distribution close to your sampling point, this is what is important and this is what is by far not the RMS value of the jitter.

47. Jitter Histogram and Distribution

Okay to analyze this point we go to analyze some talk about histograms now. When you take histograms of the jitter, normally you take a finite number of samples. So your histogram you take from measurements is necessarily bounded. But it is not necessarily Gaussian right. And the second point is that the underlying jitter physical process is normally bounded you have thermal noise which is a Gaussian distribution so it is unbounded.

48. Jitter Statistics

So once you have a distribution you can define different metrics. You can define the RMS value which we have seen before. You can now try to define the peak or the peak-to-peak value. Now we have seen the RMS up to now how to calculate for the histogram and so on. Now we will focus our self on how to look into, how to calculate the peak or the peak-to-peak value.

49. The Peak for Unbound Jitter (1/4)

Let's assume you have an unbounded jitter note. So here now I show you two examples of histograms, one is taking of a pure Gaussian jitter source, one is taken with twenty-five thousand samples - it's the blue line, and the second line is with one million samples. If you plot the histogram on a linear scale on the Y axis you don't see very much of a difference between the two. But if you plot it on a log scale you will see a very big difference you will see that the larger the number of samples you take the more the peak jitter grows, right. So if you took only twenty-five thousand samples you could say okay my peak jitter is below let's say below four, but if you go to one million samples you will find that okay my peak is actually above four. So if use this kind of histogram you will see that the peak-peak in the histogram depends on the number of hits you are using so the longer you measure the more the peak value of the jitter.

50. The Peak for Unbounded Jitter (2/4)

So we can then use two ways to calculate the peak jitter for unbounded distribution. The first thing, which is what we have seen before, is just take the histogram and calculate the maximum value of jitter minus the minimum value of jitter. This is an easy way but we have seen the result depends on the number of hits and actually it is not a very reliable it is not a very insightful kind of measurement. And the second way is a more interesting one. The second way you just calculate the RMS and we have seen how to multiply the RMS and also to have a good estimation of the RMS. So calculate the RMS and then you multiply it by a number which is Q. This number this way this result does not depend on the number of hits so that you have taken but the question is that it is only applicable to distribution which are closed to Gaussian.

51. The Peak for Unbounded Jitter (3/4)

So this is again the method okay you have distribution you calculate the signal value you multiply by your sigma value by a number Q. And the higher the Q the lower the probability of the jitter event is beyond the peak or the peak-to-peak value. Actually can be applied to any distribution, but it's the probability of error is can be easily known only for Gaussian distribution.

52. The Peak for Unbounded Jitter (4/4)

So we will see here now how the probability of error looks like. So you define your peak value as Q times sigma and the probability that your jitter event is beyond this peak value is given by statistic of Gaussian distribution using this formula: one half the complimentary error function of Q divided by square root of two. So the higher the Q you take the lower is your error probability. For instance if you want the probability of one of ten to the minus three, you take Q equals to 3. If you have a probability of ten to the minus six you have to take a Q almost five. If you want to have a probability of error less than one ten to the minus twelve you have to take a Q of seven.

53. PDF and CDF

Okay before we go on I have to explain some basic things about how to introduce another kind of function which is the probability density function and the cumulative distribution function. So if you have a random variable, you make an histogram of this random variable and then you normalize the histogram so that the area below the histogram is equal to one. What you get in this way is the so called probability density function. Now you start integrating for instance from the left this probability density function up to a point X here so this area here gives you the probability that your random variable assumes a value below this point. You plot this in another graph here and then you start integrating more and more starting from the left, and you will build another function which is the integral of your PDF starting from the left. This function will saturate to one because the area of this function is equal to one. This is the so called cumulative distribution function, and the meaning of this function is the following: each point on this function gives you the probability that your jitter event is below a given, lies on the left of a given point, right. You can do the same starting integrating from the right hand side instead of the left hand side, and you get let's say complimentary cumulative distribution function which gives actually the probability that the random variable has a higher value then a given one. Okay we will use now this CDF's so this CDF is actually equivalent name for probability of error.

54. What if Jitter is not Gaussian?

Okay so what happens if jitter is not Gaussian? Here I have generated in the black line the distribution which is obviously not Gaussian. And here on this graph the black line is the CDF, so the probability of error, derived integrating this distribution here from the right hand side. So for instance this curve says that the probability of the jitter that the jitter is higher than four is about ten to the minus two. Alright. So then if you calculate the RMS value out of this known Gaussian distribution you get a value of 2.2 and I plot it here exactly Gaussian distribution with this RMS value. What you can see here is that you cannot use this Gaussian approximation to estimate the probability of error for the jitter close to the tails - you are completely off. You see it also from the CDF function you have a completely wrong behavior.

55. Summary So Far

So we have seen that a lot of important applications are not affected by RMS value of jitter. We have seen that the tail behavior might be more important. We have analyzed some statistical properties of Gaussian distribution and we have seen that if you use this procedure of multiply RMS jitter by a factor of Q for estimating the big jitter this leads to the wrong result if the jitter is not Gaussian. So the next thing we will do is to develop a method to characterize jitter distributions which are not Gaussian.

56. Ideal Eye Diagram

So first of all I will give you motivation why do we consider distributions which are not Gaussian this happening in real life. We will analyze high speed serial systems here so what I show you here is an ideal eye diagram. The transitions here in the middle is a very sharp one and here the histogram shows a very peaked behavior.

57. Eye Diagram: Random Jitter (RJ)

If introduce some random noise on your system, now we see that the transition here has assuming some Gaussian distribution. And this is due to thermal noise, flicker noise or the many effects of uncorrelated noise sources.

58. Eye Diagram: Periodic Jitter (PJ, SJ)

But this is not the only kind of noise you can have. In this kind of system you see that we have a blur transition, a band transition. But the histogram is very bounded right. In this case this was generated by applying a sinusoidal component to the jitter. This kind of histograms you can get from system due to when you have a power supply noise issues or you have strong local RF carriers or spurious tones in the PLL. The important thing to note here is that this kind of jitter is bounded. The histogram is bounded.

59. Eye Diagram: Data-Dep. Jitter (DDJ, ISI)

Another picture of different picture of jitter happens if you have a system with a very low if you are transmitting a signal through a channel with very low bandwidth. Then it happens that the bandwidth of the channel is not high enough to let the signal settle before it turns back, for instance in this point here it goes from zero it should go to one and then back to zero, but the bandwidth is so low of the channel that it doesn't reach the high point and has to turn back before. So here in this kind of system you will see that if you plot the histogram that you have some hot spots where the transitions are concentrated but there is no other points. There is no blur transition, no diffused one.

60. Eye Diagram: Duty Cycle Distortion (DCD)

Another kind of jitter that you can have is the so-called duty cycle distortion. In this case transitions are concentrated in two hot-spots here which show equally large peaks. This is called duty cycle distortion and it is a little bit difficult to see it from this picture.

61. Eye Diagram: Duty Cycle Distortion (DCD)

But if you unfold the eye over more UI's let's say you will realize why it is called like that. And that it is typical for DDR systems. You have a clock and which clocks the data on both rising and falling edge and if the clock has a duty cycle distortion you will find that some part of the eye is smaller and some part of the eye is larger. So this is called duty cycle distortion.

62. Eye Diagram: Rise/Fall Time Asymm.

Another kind of bounded histogram that you can get is in case you have a rise full time asymmetries. Assume that your receiver is sampling the signal in the middle here, you will see one transition here and another transition here. So also in this case your distribution of the jitter show two different peaks which are bounded.

63. Eye Diagram: Multiple Jitter Sources

If you put these things together, things begin to get complicated. In this case for instance I superimposed DCD so duty cycle distortion with random jitter, and you can see that the shape of the jitter distribution that you get is absolutely not Gaussian, right. Actually the result in distribution is the convolution of the signal distributions.

64. Eye Diagram: Multiple Jitter Sources

And this one is a plot of a system, a measurement taken after 50 inches of FR4. You see that the data transitions show the deterministic patterns plus some blurred behaviors, and the histogram is absolutely not Gaussian.

65. Jitter Decomposition (1/4)

So following this reasoning we can see that the jitter is the result of the device noise, which is flicker or thermal noise, is also the result of some system non idealities like nonlinearities limited channel bandwidth, spurious tones and so on. But we have also some external disturbances like supply switching noise, crosstalk, all this kind of stuff. Normally device noise contributions come from resistors or transistors are Gaussian, so the jitter that generate is unbounded. All other contributions are generating bounded jitter like we have seen in the plots before.

66. Jitter Decomposition (2/4)

So the idea is now to consider the total jitter distribution as the combination of the deterministic components and random components. We call the deterministic components or the deterministic jitter, DJ, any jitter with bounded distribution. And we call random jitter, RJ, any jitter with unbounded distribution. So the result of RJ plus DJ is what is normally called total jitter (TJ).

67. Jitter Decomposition (3/4)

So this is just a table to give you an orientation in all this jungle of terminology. The total jitter actually can be splitted or divided in two tables, unbounded jitter this is called the RJ, or bounded jitter, DJ. The DJ, the deterministic jitter, can be divided, sub-divided into different tables like we have seen before. But the main important thing is that it doesn't matter what is the generating factor of the jitter. The main thing is that DJ is always bounded.

68. Jitter Decomposition (4/4)

Now from the probability theory it's effect from probability that the distribution of the sum of independent random variables is the convolution of the single distribution. So if we apply this to our case where we have our TJ is RJ plus TJ, it comes out that the distribution of the TJ is the convolution of the distribution of RJ and DJ. Here it shows an example: so you have an RJ which is Gaussian and a DJ which can be like a duty cycle distortion. The convolution means that you take this Gaussian you move it where the DJ is not zero, you get this two Gaussian here, you sum them up and you get the total jitter. In case of sinusoidal jitter, the DJ had a histogram of this form but the concept is the same, only in this case you have much more Gaussian that you have to sum up in the end you get such a form. The important thing to notice here is that DJ is always bounded so there will be always here the skirts of the total jitter. This part of the total jitter is due to some Gaussian component which is mainly due to RJ, right.

69. Tail Fitting (1/3)

So this lead to the idea of tail fitting. So the idea is that the assumption is that the tails of any jitter distribution means real physical system is given by RJ by random jitter so that can be fitted with a Gaussian curve. So the idea here is that we take our distribution we had before and fit the right part of the tail with a Gaussian distribution here. If you are able to fit this then you get a curve, the red one, which is describing the probability of error very accurately. What you get from the fitted are three values: the sigma of this Gaussian curve, the mean value of this Gaussian, so the point where the peak is, and of course also the amplitude you have to scale here.

70. Tail Fitting (2/3)

So now we do the same thing also on the left hand side and of the distribution, and during this kind of tail fitting we get six parameters - three for one Gaussian and three for the other. And with this three parameters we can describe the probability of error from the left tail and the right tail very accurately.

71. Tail Fitting (3/3)

Okay. So once we have done this tail fitting we can define now the peak jitter by using the same approach we use before because now we have to do with Gaussian distributions here. So the peak right can be defined as the mean value of the Gaussian distribution on the right hand side here, plus Q times the sigma value of the random distribution of the Gaussian distribution. In an analog way you can define the peak left. And then you can define the peak to peak as the difference between peak left and the peak right. This will be the difference between the main values of the two Gaussian distributions which are fitting your original measure distribution, plus Q times the sum of the two variances. So this the difference between the two mean values is the DJ while the sum of the two RMS values is the RJ and the total gives you the DJ.

72. TJ and Probability

Okay once you have the TJ actually based on the value of Q, we get back to the probability of error. So there is a given probability that the jitter is outside the TJ range and this probability of error is given by the formula that we have seen before because now we are actually dealing with Gaussian distributions.

73. Bathtub Plot (1/3)

Okay now let's see how to apply this to a tail fitting procedure to some practical applications. The applications we want to analyze now is the sampling of our X data, serial high speed data. So what we are assuming here is that we are sampling in the middle and the data are jitter here what we want to look at is how much is the our eye opening so how much can we move around the our sampling point in between here and still get a bit error rate which is reasonable. So the thing we do now is to take the distribution of the transitions and fit them with Gaussian curves, right and left. And then what we do is to plot the CDF's of these two Gaussian curves what you get here in this way is the plot is this plot here, which shows you the bit error rate depending on your sampling point. So from this kind of plot you can see that you have an eye opening for a bit error rate of ten to the minus twelve of about 0.3 UI. So the bathtub plot is used to estimate the eye opening for very low bit error rate levels.

75. Bathtub Plot (1/3)

There is one thing which we don't like for this representation and this is the fact that for Gaussian distribution the bathtub plot has some curvature. We don't like curvature. Engineers normally like straight lines. So some there is one way to convert this curvature into straight

76. Bathtub Plot (2/3): Q Scale

line and this is the so called Q-scale. The Q-scale is just another mapping, is the mapping from the probability of error into another scale which is called the Q-scale in such a way that if you have a Gaussian distribution here, and the probability of error looks like this, has some discurvature, in this Q scale it is the Y axis is distorted in such a way that this curves turn into straight line. The formula for mapping the probability of error to the Q scale is this one - I will not explain all the details - but you can see that we will convert probability of error of a Gaussian curve into a straight line. Due to this mapping different values of error probability are mapped to different values of Q. For instance for a probability error of ten to the minus twelve you have to look in the Q scale at the value of minus seven.

77. Bathtub Plot (3/3)

Okay now this shows exactly the same plot as before where we replaced the Y axis with a Q-scale, we re-mapped it. So what we had before like curved lines are now looking exactly straight lines here. And this is also a nice way to see this is also used to understand when the RJ contribution starts to dominate, because as soon as the bathtub curves and this Q-scale gets linear you know that we have to do with the Gaussian distribution. And so this is the point where the RJ, there is no DJ any longer, and the whole contribution is determined by the RJ.

78. Bathtub Example (1/4)

Okay I will now show you an example of two systems - we have a blue system and a red system. There are the histograms taken from the two systems of course over ten thousand hits. Although the histograms are different it looks at the first site that the peak- to-peak value could look the same. it looks like 0.3 in both cases right. Now let's have a look at the bathtub plots taken for these two curves here.

79. Bathtub Example (2/4)

You see they have two different behaviors. The red system has a steeper slopes as the blue one. So you could imagine that if you go to lower bit error rates here, the red system seems to be better - it allows you a wider eye opening.

80. Bathtub Example (3/4)

Indeed if you measure for ten million hits and build the bathtub curves, you see that red system has definitely a larger eye-opening for low bit error rates that the blue one. Anyway from this kind of graph it is not easy to predict what will be the eye-opening at the bit error rate of ten to the minus twelve because both curves have curvature right. So now what we do is to go from to transfer our self to the Q-scale mapping.

81. Bathtub Example (4/4)

If you map the data we had before the Q-scale you will recognize immediately that the blue system has a very straight line. This means that the blue system is completely RJ dominated, dominated by a random jitter components. So then you can really fit this straight line with these measure points with a straight line on both sides and you will see that they meet exactly at minus Q equal to minus seven. This is the point which corresponds to a bit error rate of ten to the minus twelve. So for the blue system there is no eye opening at ten to the minus twelve. For the red system on the other hand you see that we have some DJ components here because we don't have the straight lines but at some point the jitter gets RJ dominated and you can then fit the last part of the bathtub curves with straight line, and you realize that now the red system even though it looks similar to the blue system the first time, it does a very, very different behavior at low bit error rate. It allows you an eye-opening of almost 30%. Okay this is just to show you how powerful this bathtub method is and how to use the Q-scale.

82. Spurious Tone in Spectrum (1/4)

So the last topic I want to cover today and then we are done, is how to treat spurious tone in the spectrum of a signal. So spurious tones indicate periodical phase modulation. And any periodic signal can be decomposed in sinusoidal components. And each sinusoidal components then contributes to bounded jitter and then contributes to DJ. Now we want to analyze what is the relationship between the spectrum that we see and the DJ.

83. Spurious Tones in Spectrum (2/4)

So this is again a simplified version of what we have seen before. We have the carrier and the phase modulation at the frequency FM from the carrier. This phase modulation will move the jitter up and down as sinusoidal way, and it will generate an histogram of this form which gives you then a given DJ right. So now

84. Spurious Tones in Spectrum (3/4)

what is the relationship between the DJ and the spectrum? So now if we assume then a signal of this form where we have sinusoidal signal of omega zero T plus a phase modulation at the given amplitude A. This can be decomposed into basal functions so J zero and the J one are the basal functions, so that you can express actually you can find the relationship between the difference between the carrier of the spurious levels which are called spurious-to-carrier ratio and you can find the relationship between the spurious-to-carrier ratio and the value A. And this is a very simple relationship it's the SCR is given by twenty log ten of A over two. And now we have this relationship so we know that A is the amplitude of the phase modulation which is related to the DJ so we can calculate now A starting from the SCR..

85. Spurious Tones in Spectrum (4/4)

and converting into DJ. And you will find the following formula. The DJ presses seconds is just two over pi F zero, ten to the power of SCR divided by twenty. And if you want to express the DJ in UI F zero will vanish and you get a very easy formula. So this formula allows you to start to calculate the DJ of a sinusoidal phase modulation starting from your spectrum block. On this table I tabulated some values of the DJ corresponding to different SCR values. As you can see also DJ depends only on the SCR and the F zero but it doesn't depend on the modulation of the frequency.

86. Summary

Okay so this leads me to the conclusion of my tutorial. We have seen the definition of several jitter types, measurements in time and frequency domain, relationship between jitter and phase noise. We have looked at jitter in typical applications. We have analyzed jitter distribution for both Gaussian and non-Gaussian distributions. Jitter the composition in RJ and DJ. And then we have had a look in more detail into DJ in high speed communication we have worked with tail fitting and bathtub plots and how to extrapolate eye opening using bathtub plots for very low bit error rate.

87. References

I included as reference one slide with some literature. These are just some I would say some key references which allows you to get into the topic of course, if you want to be more into depth, there are many other valuable contributions. But these are from my point of view they are first one you should have a look at if you want to get into this topic. Okay this is the last of my slides I would like to thank you very much for your attention. Thanks a lot.