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Hardware Efficient Maximum Sequence Length Digital MASH Delta Sigma Modulator

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This Talk

Question: Is it possible to achieve the maximum sequence length in a MASH DDSM without dithering or seeding? If the answer is yes, then there would be a huge reduction in hardware and consequently in power consumption when compared to state of the art MASH structures.

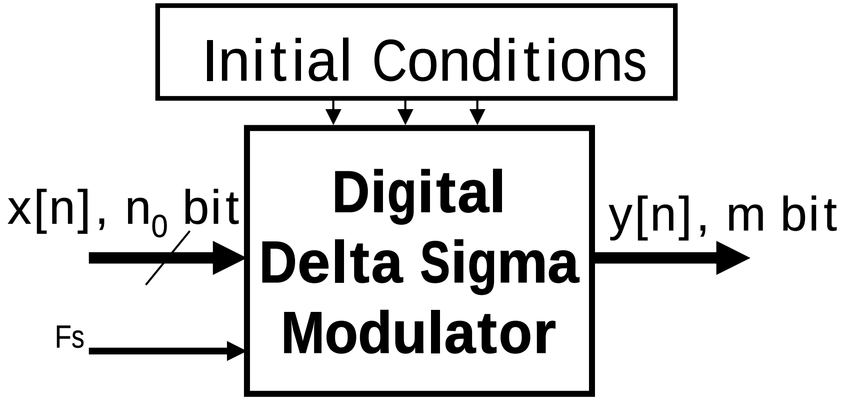
Answer: Yes! We have designed a MASH DDSM structure that yields the maximum sequence length for all inputs without dithering or seeding.

Outline

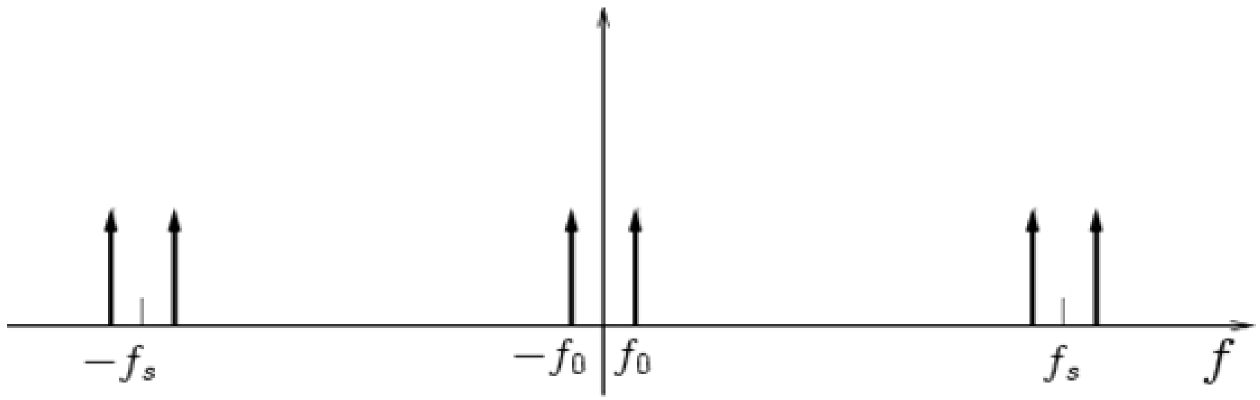
- Introduction to the Digital Delta Sigma Modulator (DDSM) and Multi Stage Noise Shaping (MASH)
- A challenge encountered with a constant input (a short sequence issue)
- State of the art solutions to guarantee long sequences
- A **maximum** sequence length MASH DDSM
- Simulation and measurement results
- Conclusions

Introduction to the DDSM

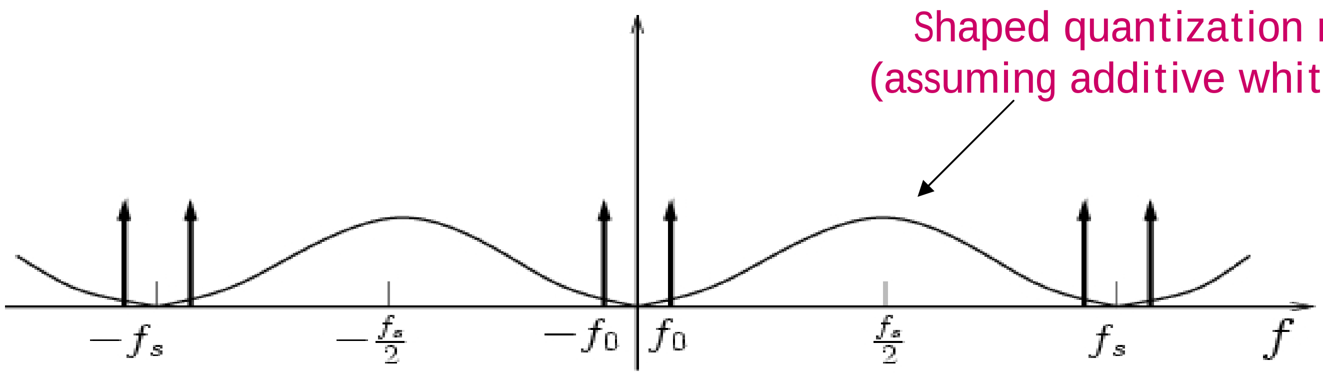
- The DDSM is widely used in frequency synthesis applications, where fine tuning is needed.
- Given an n_0-bit input sequence x[n], the DDSM delivers an m-bit output sequence y[n], where $m<n_0$.
- While the input spectrum appears at the output, the spectrum of the resulting quantization error (so-called quantization noise), y[n]-x[n], is shaped by the DDSM.



Noise Shaping Property of the DDSM (1)

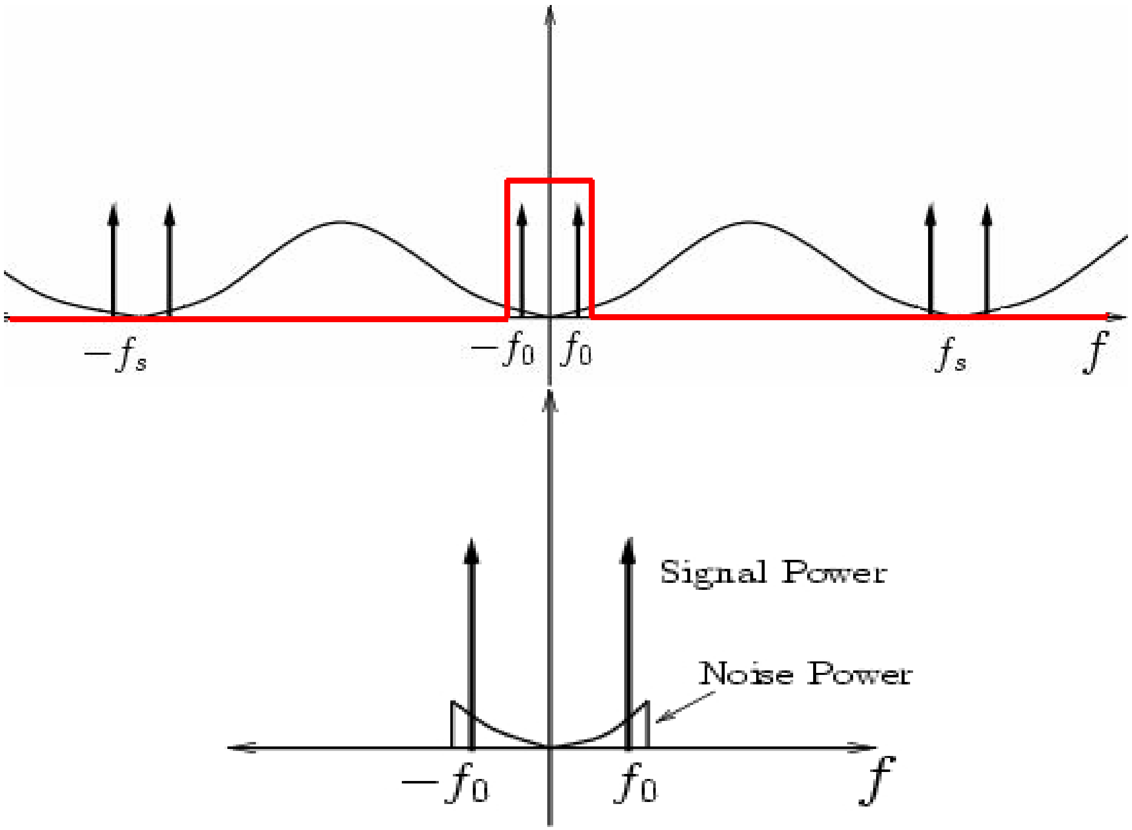


x[n] spectrum



y[n] spectrum

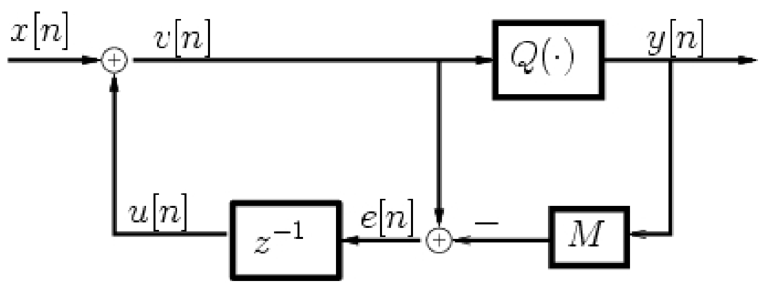
Noise Shaping Property of the DDSM (2)



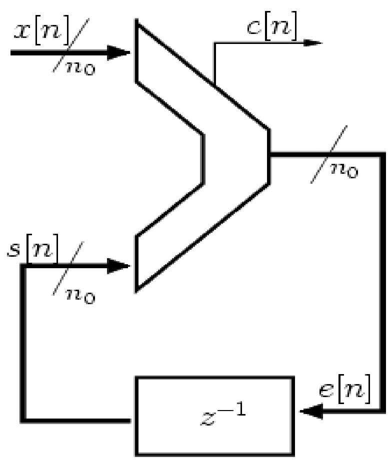
DDSM output spectrum after low pass filtering (enlarged)

Error Feedback Modulator (EFM)

- The first order digital EFM1 (m=1), simply an accumulator, can perform the described DDSM operation if the quantization noise of the quantizer (e[n]=c[n]-x[n]/2^(n_0)) assumed to be an additive white noise.
- A first order noise shaping $(1-z^{-1})E(z)$ can be achieved.



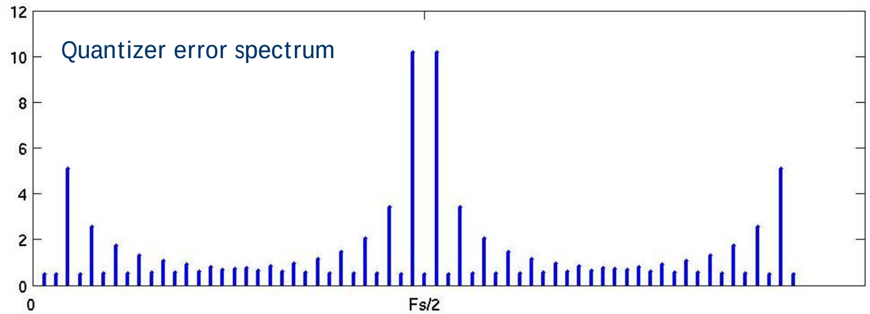
EFM1

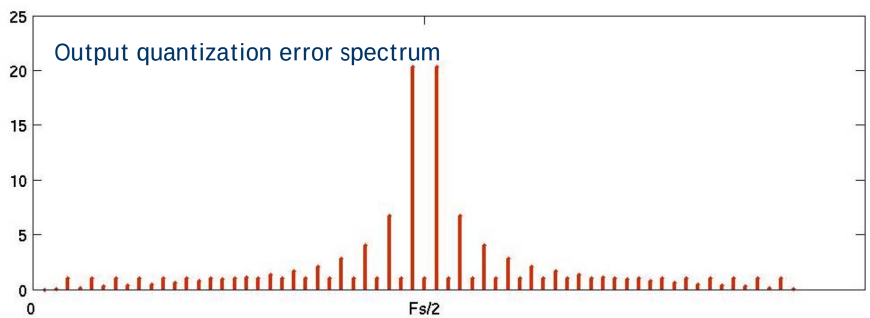


Accumulator (EFM1)

Is white noise assumption correct in the EFM?

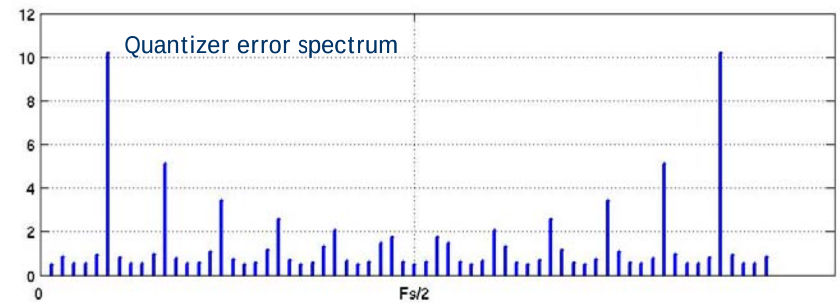
Quantizer error spectrum is not white.

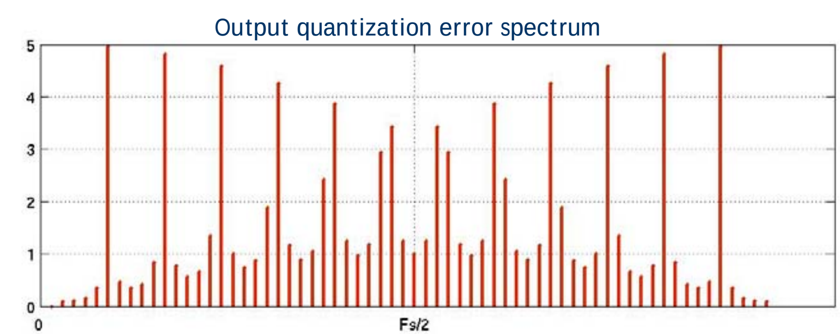




$x[n]=X=31,\ n_0=6$

$x[n]=X=5,\ n_0=6$



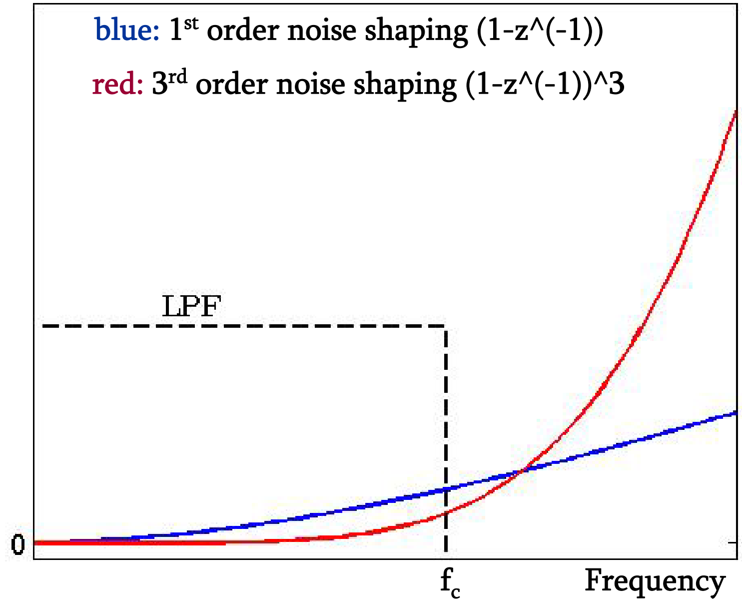


First order EFM fails to perform proper noise shaping.

Motivations to Use Higher Order Noise Shaping

- In higher order modulators the white noise assumption of the quantization noise can be more realistic "under some specific conditions", (probably because of more complex structure), when compared with the first order case.

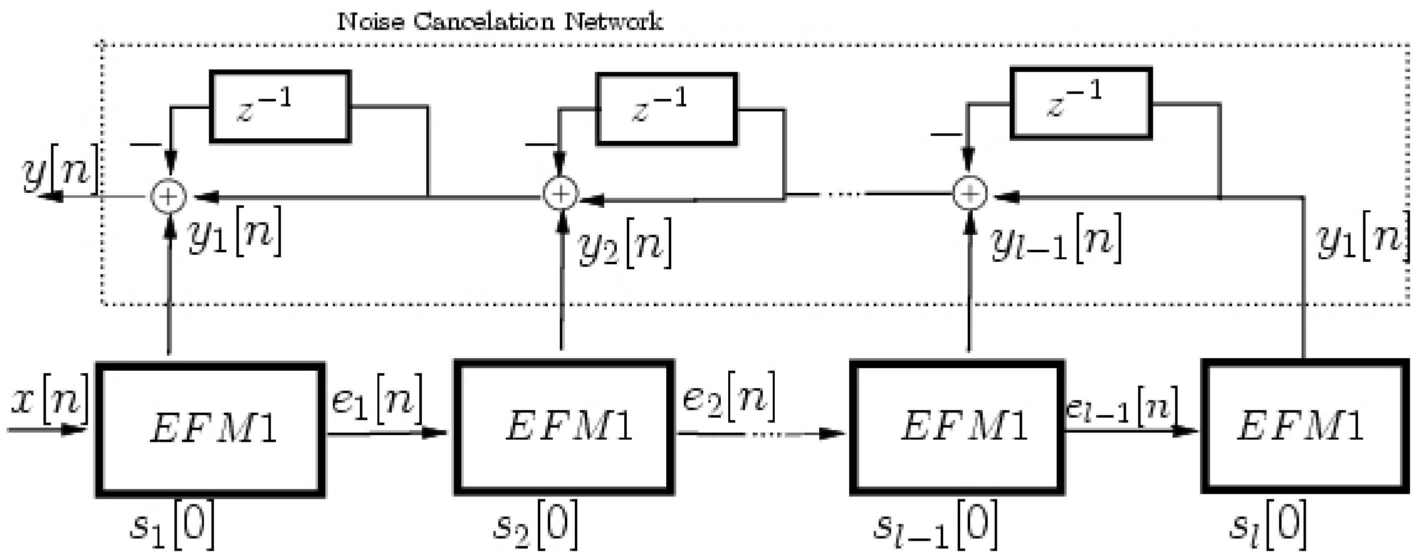
- Higher order noise shaping, yields better in-band SNR.



In-band portion of the spectra is shown.

MASH Structure

The digital MASH structure is one of the popular solutions to perform the above operations.



The Digital MASH Structure

MASH Structure (cont'd)

- A higher order noise shaping can be achieved.

$$(1 - z^{-1})^{l}$$

- The quantization error sequence is long and also is random under some specific conditions.
- Despite the more complex structure it still suffers from the problem of short sequences.

A Challenge

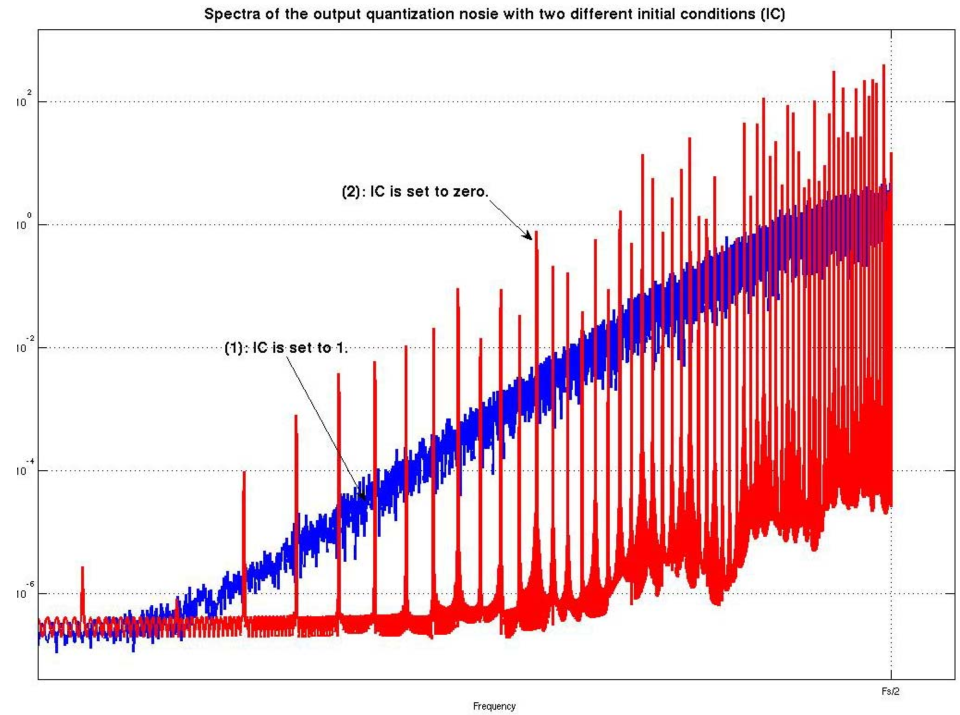
- The figure illustrates the effect of a short sequence in a third order MASH DDSM.
- With exactly the same structure and the same input but with different initial conditions (IC) the spectra are completely different. (1) is desirable.

Simulation conditions:

Input = 256 (decimal),

Accumulator word lengths (n_0) = 14

(1) And (2) are the spectra of the quantization noise.



How we can get the maximum sequence length?

Current Solutions (1)

Dithering

- It increases the sequence length by randomization. However, it requires additional hardware; and, it increases the in-band noise when used in fractional-N PLLS.

Prime Interval Quantization

- It guarantees a minimum sequence length for all MASH modulator orders equal to the prime intervals. While it avoids the need for seeding and dithering, it does not yield the maximum sequence length and adding stages does not increase the sequence length.

Current Solutions (2)

Seeding

- *Kozak and Kale's* (2004) analytical method suggests an odd initial condition on the first stage of the MASH.
- *Borkowski* et.al.'s empirical FSM observations (2005) also suggest an odd initial condition on the first stage of the MASH. The lengths of the sequences were quantitatively recorded for the first time. A design methodology has been proposed.
- *We* have confirmed and extended analytically a part of their empirical results (calculating the sequence lengths).

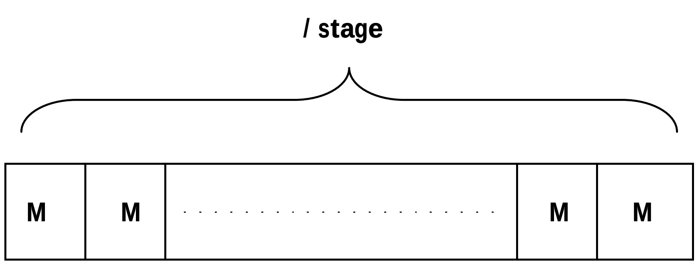
Quantitative Results of the Sequence Length

- In the case of the MASH including n_0-bit similar internal accumulators, a sequence length can be guaranteed, if the accumulator of the first stage is set to an odd value.

Modulator Order (l)	Guaranteed Sequence Length
2	$2^{(n_0-1)}$
3	$2^{(n_0+1)}$
4	$2^{(n_0+1)}$

Open Problem

- Considering $M=2^{n_0}$ states for each stage of the MASH, why cannot we achieve the maximum sequence length of $M^l=2^{(l n_0)}$ for an l^{th} order MASH, where n_0 is the accumulator size?



All possible states are M^l.

Our Solution

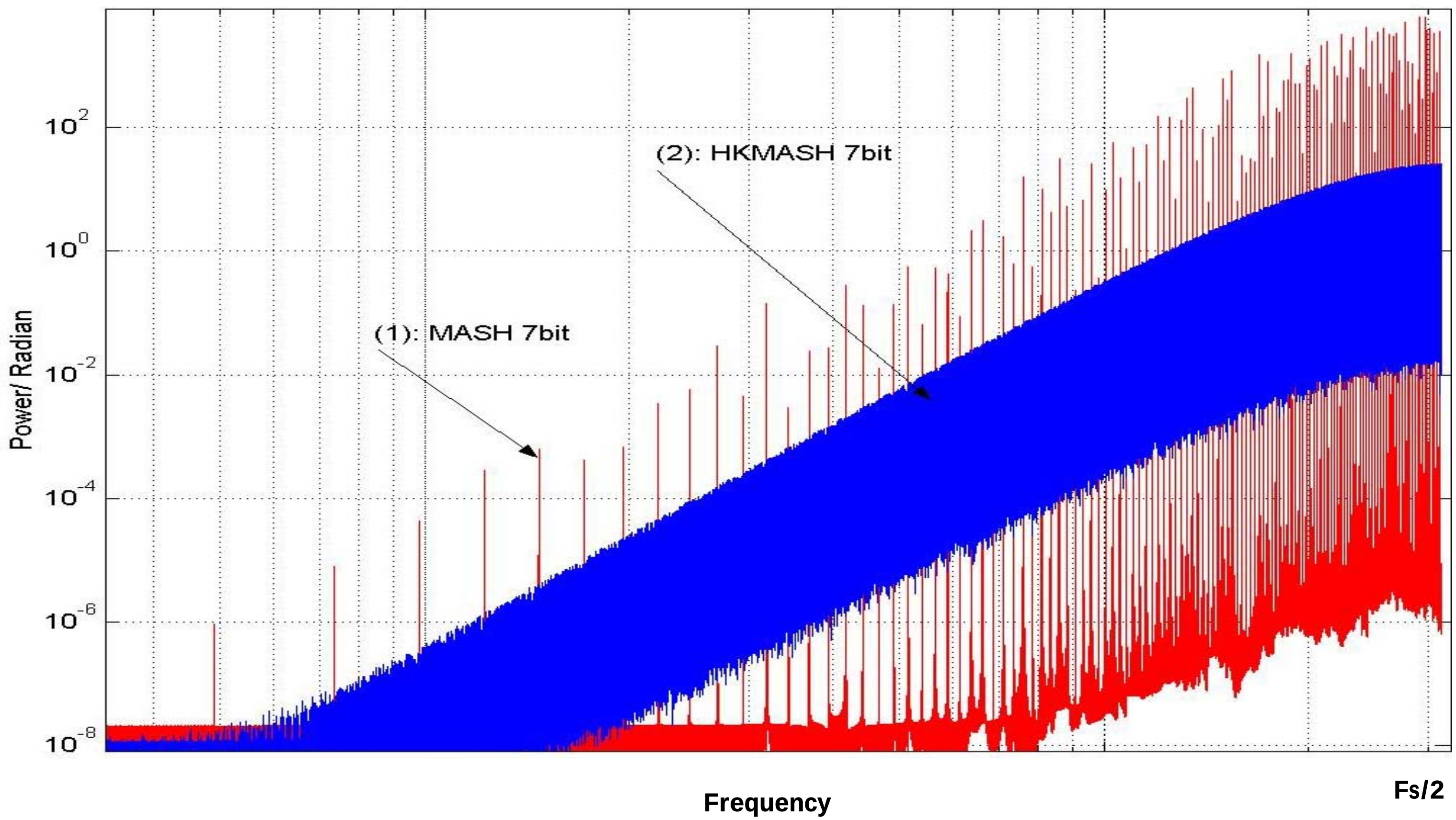
- The sequence length is guaranteed by our circuit to a value close to the ≈ $2^{(n_0)}$ for all inputs in case of a first order EFM modulator.
- The good news is that when the above technique is incorporated to a MASH structure, the sequence length is increased by the factor ≈$2^{(n_0)}$ for adding any similar first order stage to the MASH.
- Therefore, we have a new circuit which yields the guaranteed sequence length ≈$2^{(l n_0)}$ for all inputs without seeding and dithering.

Comparison of the sequence lengths

Comparison of the sequence lengths of state of the art MASH structure with our new MASH

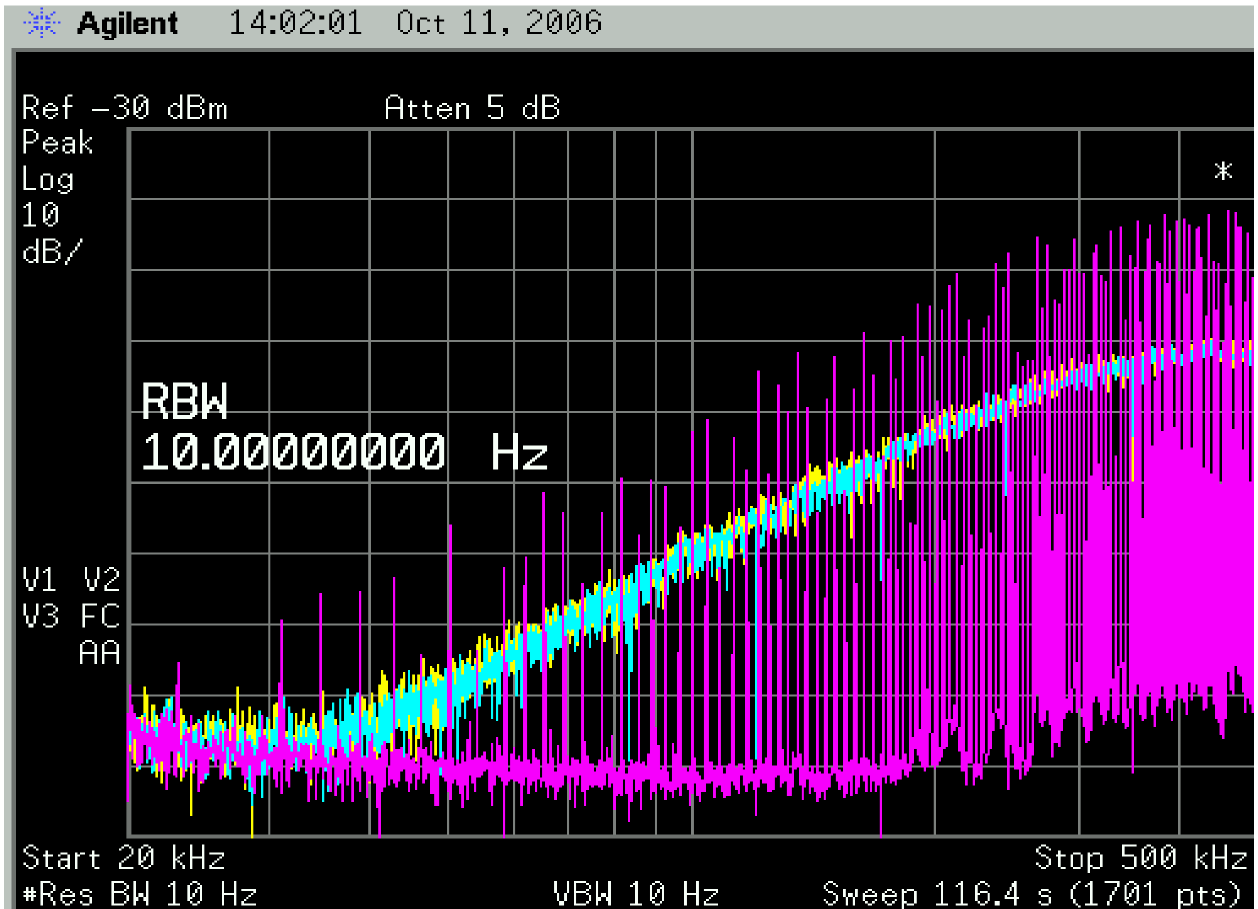
Modulator Order (l)	State of the art work *guaranteed sequence lengths* *for odd initial condition on the first stage*	Our circuit *guaranteed sequence lengths* for all inputs and all initial conditions
2	$2\text{\textasciicircum}(n_0-1)$	$\approx 2\text{\textasciicircum}(2n_0)$
3	$2\text{\textasciicircum}(n_0+1)$	$\approx 2\text{\textasciicircum}(3n_0)$
4	$2\text{\textasciicircum}(n_0+1)$	$\approx 2\text{\textasciicircum}(4n_0)$
5	$2\text{\textasciicircum}(n_0+2)$	$\approx 2\text{\textasciicircum}(5n_0)$

Simulation Result



MASH: 7 bit (the best case)
New MASH: 7 bit

Measurement Result



Blue: 7 bit HK-MASH, IC1,2,3=0

Yellow: 14bit MASH, IC1=1, IC2,3=0

Purple: 7 bit MASH, IC1=1, IC2,3=0

Discussion

- Simulation and measurement results confirm the spectral improvement by a remarkable increase in the sequence length.
- For the same spectral purity compared to the conventional solutions, our structure can be used with significantly less hardware and power consumption.
- There is no need to care about setting the initial conditions.
- There is no need for dithering.
- The performance is guaranteed mathematically for all inputs and all above claims.
- The transfer function is the same as for the conventional MASH.

Conclusion

- A maximum sequence length MASH DDSM has been developed.
- No dithering and seeding are required and the performance is guaranteed for all inputs.
- The performance has been confirmed mathematically, by simulation and by measurement.

Acknowledgment

- ✓ Funded by SFI

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Thank you for listening