

# A/D Converter Fundamentals and Trends

(IEEE CICC 2017 Tutorial)

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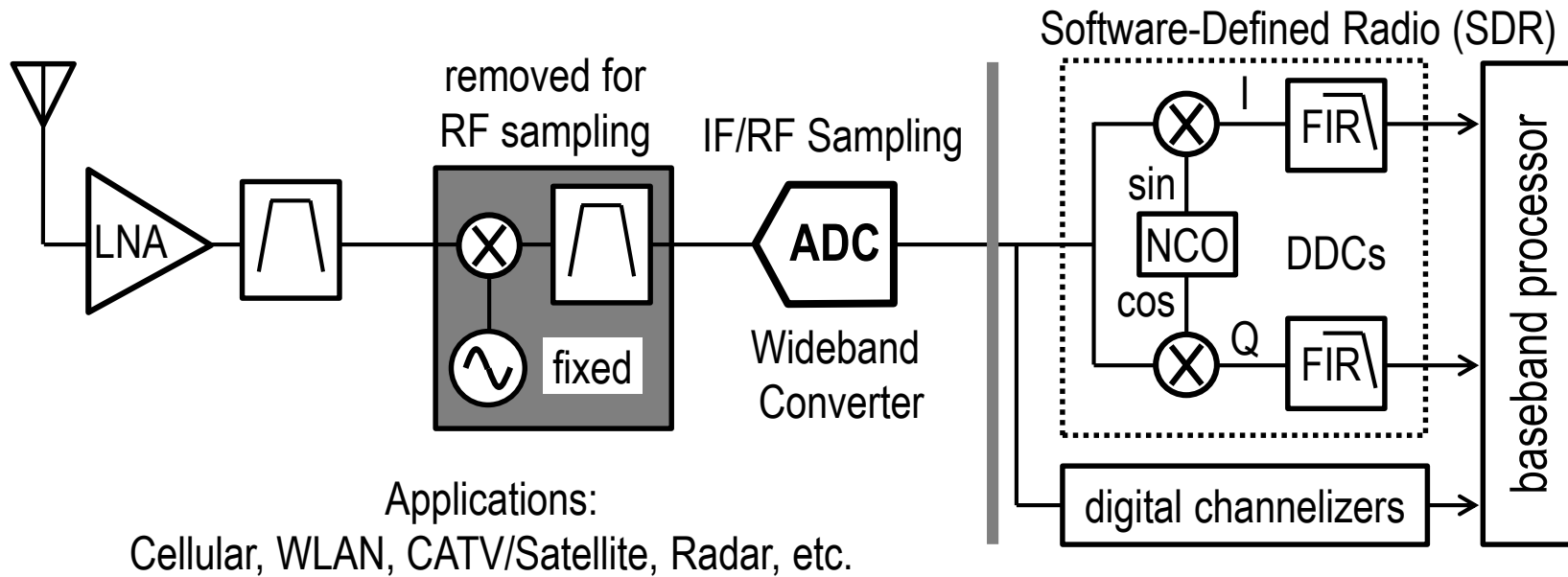
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# Agenda

- Applications and Systems
- Algorithms and Architectures
- Circuit Implementations
- Imperfections and Corrections
- Metrics and Measurements
- Evolutions and Trends
- Closing Remarks

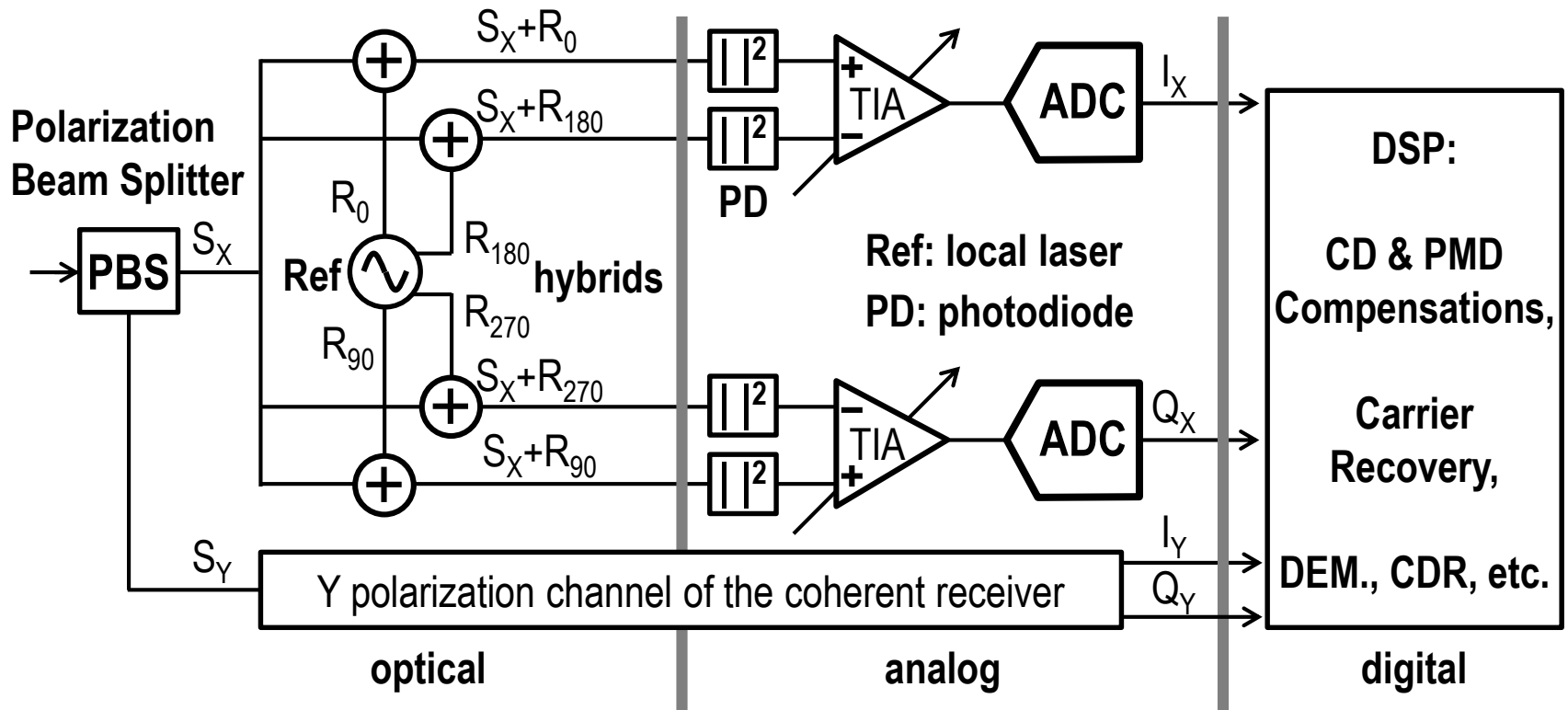
# Applications and Systems

# Full Band Capture and Software-Defined Radio



- Multi-GS/s ADCs for IF/RF full-band/multi-band capture of hundreds of channels
- High ADC dynamic range for DSP to sort out weak signals from in-band blockers
  - SFDR  $\sim 80\text{dB}$  & SNR  $\sim 60\text{dB}$  for GSM base station applications [Pan, JSSC 2000]
- High power, but gap closing with heterodyne for  $f_t \gg f_{\text{nyquist}}$  [Gomez, TCAS 2016]
  - Direct sampling (DS) more efficient than heterodyne with carrier aggregation

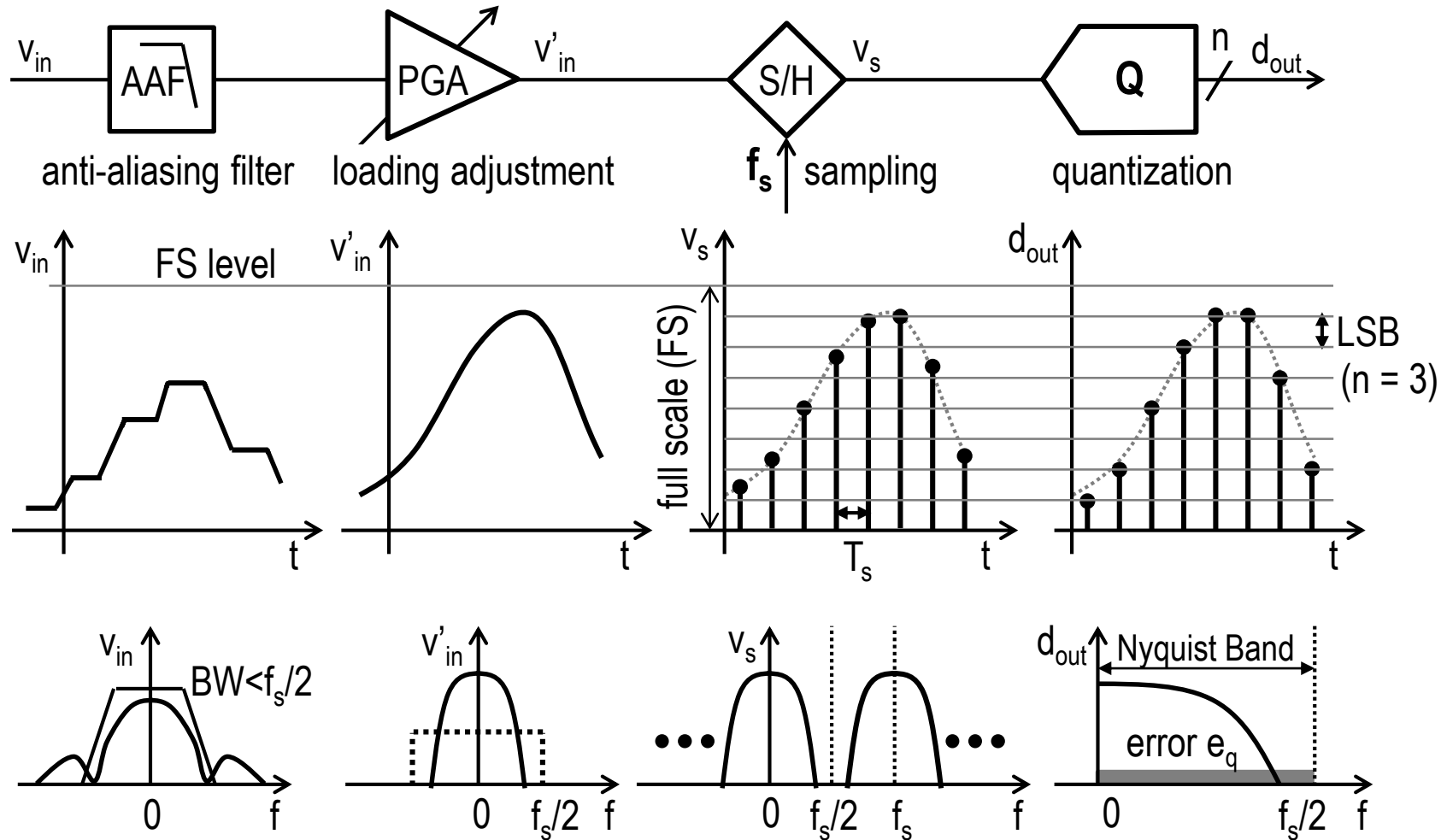
# DSP Based Wireline Receivers



Applications: coherent optical communications (100G and higher)

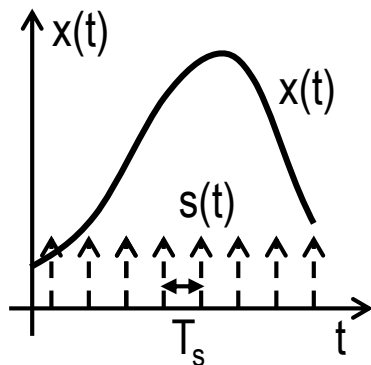
- Ultra high-speed ( $>10\text{GS/s}$ ) 5~8b ADCs and DSPs enable long-haul optical transmission beyond 10Gb/s per lambda [Crivelli, TCAS-I 2014].

# A/D Conversion: A System Perspective

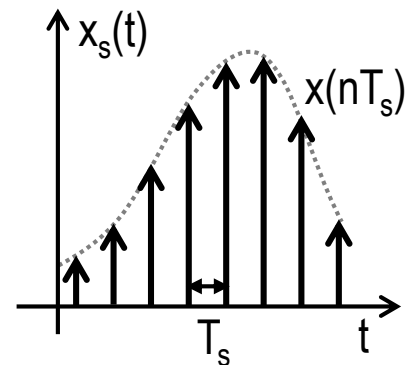


- Sampling and quantization operations are permutable with each other.

# Sampling Theorem

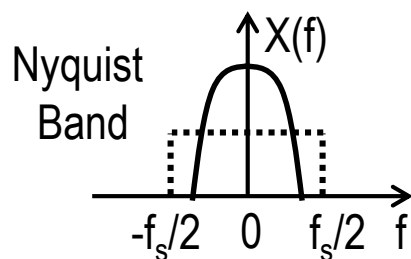


$$x(t) s(t) \rightarrow s(t) = \sum \delta(t - nT_s)$$

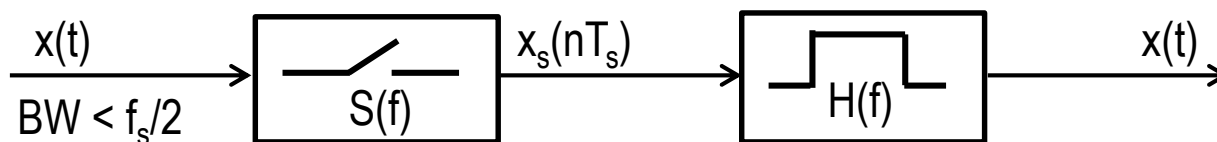
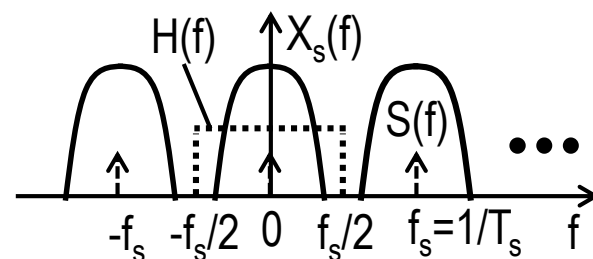


$$\text{F.T.} \updownarrow x(t) = \sum x(n)h(t - nT_s)$$

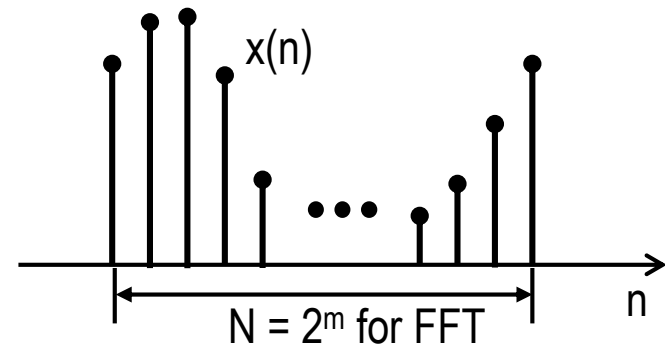
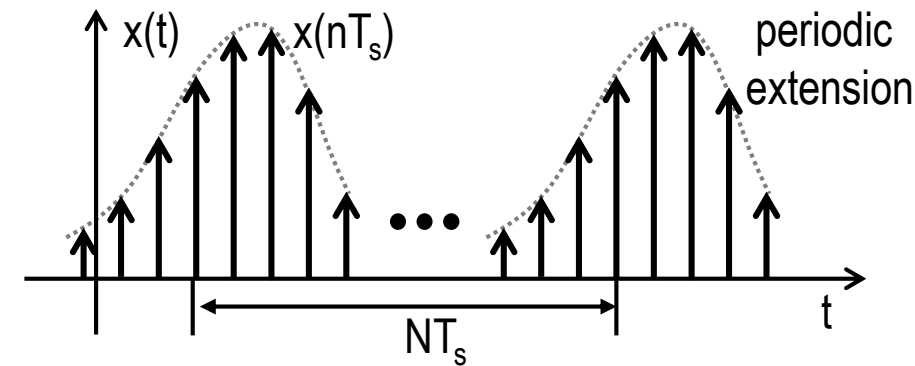
$$X_s(f) = X(f) * S(f) \updownarrow \text{F.T.}$$



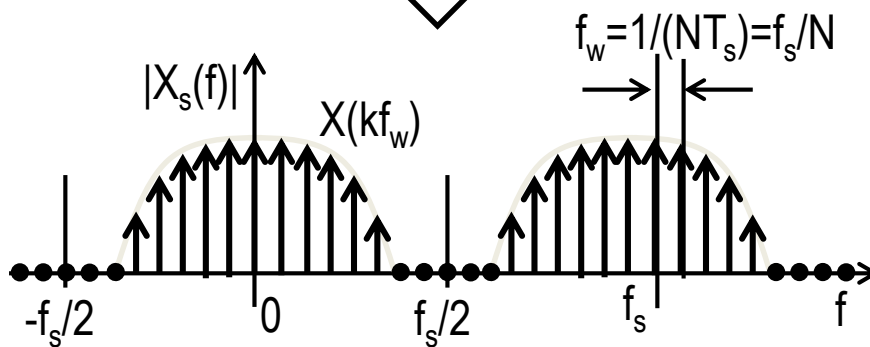
$$X_s(f) H(f) \leftarrow$$



# Discrete Fourier Transformation (DFT)



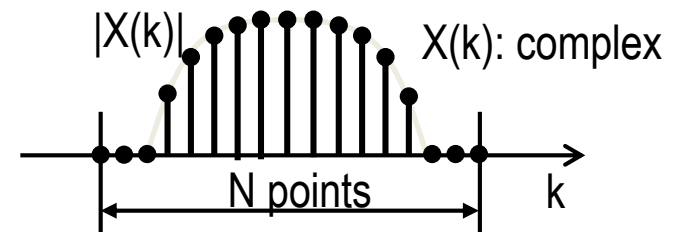
F.T.  $\updownarrow$



D.F.T.  $\updownarrow$

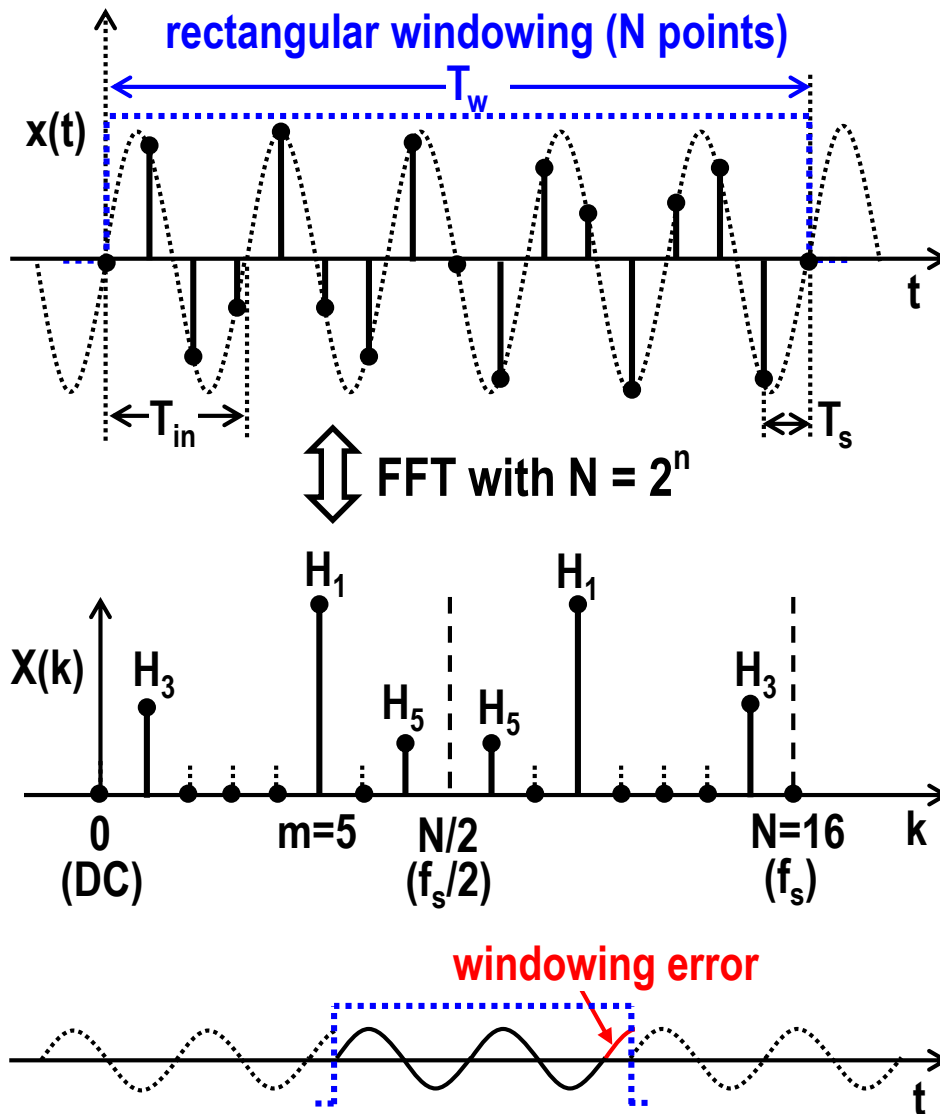
$$x(n) = \sum_{k=0}^{N-1} X(k) e^{j\omega_k n}$$

$$X(k) = (1/N) \sum_{n=0}^{N-1} x(n) e^{-j\omega_k n}$$



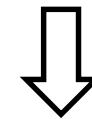
- Important tool for spectrum analysis of finite signal samples
- Fast Fourier Transformation (FFT) algorithms for  $N = 2^m$

# Coherent Sampling of Periodical Signals



N-Point Coherent Sampling:

$$T_w = N T_s = m T_{in}$$

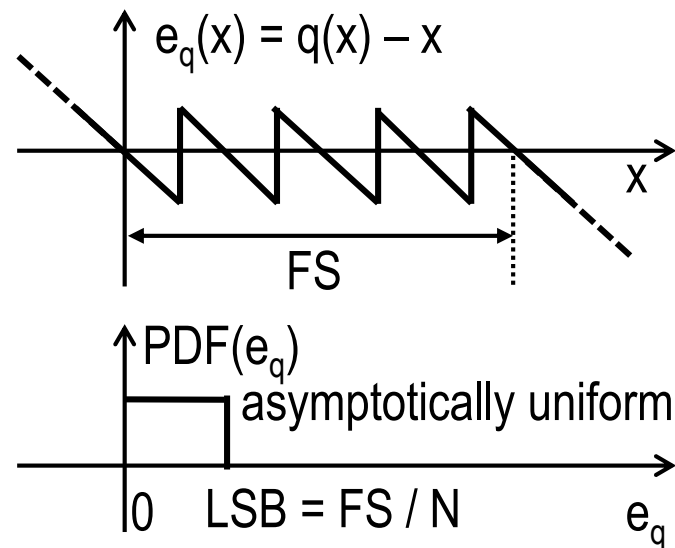
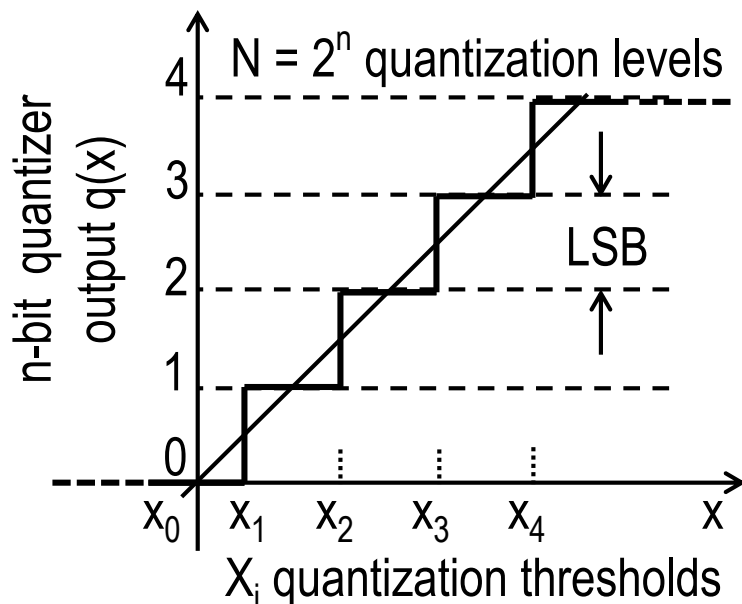


$$f_{in} = (m/N) f_s = m f_{bin}$$

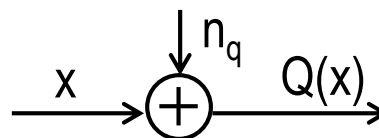
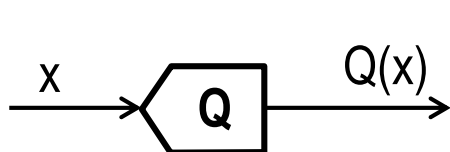
$$(f_{bin} = 1/T_w)$$

- Each  $H_i$  falls in one bin with coherent sampling.
- Less  $H_i$ 's fall in same bin with prime ratio  $m = f_{in}/f_{bin}$ .
- Windowing error causes spectral leakage to all bins.

# Quantization and Error Characteristics



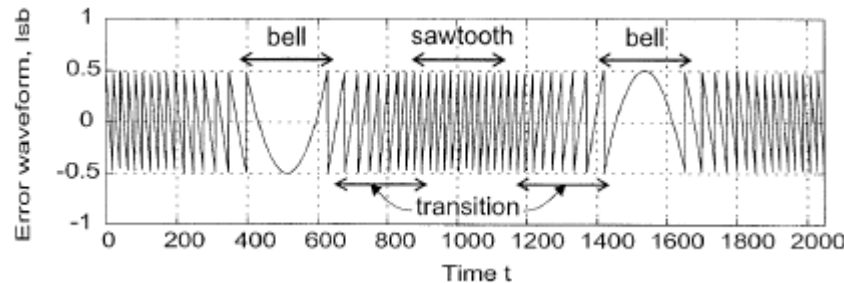
$$\text{Error Power } P_e(x) = \int (e_q)^2 \text{PDF}(e_q) de_q = \text{LSB}^2 / 12$$



- Quantization error is usually approximated as additive noise
  - Asymptotically uniform PDF, independent of  $x$  for  $x(\text{rms}) \gg \text{LSB}$
  - Converted to noise  $n_q$  by dithering of similar or higher magnitude

# Error Waveform & Spectrum of Quantized Sinewave

$$e_q(t) = q(A\sin(2\pi f_{in}t)) - A\sin(2\pi f_{in}t)$$

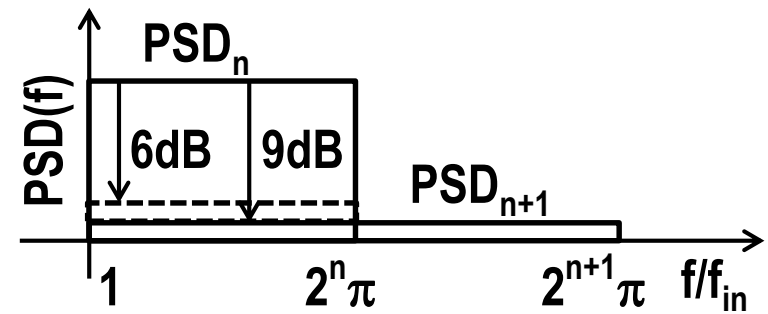
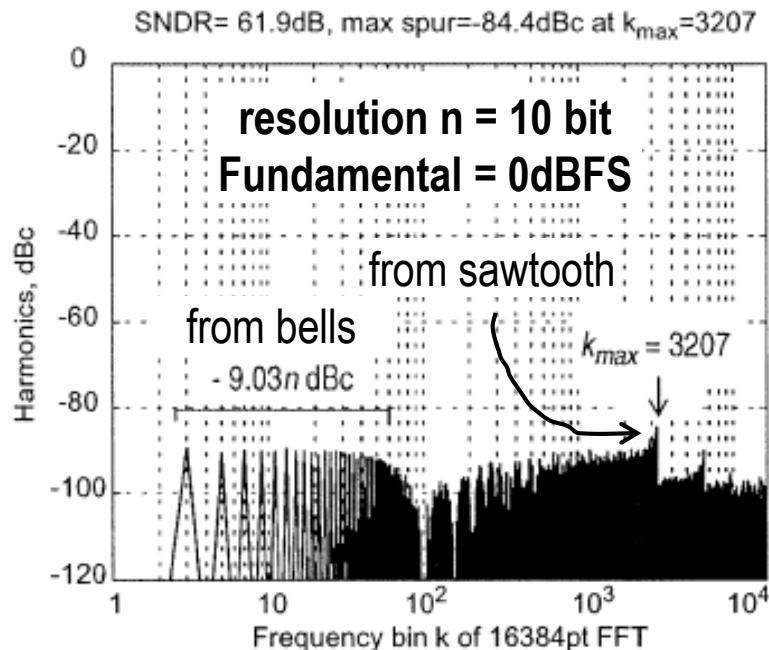


$$\text{Sawtooth period} \sim \text{LSB} / (2\pi f_{in} A) = 1 / (2^n \pi f_{in})$$

$$\text{Max harmonic freq.} \sim (2^n \pi) f_{in}$$

$$\text{Max harmonic order } k_{\max} \sim 2^n \pi$$

$$\text{Max harmonic level} \sim -9n + c \text{ (dBFS)}$$



- Quantization causes very high-order distortion (spurs).
- Spur level actually rises as signal amplitude decreases.**
- All spurs fold within Nyquist band as a result of sampling.

[Pan, TCAS-I 2004]

# Ideal ADC SNR & SFDR, and Real ADC ENOB

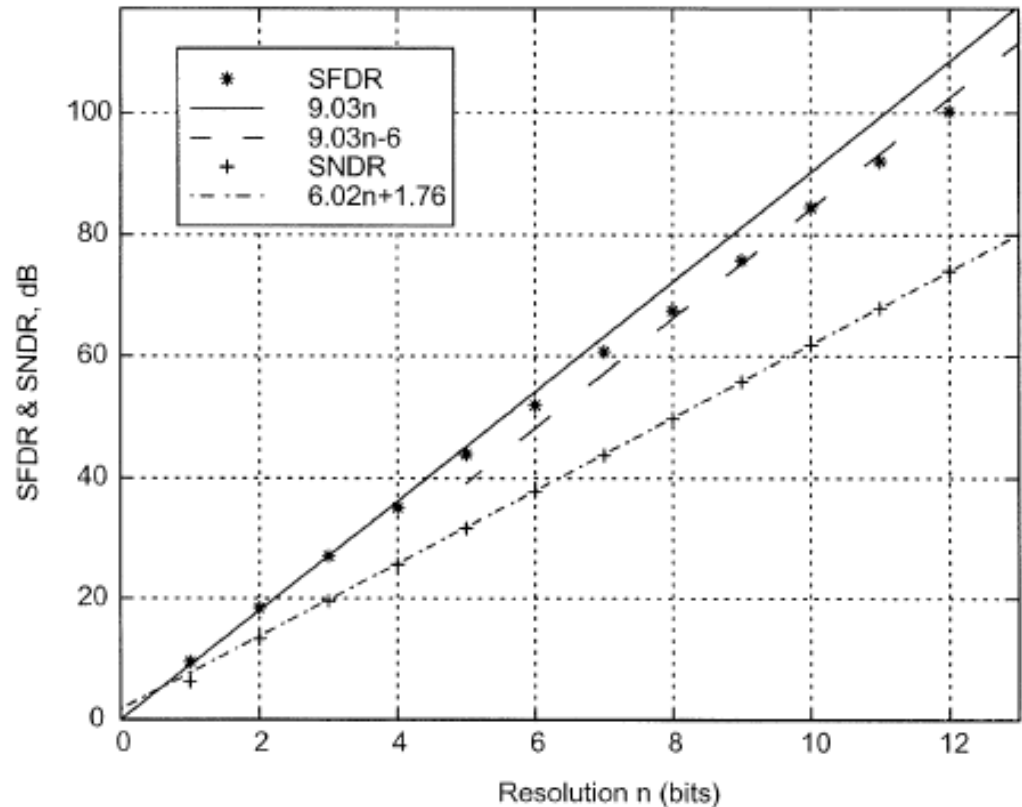
$$\begin{aligned}\text{Single-tone Signal Power } P_s \\ &= FS^2 / 8\end{aligned}$$

$$\begin{aligned}\text{Noise \& Distortion Power } P_e \\ &= \text{LSB}^2 / 12\end{aligned}$$

$$\begin{aligned}\text{Single-tone Spur Level } P_d \\ &\approx -9n \text{ dBFS}\end{aligned}$$

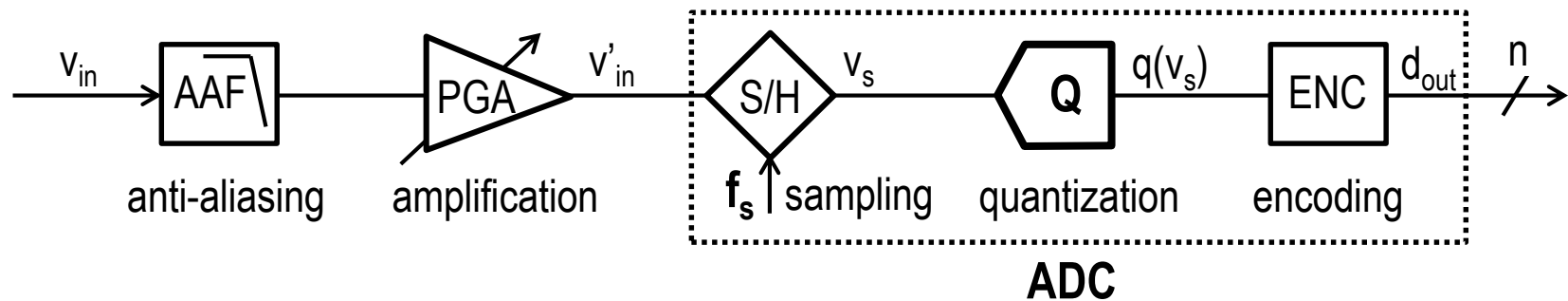
$$\begin{aligned}\text{SNDR} &\equiv P_s / P_e \\ &= 6.02n + 1.76 \text{ (dB)}\end{aligned}$$

$$\begin{aligned}\text{SFDR} &\equiv P_s / \max. P_d \\ &= 9.03n + c \text{ (dBFS)} \\ (c \approx -6)\end{aligned}$$



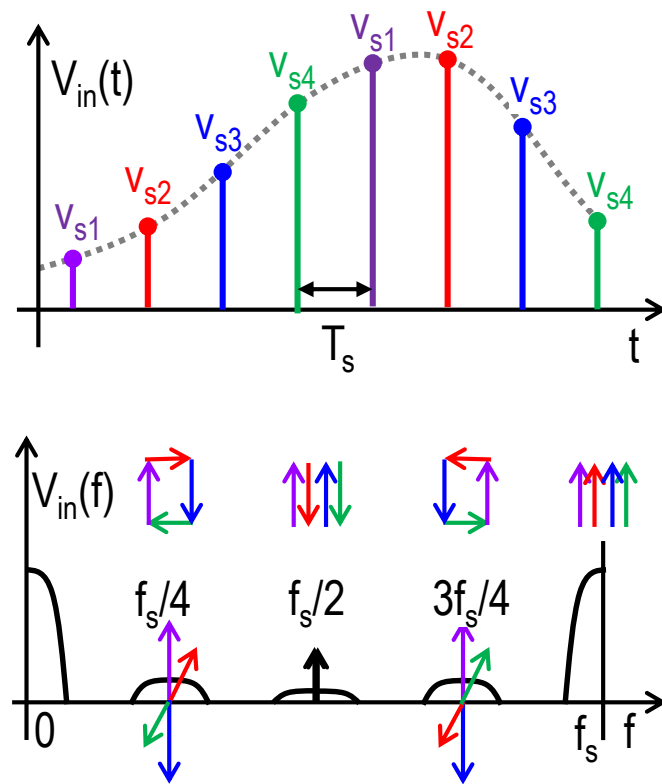
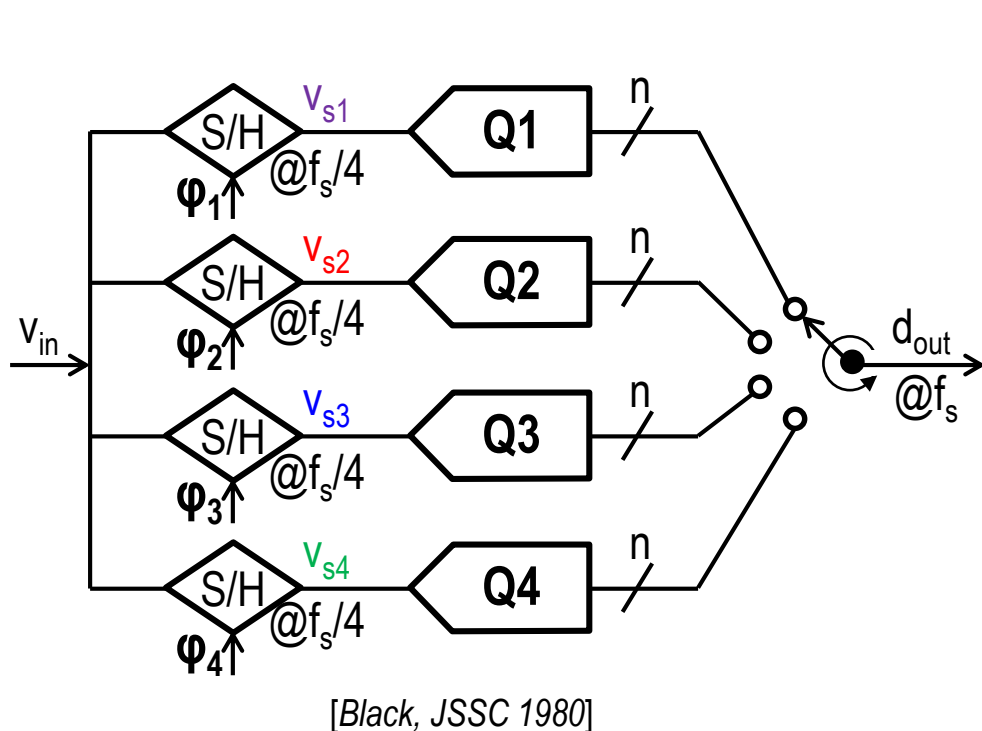
For real quantizers,  
Effective Number of Bit (ENOB) = (SNDR – 1.76) / 6.02

# A/D Converters and Systems



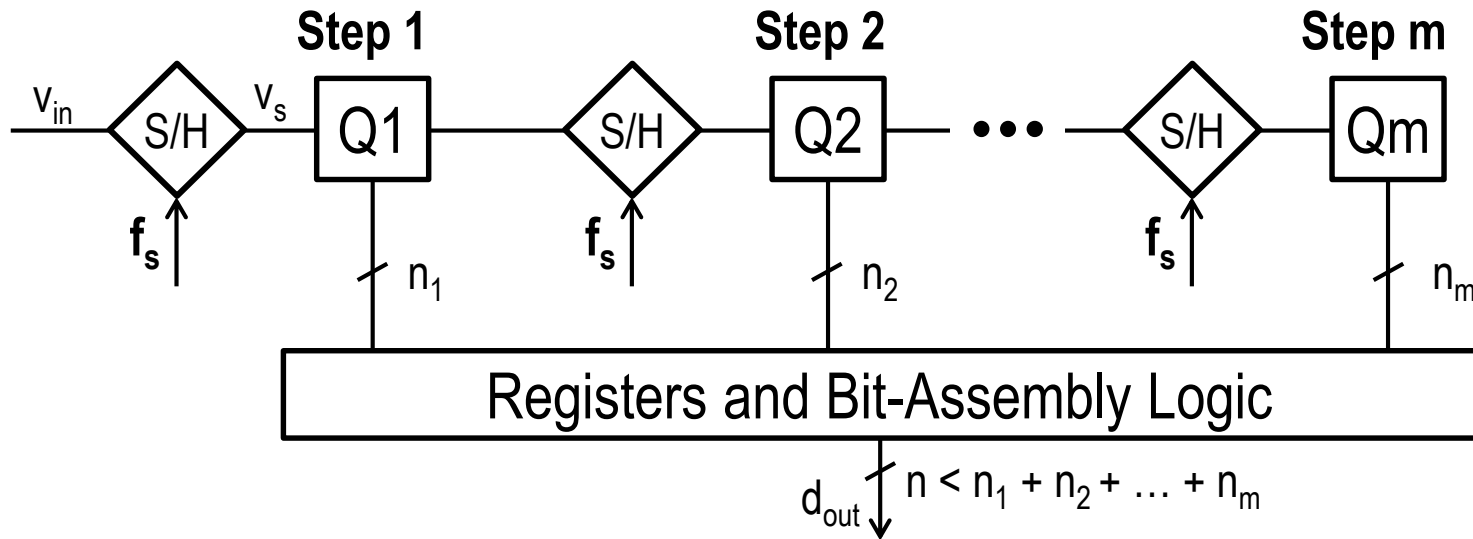
- An ADC consists of sampling, quantization, and encoding
  - AAF & PGA in different design categories or merged with S/H
  - Operation sequence permutable, but usually same as shown
- In a narrow sense, an ADC consists of only quantization.
  - S/H merged with Q or eliminated for clockless ADCs
  - Encoder merged with Q or included in succeeding DSP
- In a wide sense, an ADC is a compound system that consists of one or more quantizer cores and some auxiliary functionalities for enhancing conversion speed, resolution, efficiency, etc.

# Sampling Rate Multiplication by Time-Interleaving



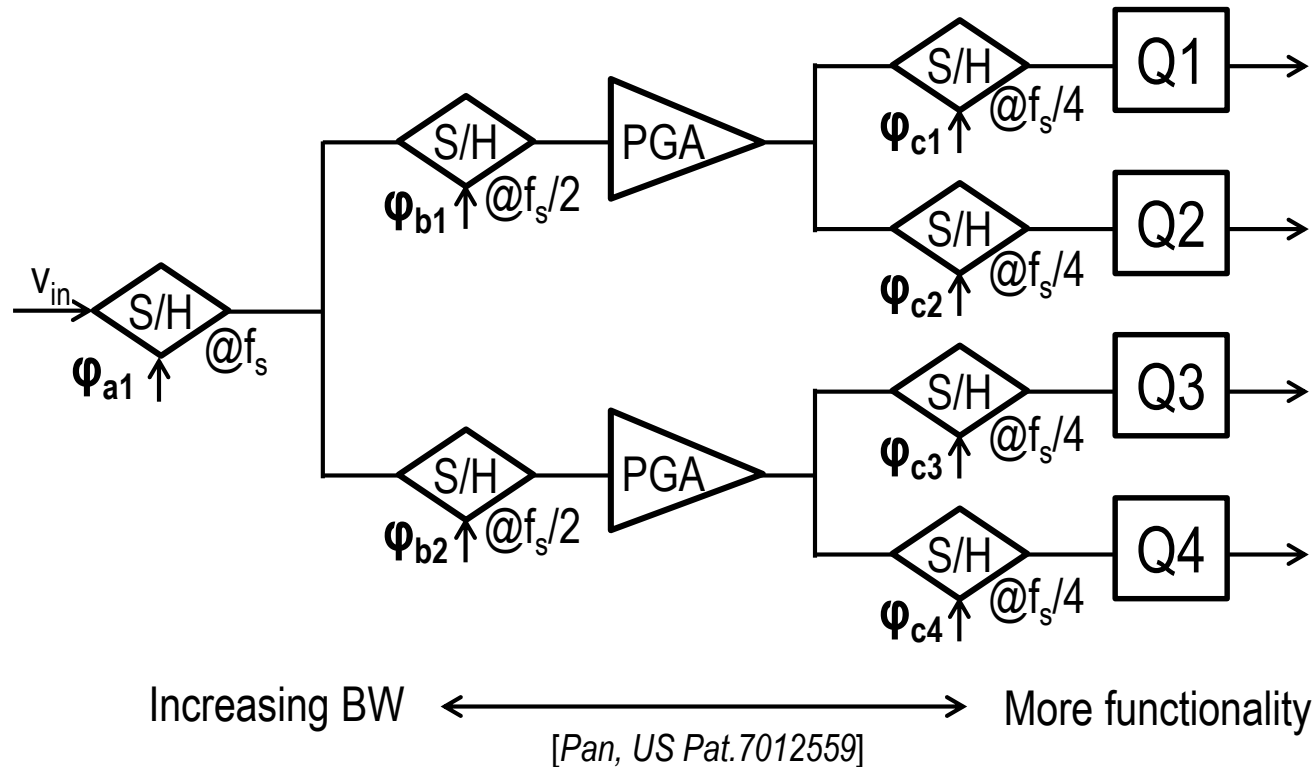
- Conversion rate linearly increases with  $m$  = number of time-interleaved ADCs.
- S/Hs are bottleneck for Nyquist operation as the tracking BW must scale with  $m$ .
- Errors arise from kickback and channel mismatch in offset, gain, BW, timing, etc.
- Error correction can be merged with succeeding digital signal processing.

# Conversion Rate Multiplication using Pipelining



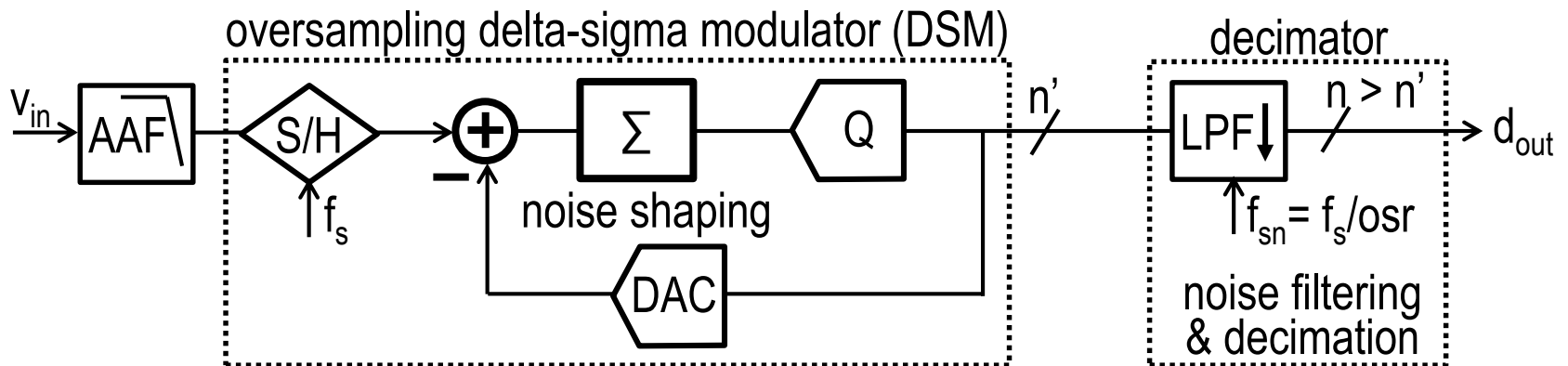
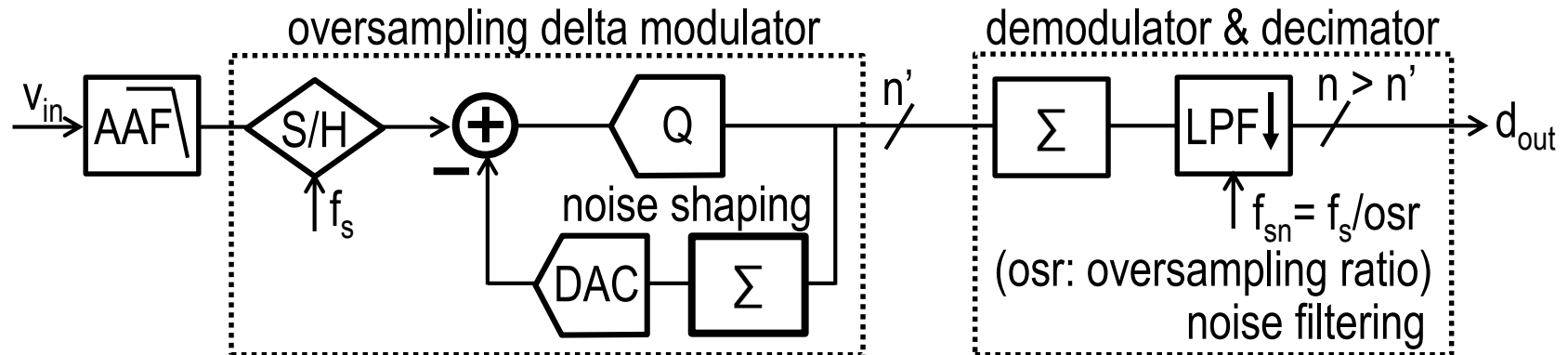
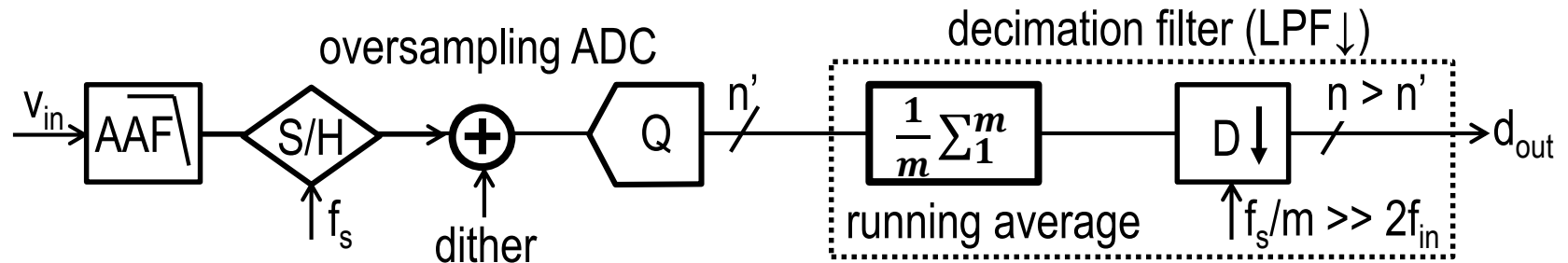
- Conversion rate linearly increases with  $m$  = number of pipeline stages, which decouple quantization throughput from latency.
- Sample-and-holds for pipeline storage must settle to full accuracy at full speed, best suited for implementation with switched capacitor circuits in CMOS technology [Lewis, JSSC 1987].
- Accurate settling is challenging esp. in deep submicron CMOS.

# Hierarchical Parallel and Pipelined A/D Conversion

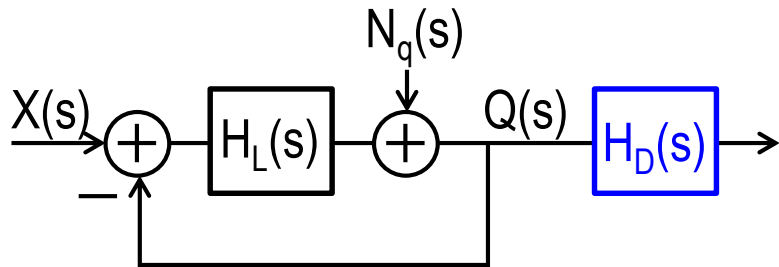
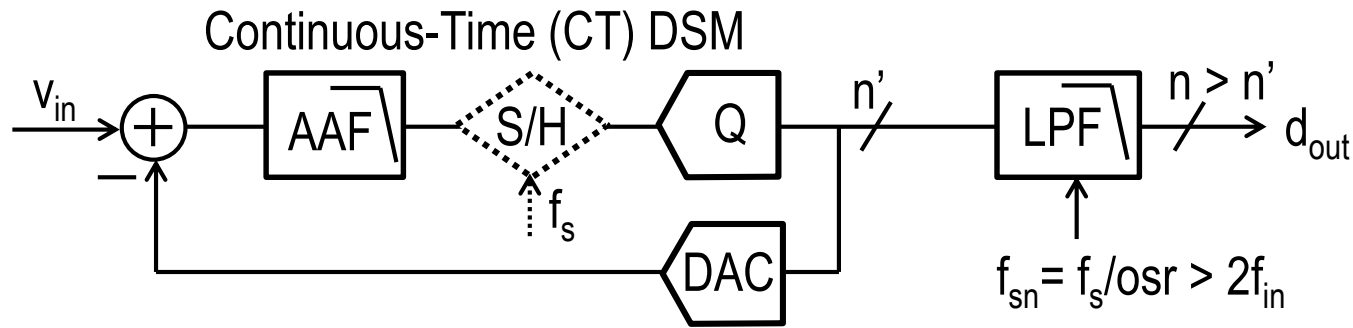


- BW and functionality grow in opposite directions in the hierarchy.
- Number of head-end channels is minimized for max. BW.
- Phase alignment necessary among multi-rate multi-phase clocks.
- Clock jitter and skew cause little error after first stage S/H(s).

# Resolution Boost by Oversampling & Noise Shaping



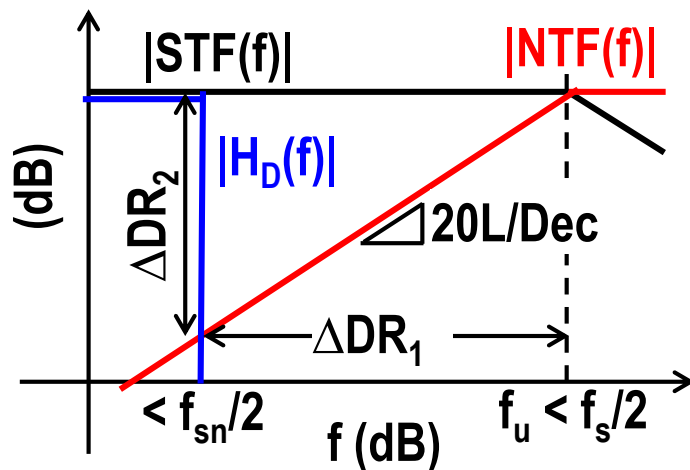
# Noise and Signal Transfer Functions (NTF & STF)



For  $H_L(s) = s_u/s$

$$\text{STF}(s) = Q(s) / X(s) = 1/(1+s/s_u)$$

$$\text{NTF}(s) = Q(s) / N_q(s) = (s/s_u)/(1+s/s_u)$$



$$\Delta\text{DR}_1 \sim 10 \log(\text{OSR})$$

$$\Delta\text{DR}_2 \sim 20L \log(\text{OSR})$$

$$\Delta\text{DR}(\text{dB}) \approx \Delta\text{DR}_1 + \Delta\text{DR}_2 \approx (6L+3) \log_2(\text{OSR})$$

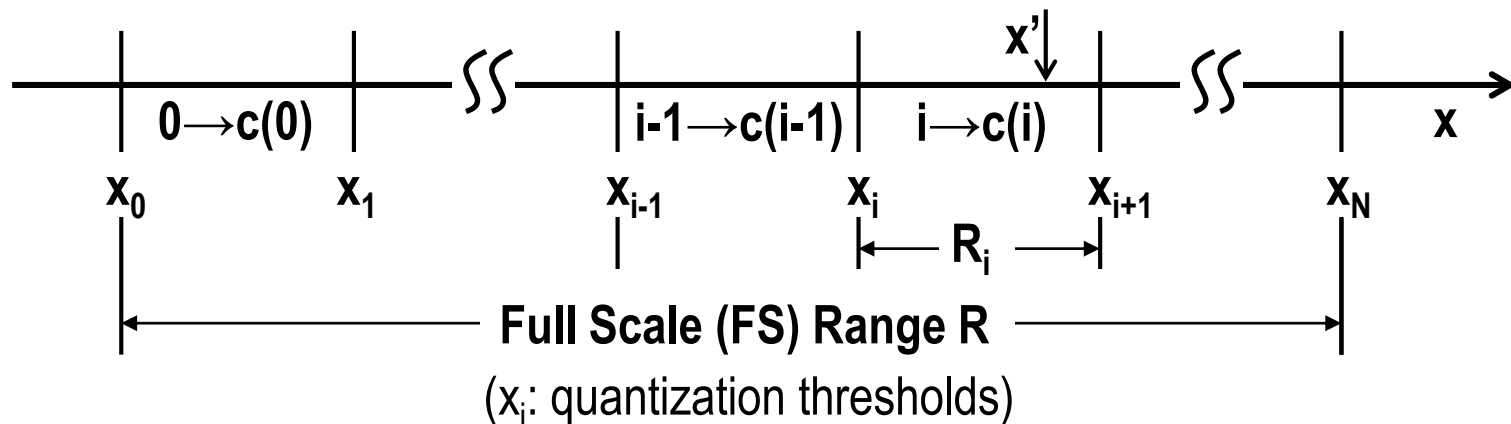
$$L = H_L(s) \text{ order; } \text{OSR} = f_s / f_{\text{sn}}$$

# Algorithms and Architectures

# Quantization $Q(x)$ as a Search Process

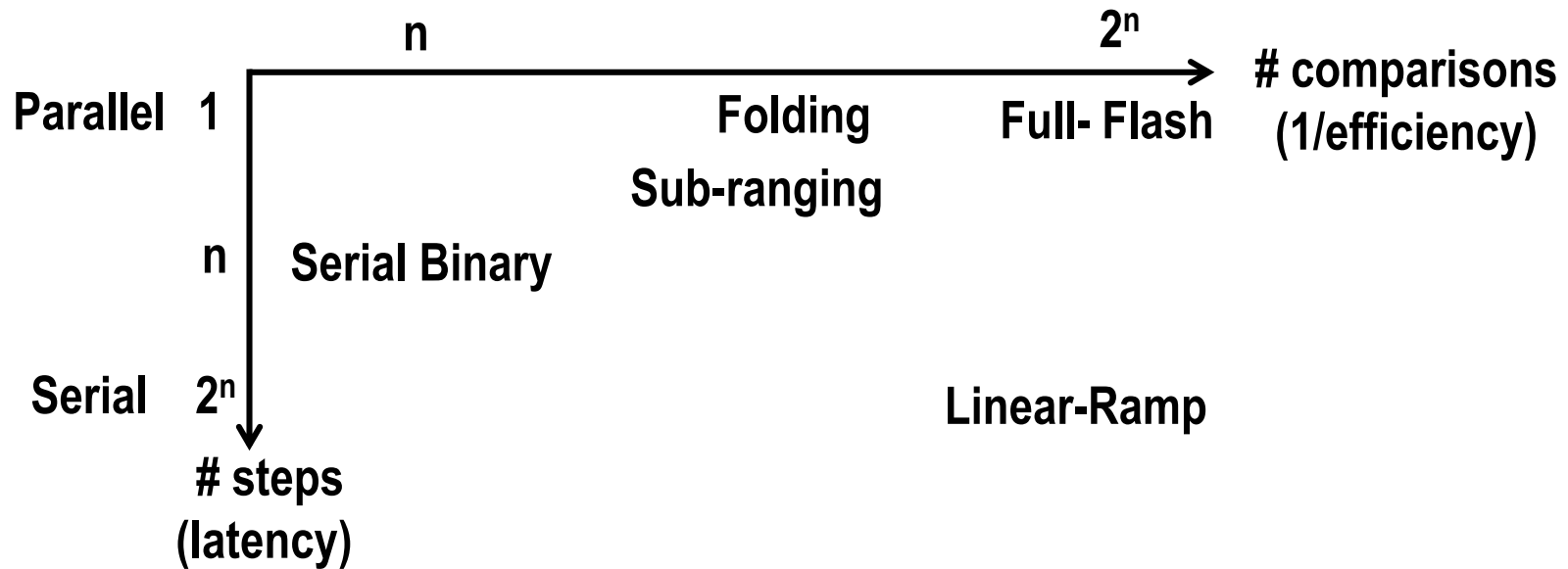
$$Q(x) = i \rightarrow \text{codeword } C(i) = \{s_1 s_2 \dots s_m\}$$

(e.g.  $\{C(0), C(1), \dots, C(7)\} = \{000, 001, \dots, 111\}$ )



- $Q(x) = i$  for  $x \in R_i, i = 0, 1, \dots, N-1; N = 2^n$  for  $n$ -bit quantizer.
- Sub-range  $R_i = x_{i+1} - x_i = R/N = \text{LSB}$  for uniform quantization.
- A quantizer searches for the  $R_i$  in which a given  $x$  falls.
- An algorithm drives the search based on  $(x, x_i)$  comparison.
- An optional encoder  $C$  maps  $Q(x) = i$  to a codeword  $C(i)$ .
- $Q$  and  $C$  can be merged to resolve each code symbol  $s_k$  directly.

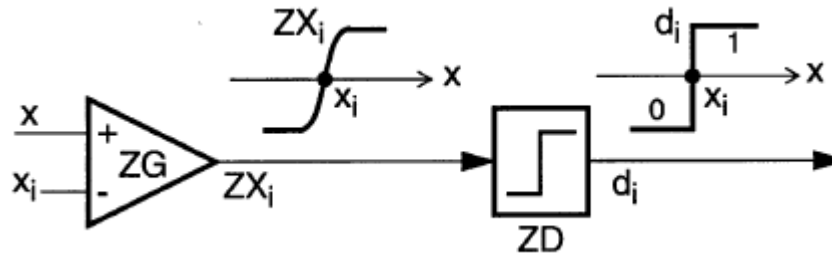
# Search Algorithms for Quantization



- Parallel brute-force (full-flash) : one step,  $\sim 2^n$  comparisons
- Serial brute-force (linear ramp up/down):  $\sim 2^n$  steps and comparisons
- Serial binary search (1b/step):  $n$  steps and comparisons
- Coarse-fine search (sub-ranging):  $1 < \text{steps} < n$ ,  $n < \text{comparisons} < 2^n$
- Search efficiency (# comparisons) dominates power efficiency

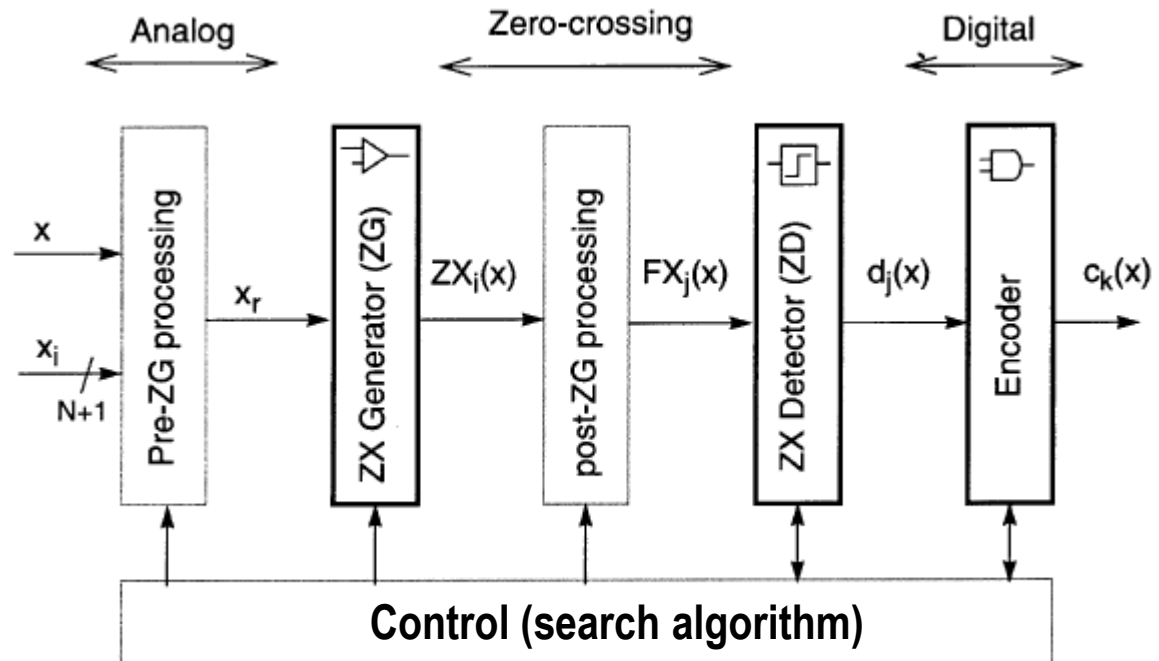
# Quantization Elements and Generic Architecture

**1b quantizer  
(comparator)**



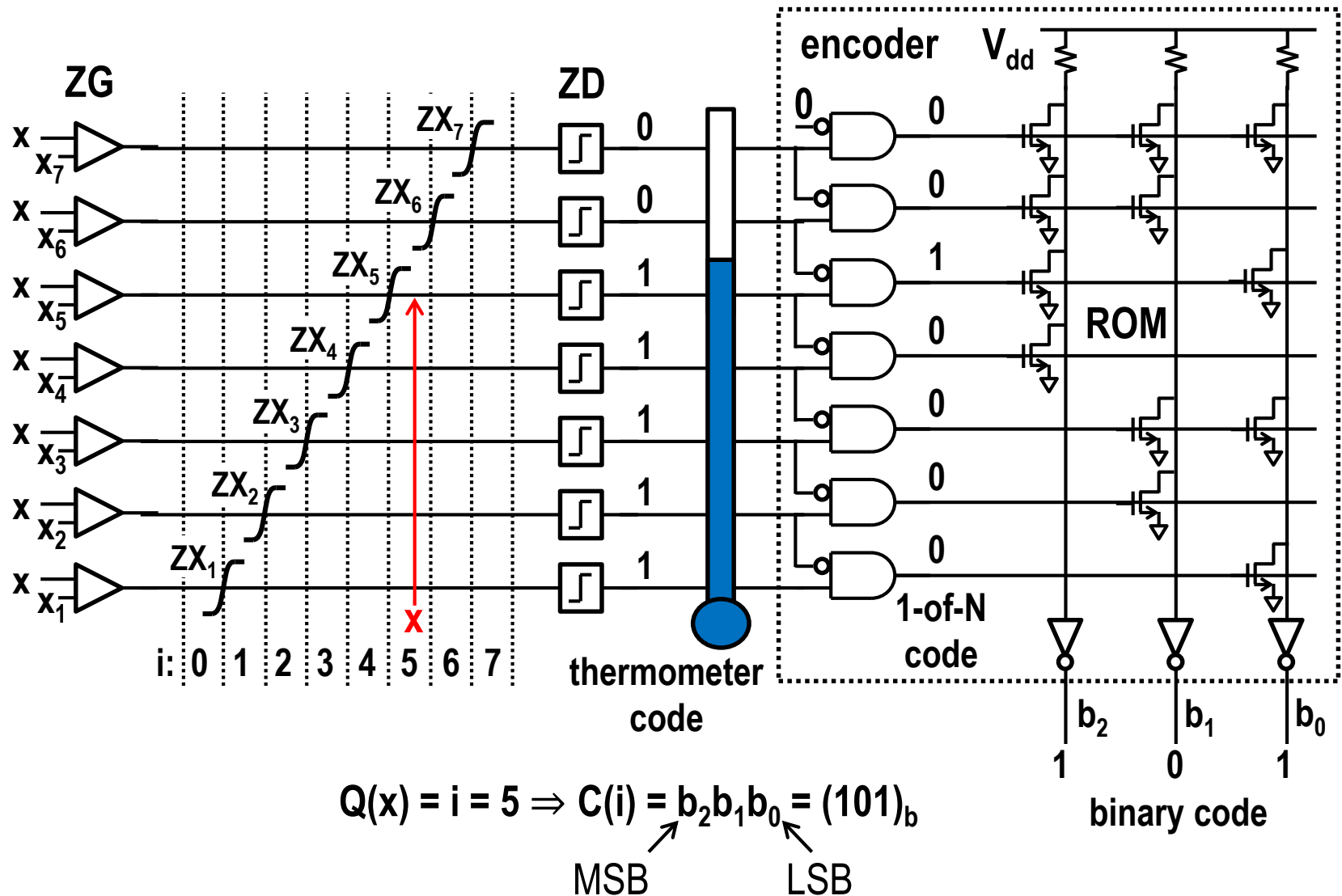
**generic  
quantizer**

[Pan, TCAS-I 2004]

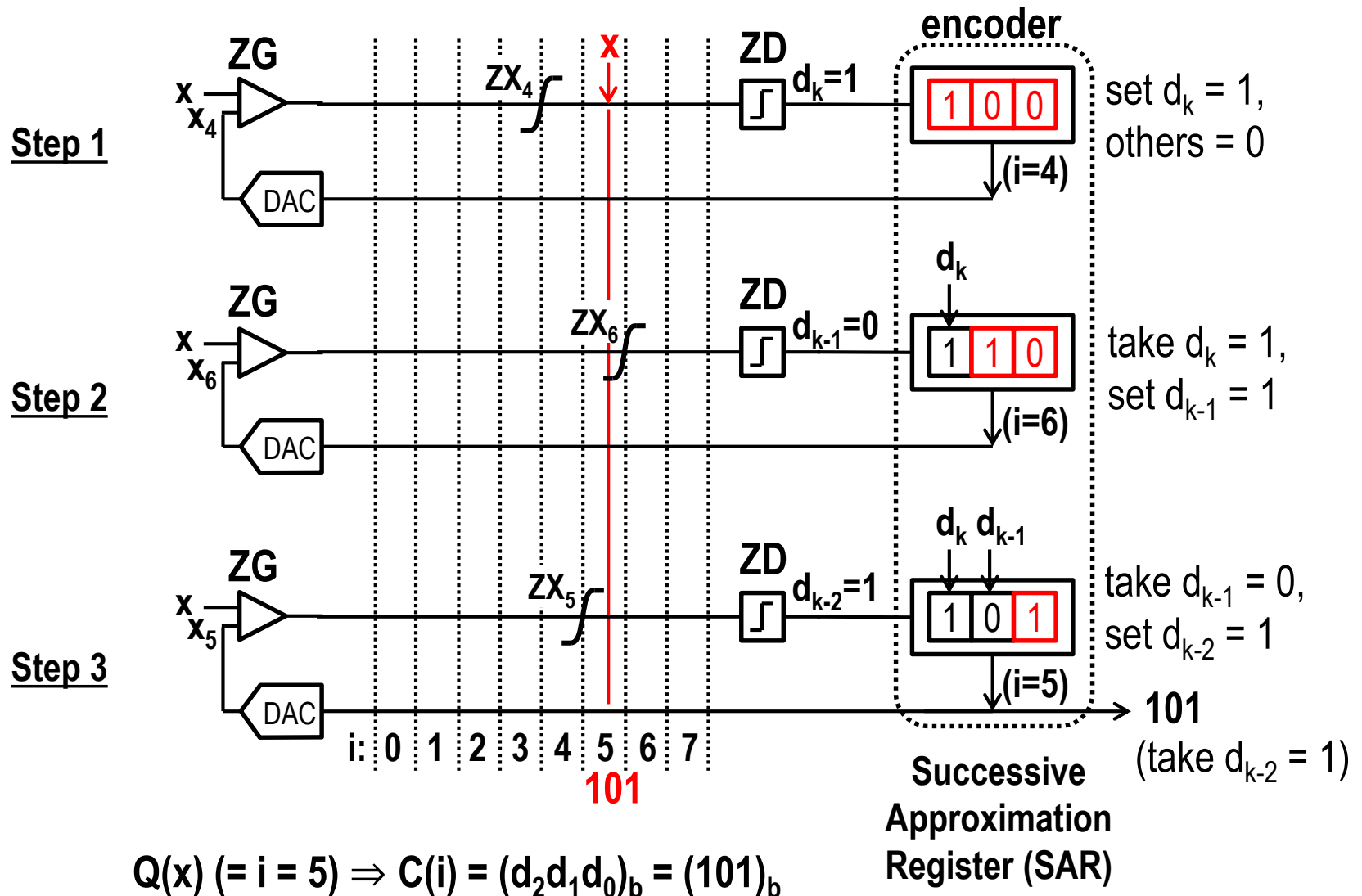


- Quantizer core elements: ZX generation (ZG) and ZX detection (ZD)

# Parallel Brute-Force Search: Full-Flash

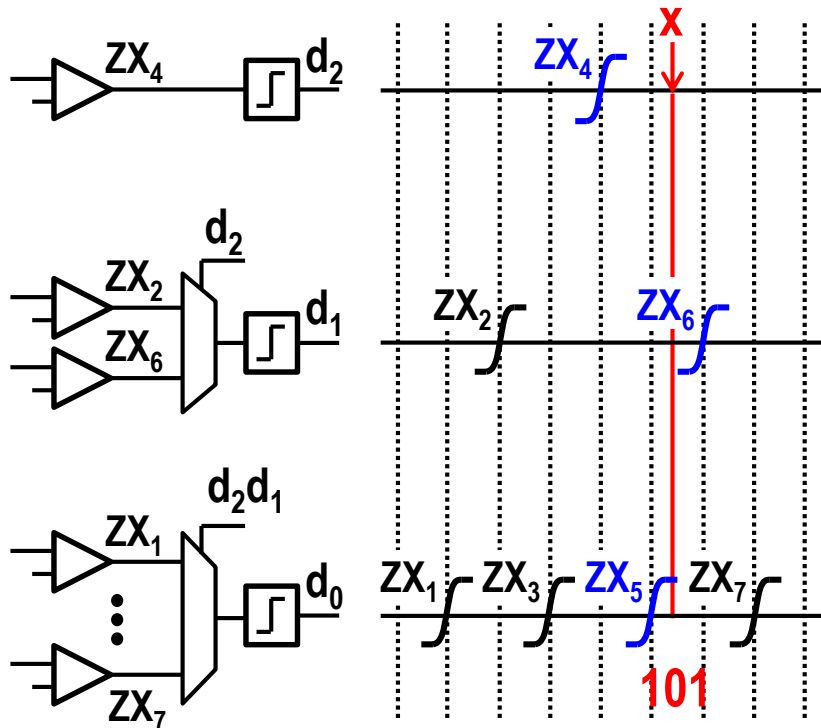


# Serial Binary Search: Successive Approximation

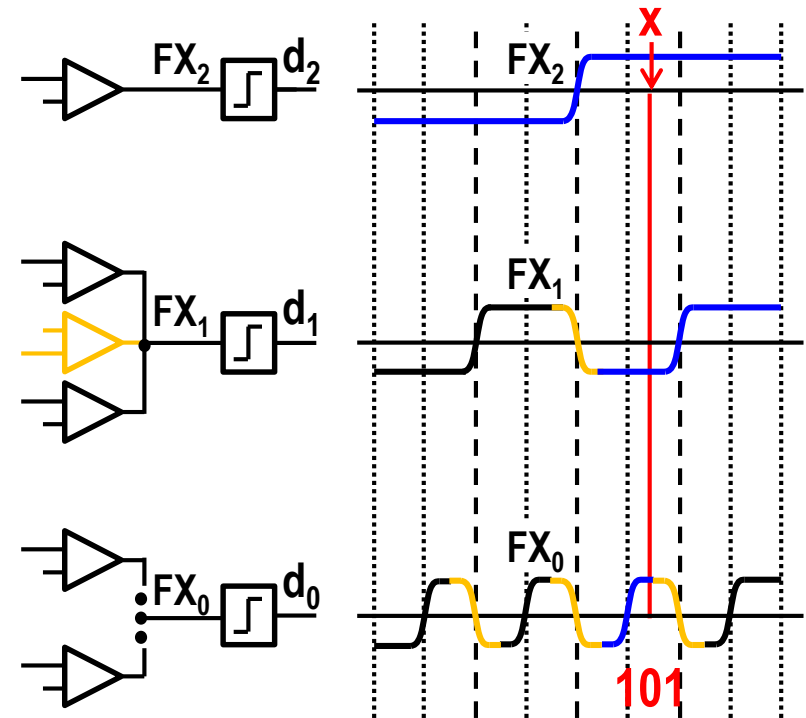


# Parallel Binary Search: Analog Encoding

n-step serial binary search  
by multiplexing ZXs

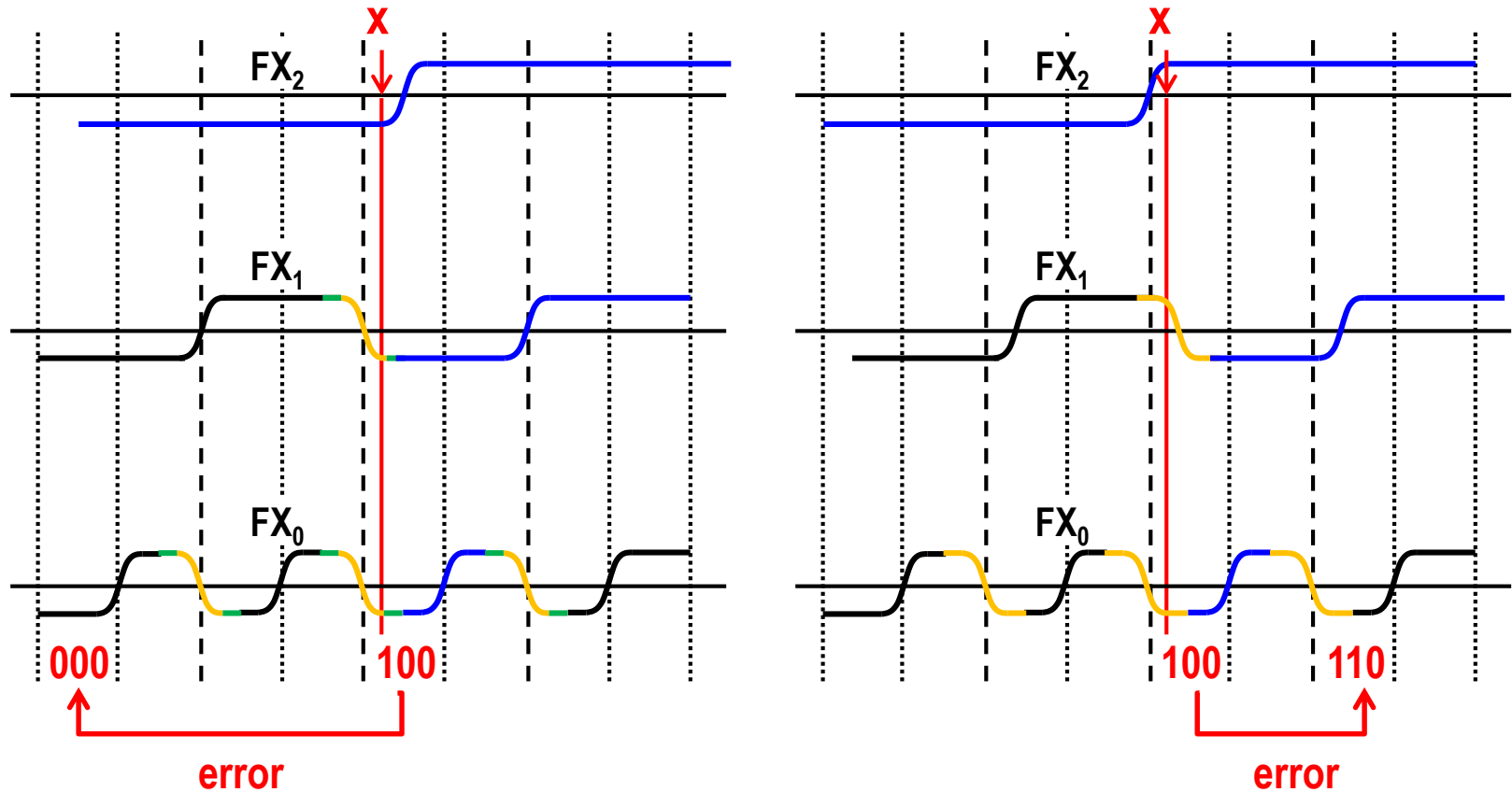


One-step parallel binary search  
by signal folding



- Signal folding merges ZG and encoding → analog encoding.
- Signal folding is equivalent to automatic multiplexing of ZXs.
- However, there is algorithmic conflict among redefined ZXs.

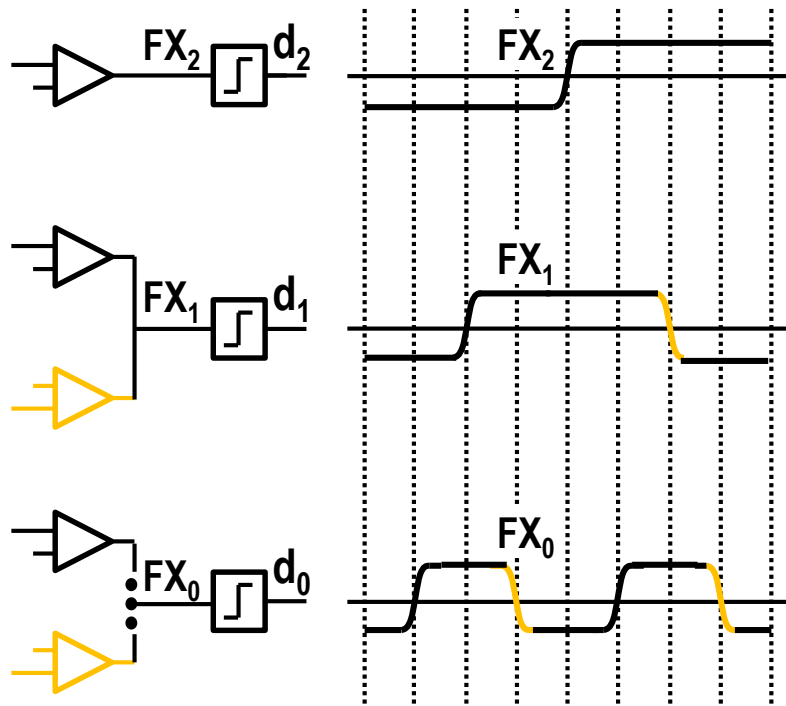
# Potential Algorithmic Conflict in Binary Search



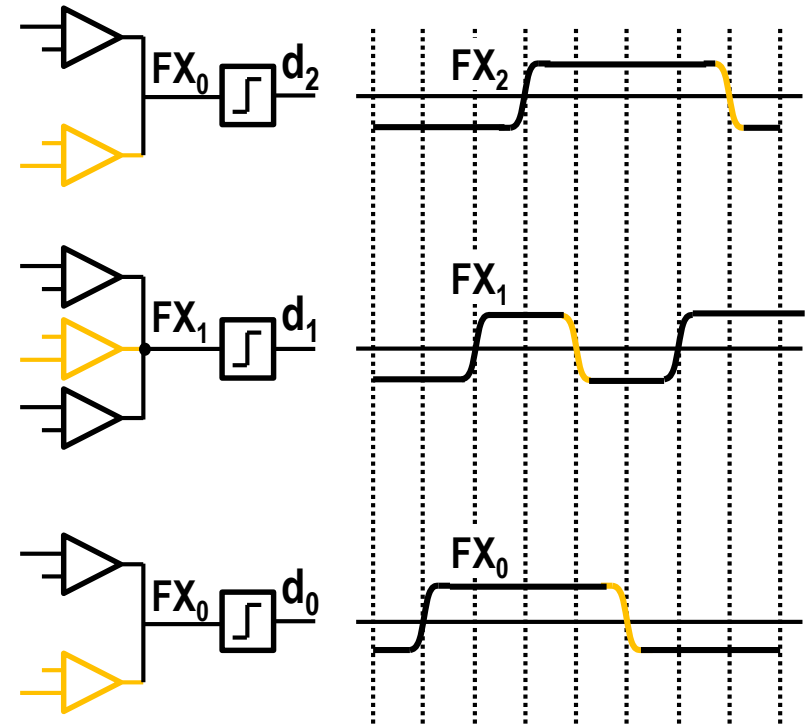
- Midpoint  $ZX$  is redefined in each parallel channel causing alignment issue, potential algorithmic conflict and large quantization error.

# Binary Search Without Algorithmic Conflicts

One-step parallel binary search  
without redefining ZXs  $\rightarrow$  Gray code

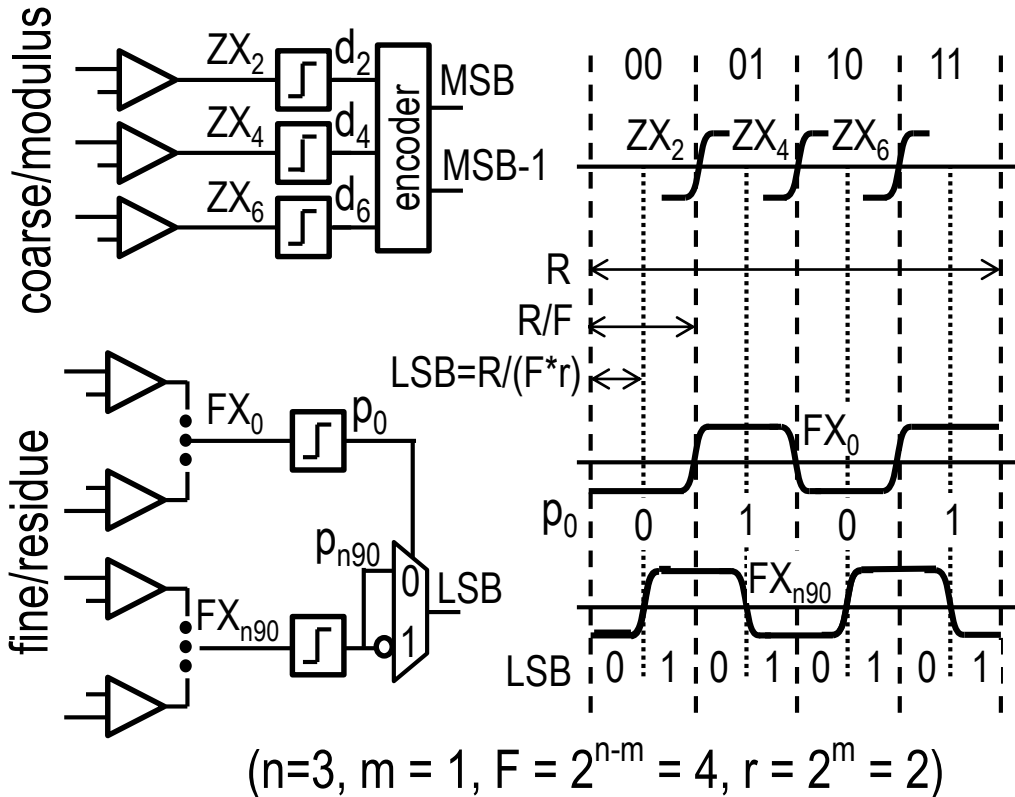


An arbitrary Gray code variant  
(one bit flip between adjacent  $R_i$ 's)



- Gray codes are robust without redefining ZXs.
- One step, total  $2^n$  ZGs but  $n$  instead of  $2^n$  ZDs.

# Parallel Coarse-Fine Search: Modulus-Residue Code



Bit-Sync (align redefined ZXs):

$$MSB-1 \xrightarrow{s=1} (MSB-1)_s$$

$$MSB \xrightarrow{s=1} (MSB)_s$$

where  $s = (MSB-1) \oplus p_0$   
(see next page)

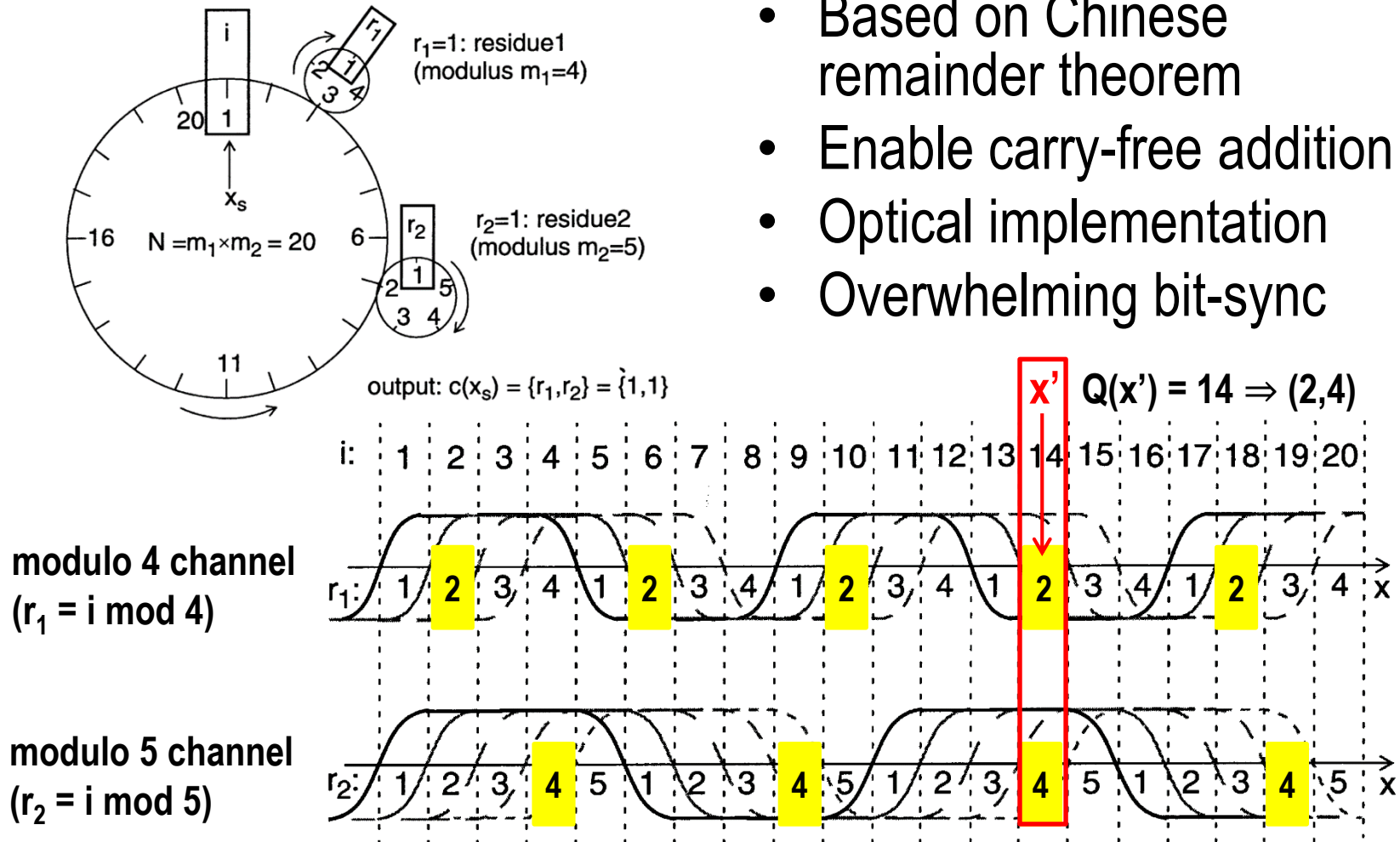
Output Binary Code:

$$C = (MSB)_s (MSB-1)_s (LSBs)$$

- $(F-1)$  ZXs (e.g.  $ZX_{2,4,6}$ ) in coarse channels divide  $R$  into  $F$  folds.
- Fine foldings (e.g.  $FX_0$ ,  $FX_{90}$ ) divide each fold ( $R/F$ ) into  $r$  LSBs.
- Bit-sync aligns coarse ZXs to fine channels. [van de Grift, JSSC 1987]

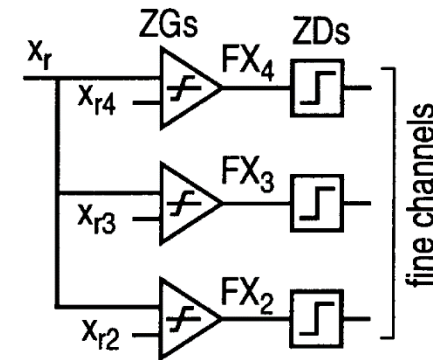
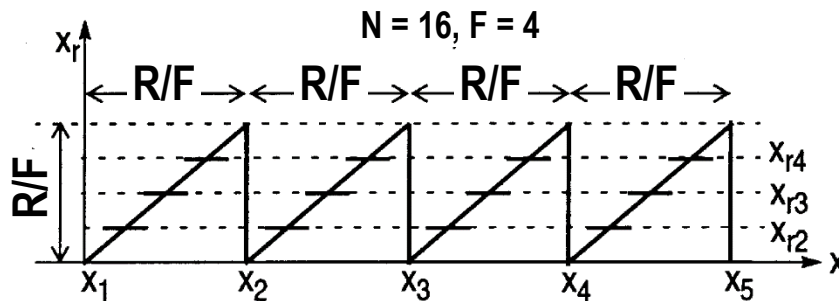
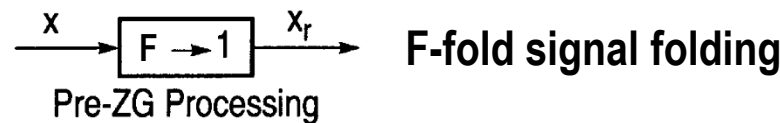
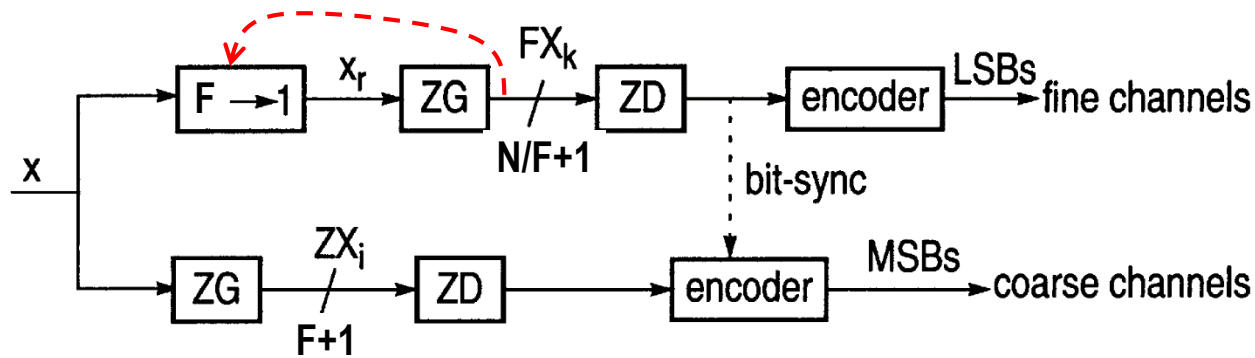
# Parallel Residue Search: Residue Encoding

- Based on Chinese remainder theorem
- Enable carry-free addition
- Optical implementation
- Overwhelming bit-sync



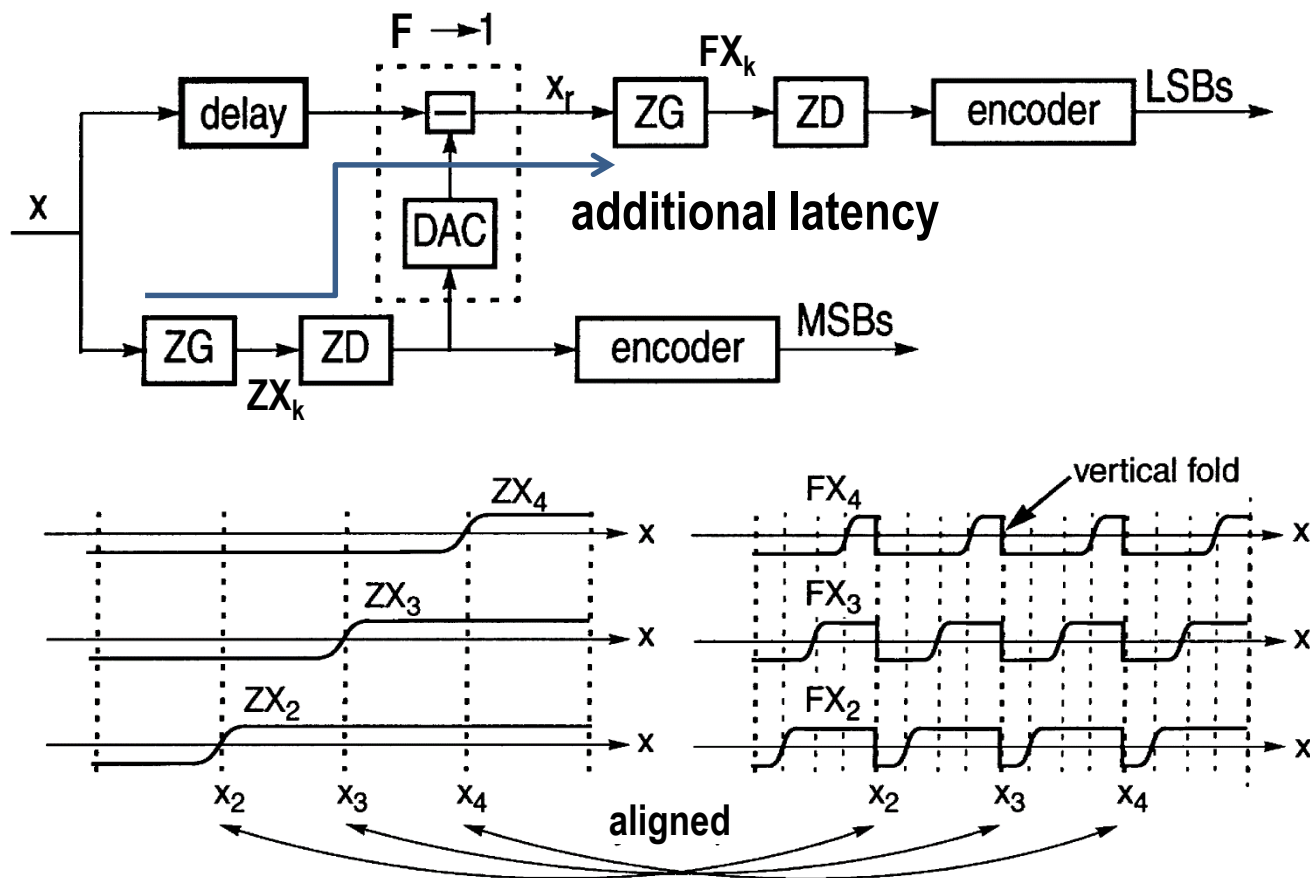
- Each symbol represents an independent partial search/quantization.
- Each encoding system represents a partition and search algorithm.

# Pre-ZG Folding Reduces ZG Complexity



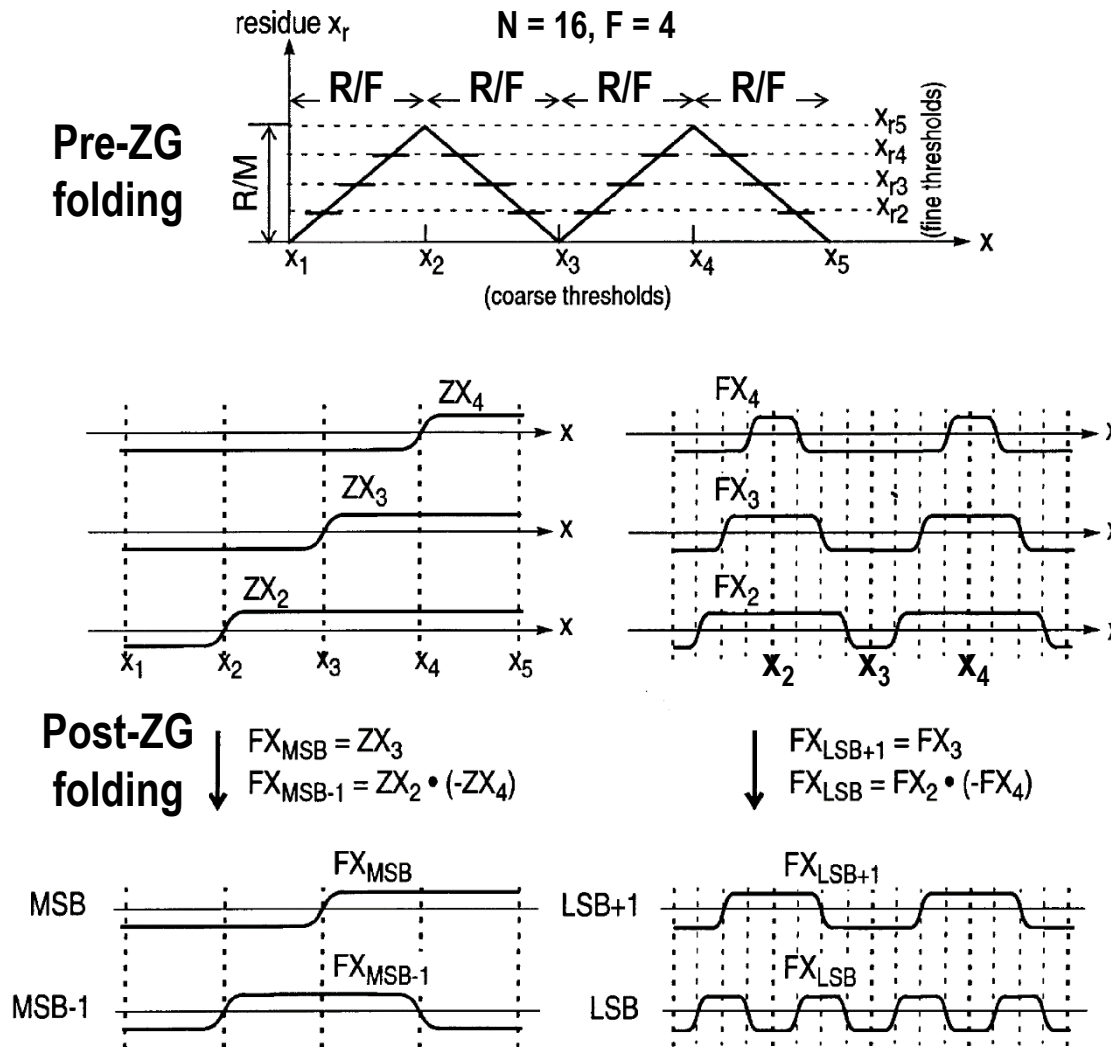
- $F \rightarrow 1$  mux(folding) moved before ZG to reduce both ZGs and ZDs by  $F$
- Overhead: folding linearity, coarse quantizer to resolve  $1 \rightarrow F$  ambiguity

# Serial Folding without Bit-Sync Issue



- Serial folding operation ensures bit sync at the cost of latency and matching.
- Analog delay can be avoided with a S/H driving both coarse and fine channels.
- Coarse threshold error does not matter if folding signal falls in fine quantizer FS.

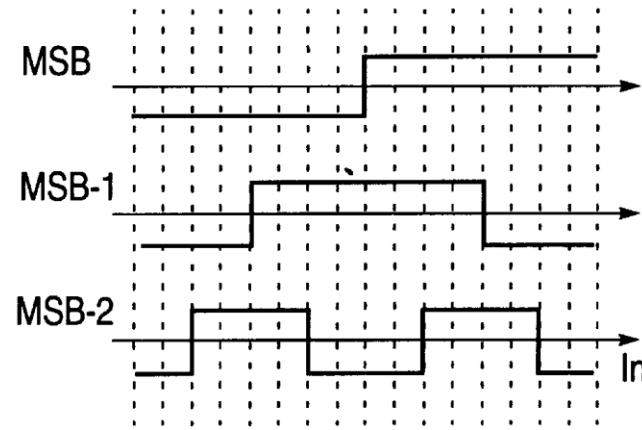
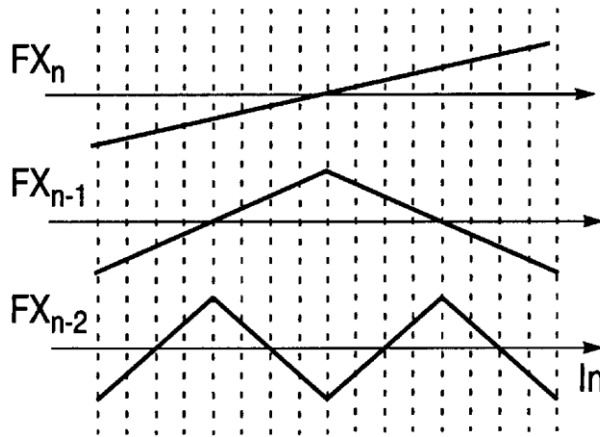
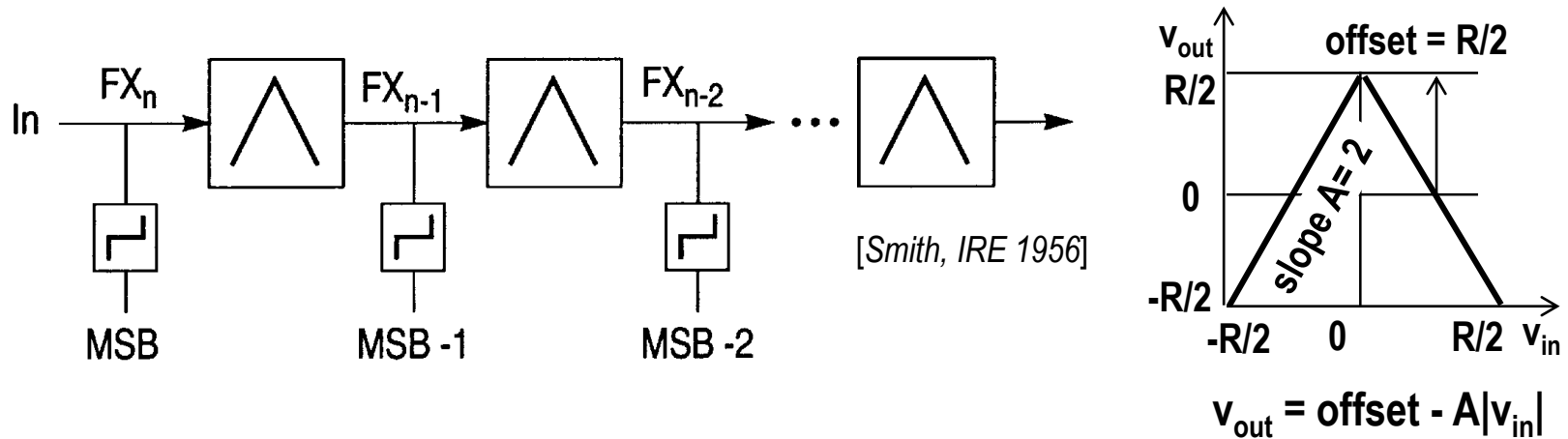
# Parallel Folding without Bit-Sync Issue



[van de Plassche, JSSC 1988]

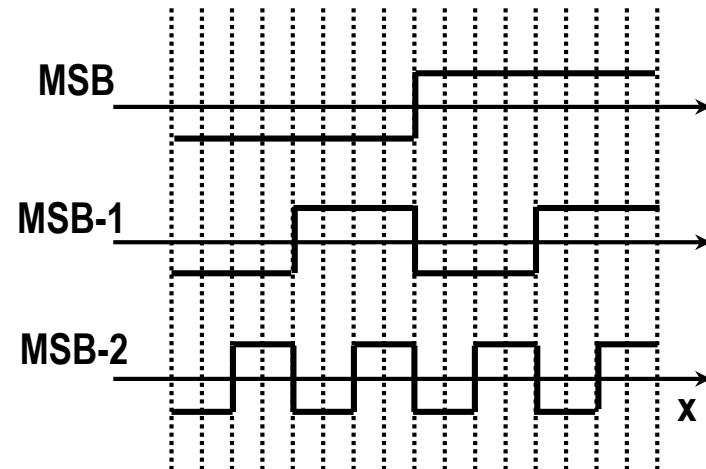
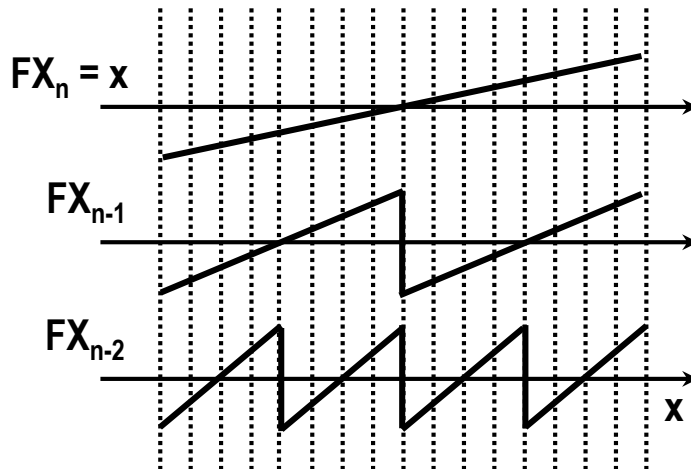
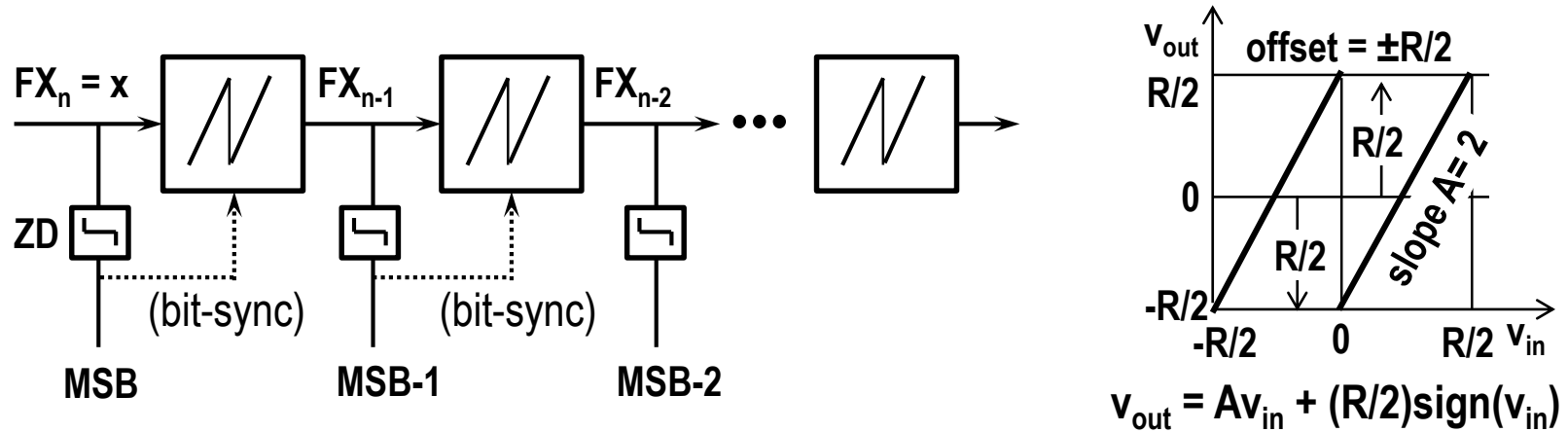
- Parallel coarse-fine channels: low latency
- No vertical folding: robust quantization
- Gray code: no bit-sync and low complexity
- Rounded folding tip: linearity bottleneck

# Ripple Through Folding for Gray Coding



- Efficient discrete (rectifier based) ADCs for PCM, video and radar.
- $FS_{out}(i) = 2 * \text{offset}(i)$  should match  $FS_{in}(i+1) = 4 * (\text{offset}(i+1) / A(i+1))$ .
- $A = 2$  is not required as long as  $\text{offset}(i+1) = (A(i+1) / 2) * \text{offset}(i)$ .

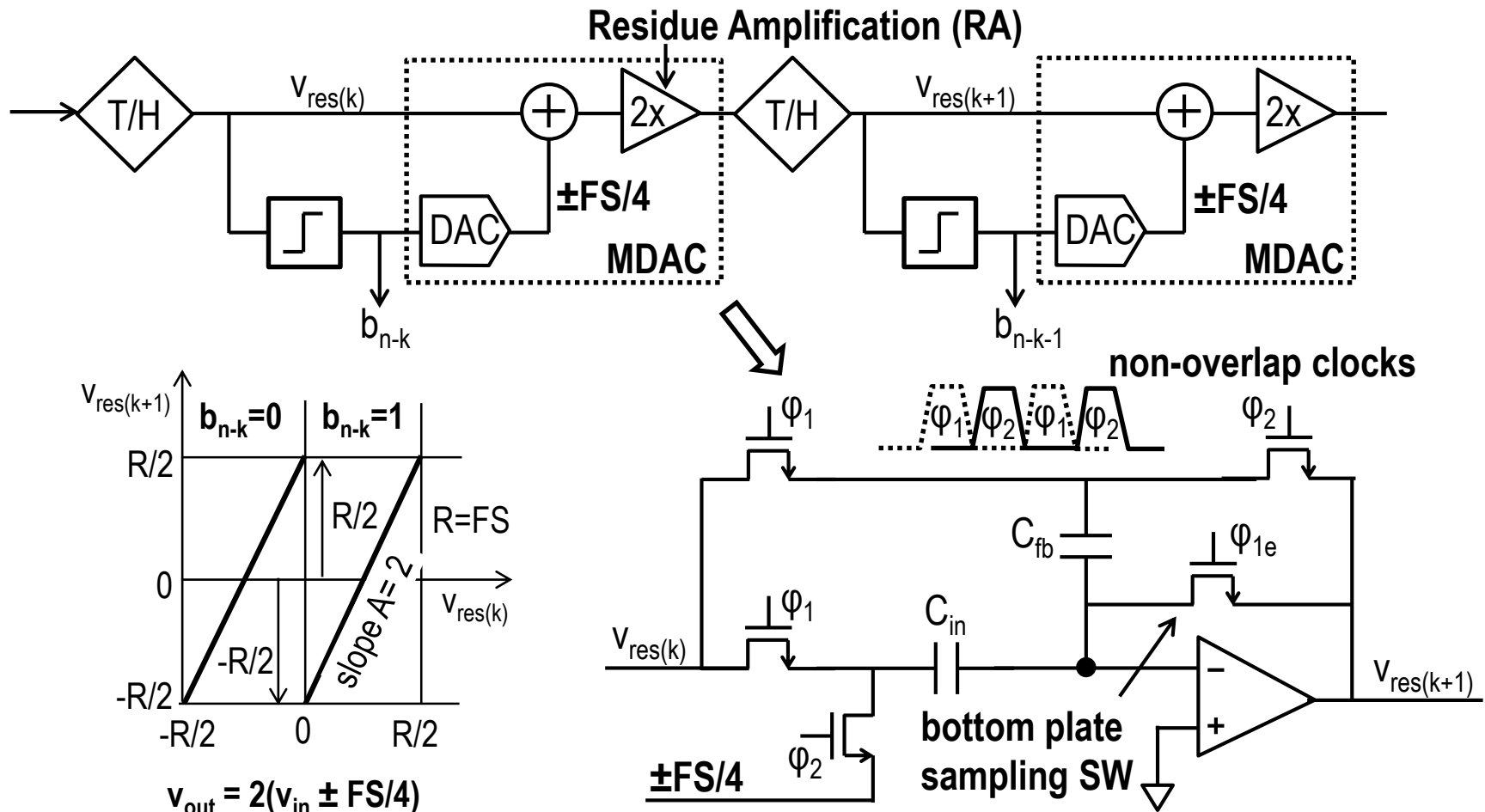
# Serial Folding for Binary Coding



- Vertical folds requires the ZD-like decision and serial operation.

# Pipelined Folding Using MDAC Summation

A 1b/stage Pipeline ADC using Multiplying D/A Converters (MDACs)

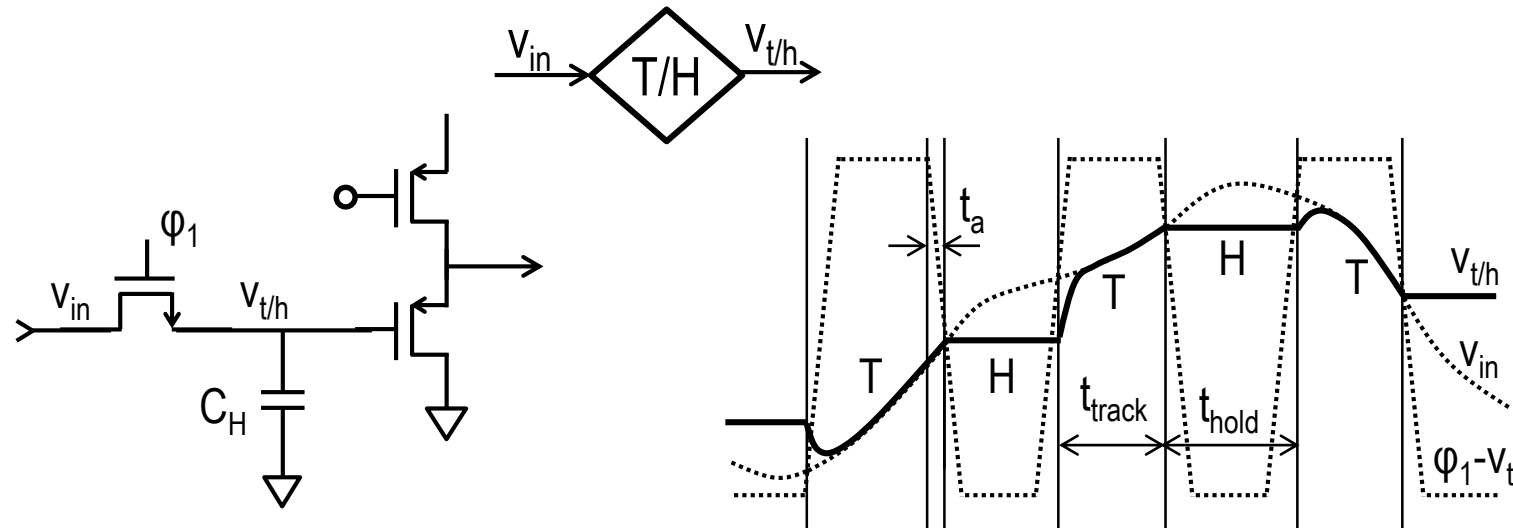


# Discussions on Signal Folding

- Signal folding reduces quantizer complexity using  $F \Rightarrow 1$  operation on the analog signal (analog folding) or the ZX signals (ZX folding),
- The  $F \Rightarrow 1$  runs automatically involving no regeneration in parallel search based quantizers, preserving low latency and high speed.
- Automatic folding is popular in bipolar process where rectifiers are readily available to fold signals independent of any serial detection.
- Ripple-through cascaded V-shape folding was the only practical option for high speed A/D converters before the advent of IC.
- Folding involving serial regeneration is popular in CMOS where switch cap circuits are readily available to pipeline the serial folding.
- However, analog folding suffers nonlinearity at the folding tips; ZX folding suffers mutual loading & interference among the ZX signals.

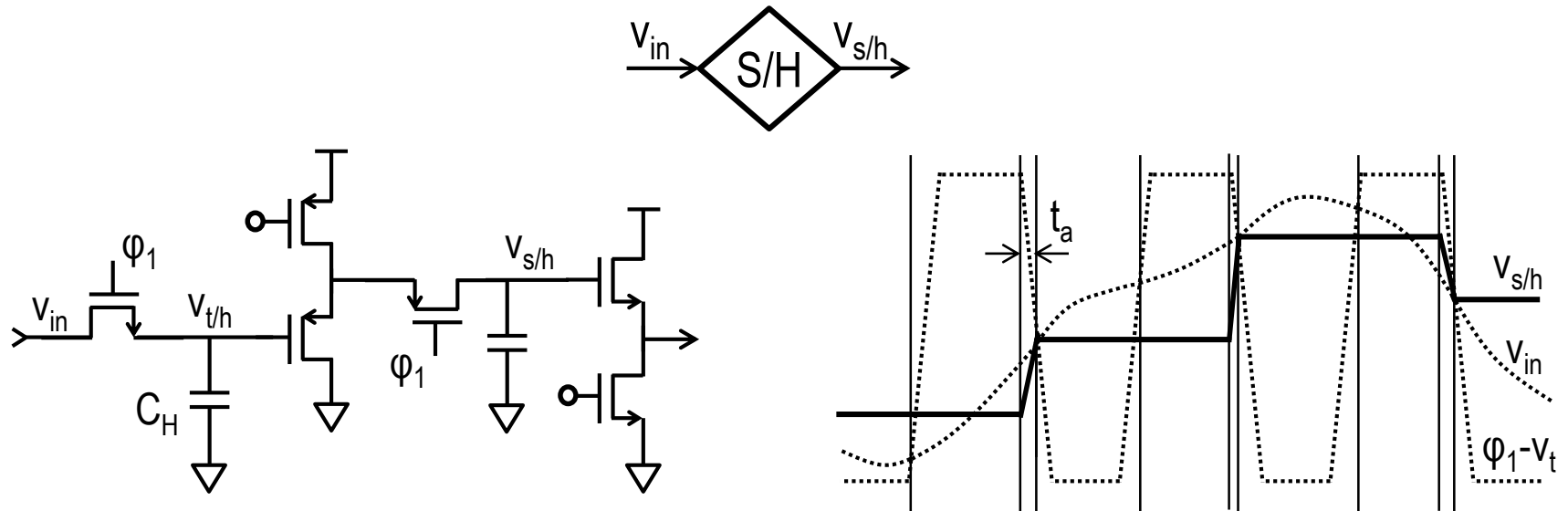
# Circuit Implementations

# Open-Loop Track/Hold (T/H)



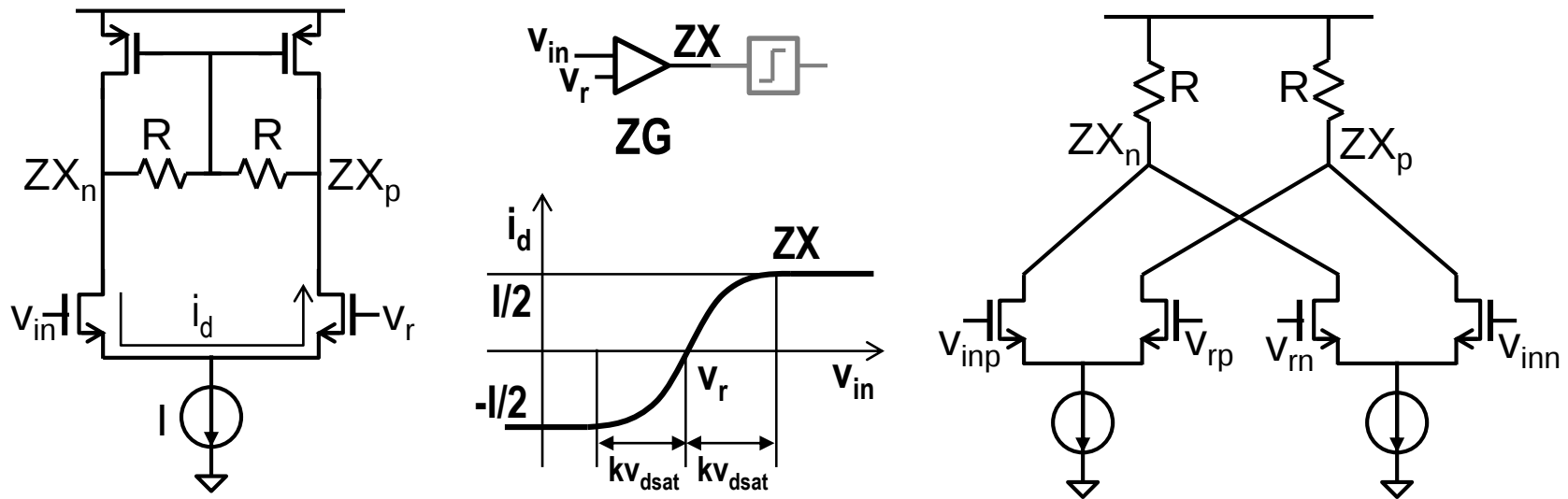
- Sampling aperture time  $t_a \ll 1/f_{in}$  limited by clock edge.
- Tracking  $BW_{track} > f_{in}$  limited by MOSFET  $f_T$ , power, etc.
- Very high speed but resolution limited by errors from charge injection, aperture time/jitter, switch and source follower nonlinearity, etc.

# Open-Loop Sample/Hold (S/H)



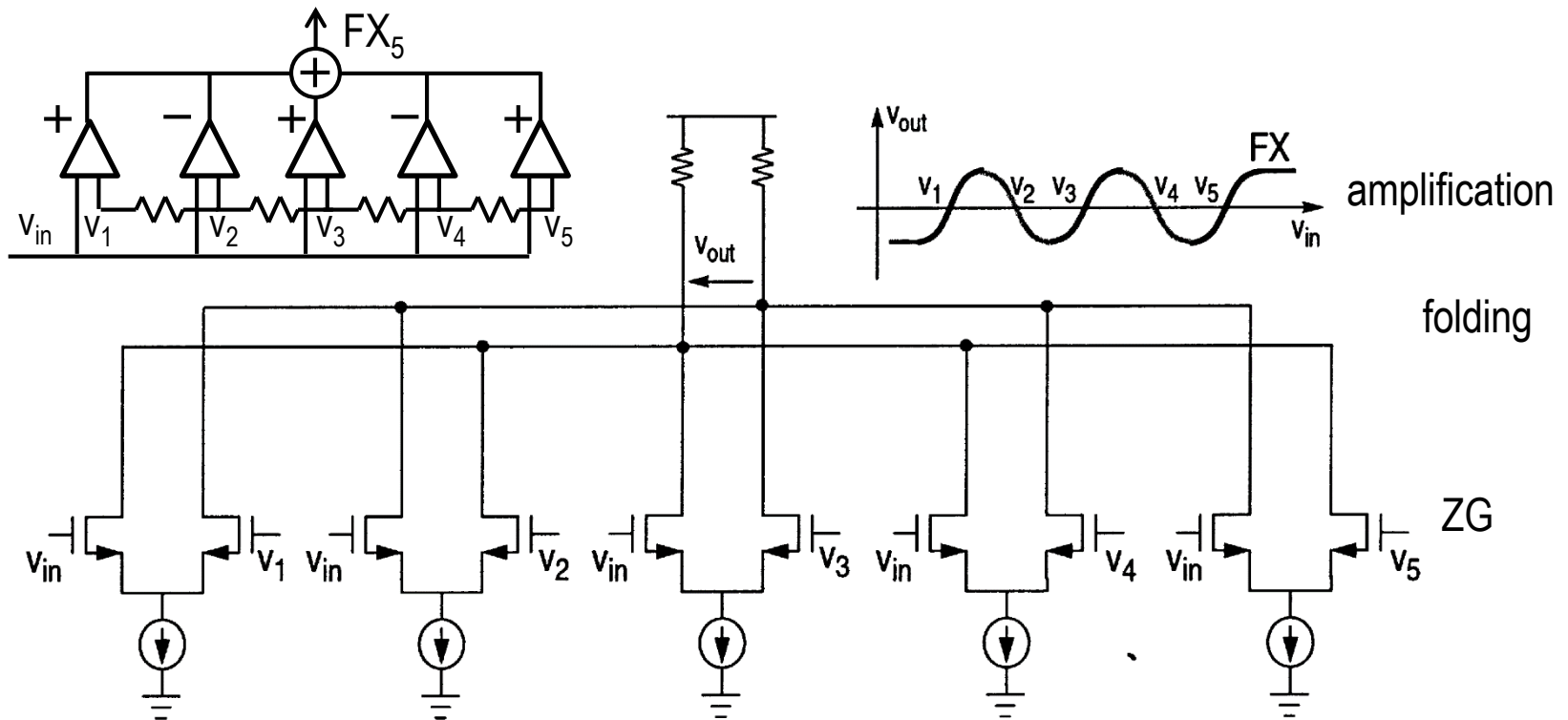
- Doubled settling time for succeeding quantizer
- Doubled power and noise from double T/Hs

# Difference Amplifier (Preamp) for ZX Generation (ZG)



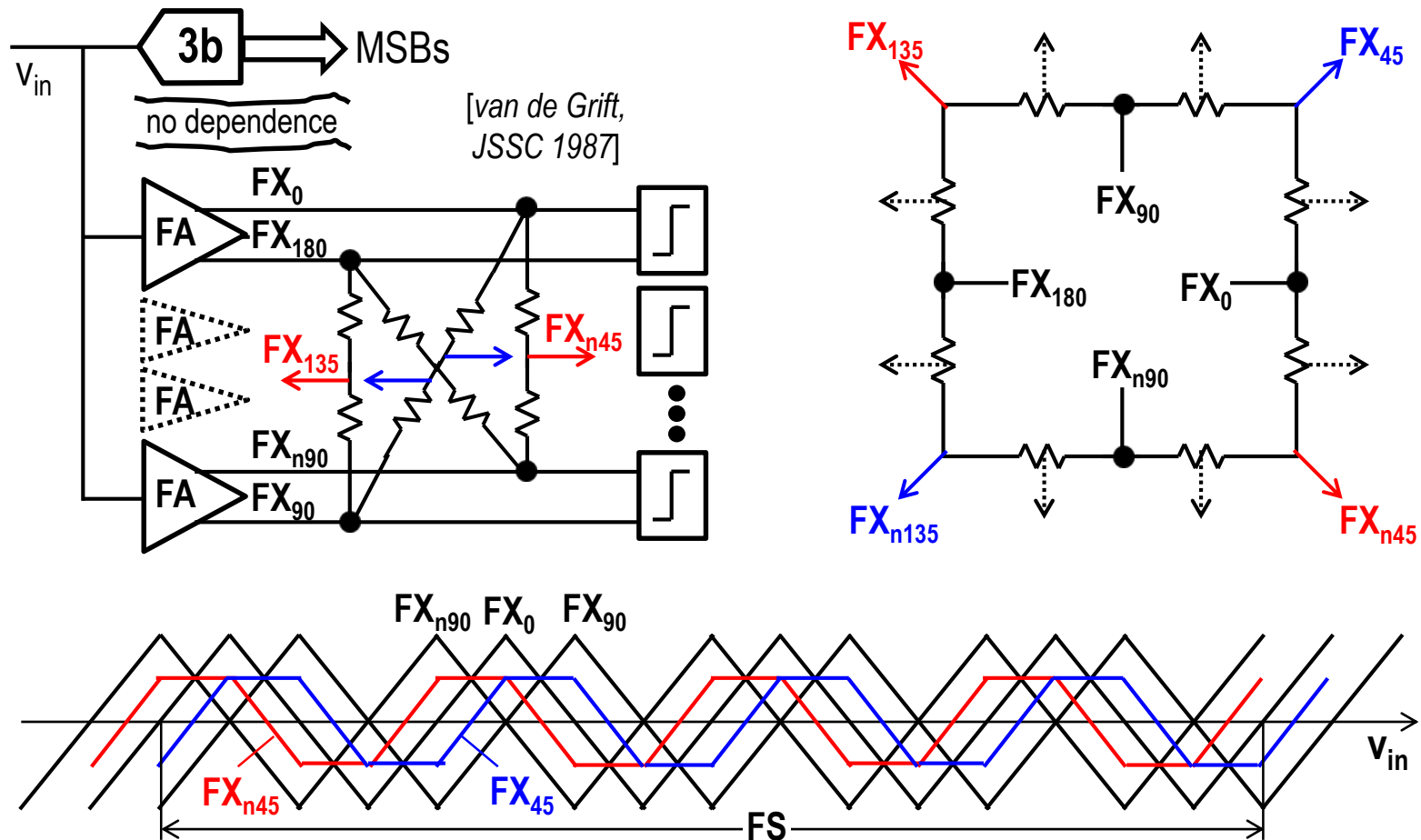
- Diff. output current  $i_d$  clips to  $\pm I/2$  for  $|v_{in} - v_r| > kV_{dsat}$ ,  $k \sim \sqrt{2}$ .
- Diff. input and reference are applied to input diffpairs in p-p and n-n combinations to avoid ZX error from the current clipping.
- Preamp merges amplification with ZG to mitigate ZD errors by buffering ZD kickback and reducing input referred ZD offset.

# Post-ZG Folding by Summation



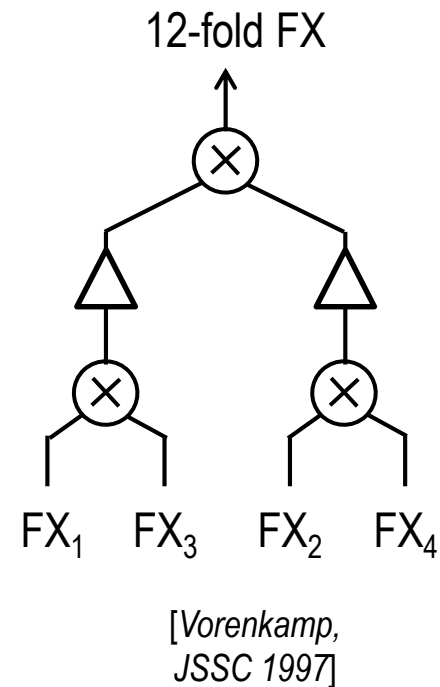
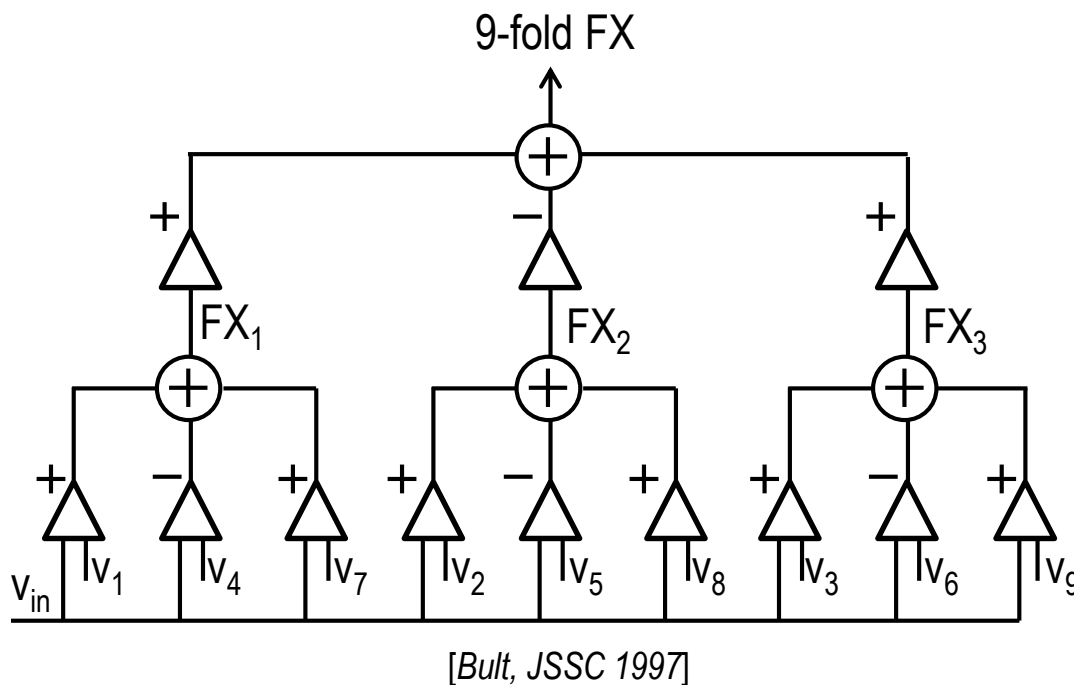
- Merged ZX generation, amplification, and folding
- Ideally, no interaction among odd number of folds (ZXs)
- Subject to all error sources and mutual loading

# Additional Folding Signals from Interpolation



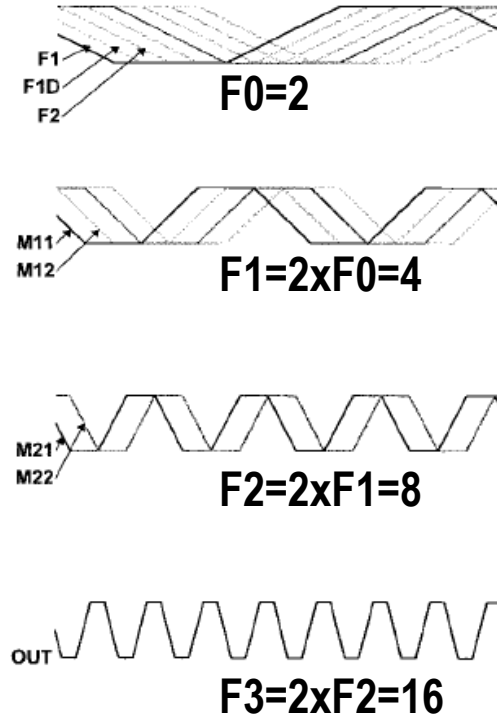
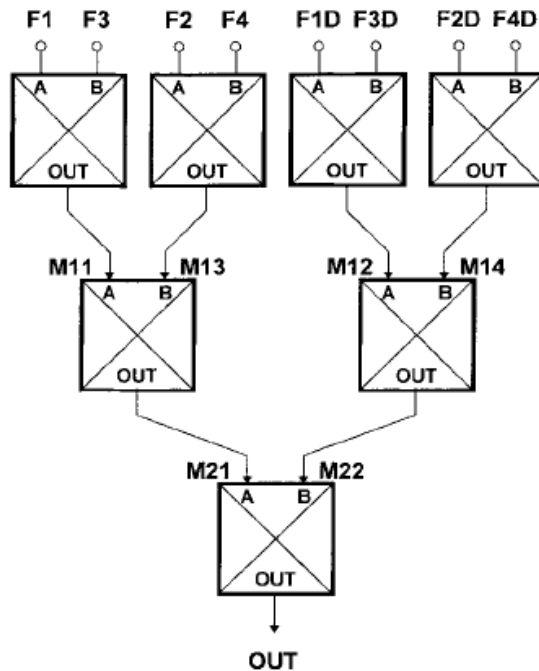
- No interaction between the folding signals
- No linearity requirement for 2x interpolation

# Multi-Step Folding

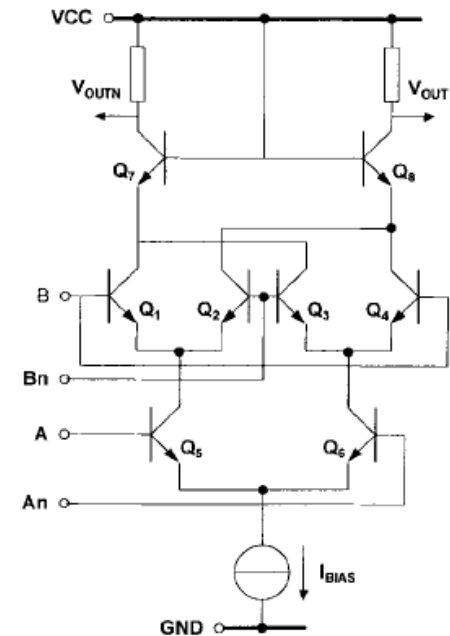


- ZXs are combined in multiple steps to avoid excessive output loading.
- Multistep folding is merged with multistep amplification for better BW.
- Pipelining is possible but is against the purpose of “flash” operation.

# Post-ZG Folding by Multiplication

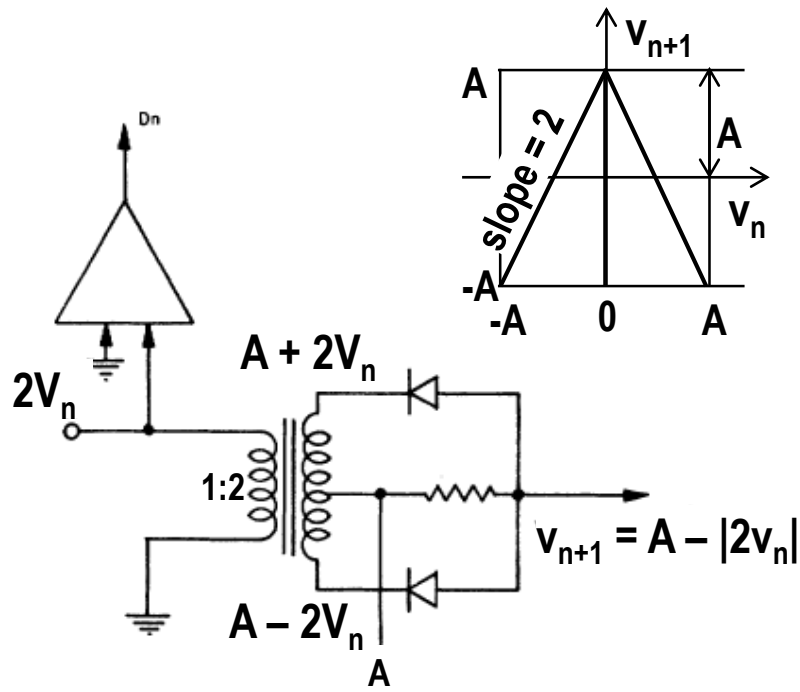


[Vorenkamp, JSSC 1997]

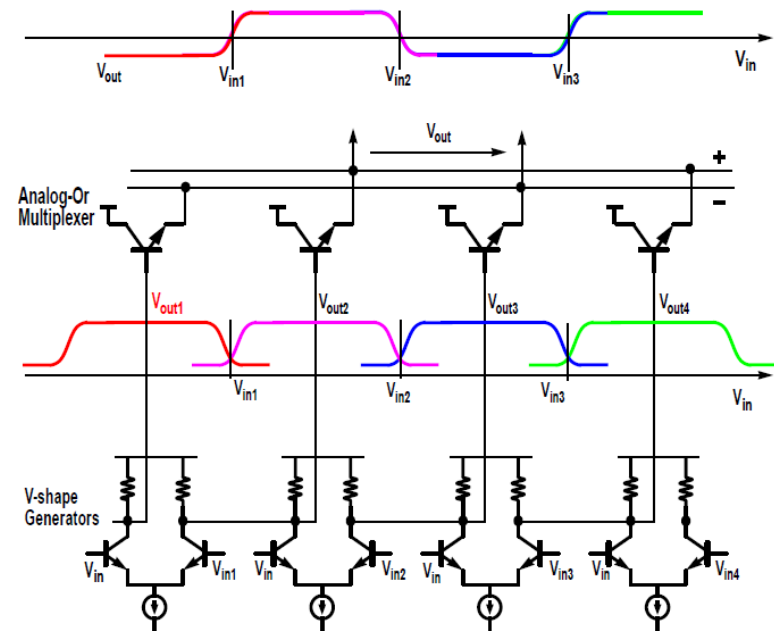


- Multiplier (e.g. Gilbert Cell) output preserves all input ZXs.
- Cascaded multiplications combine folding signals into one.

# Folding with Min/Max Follower(s)



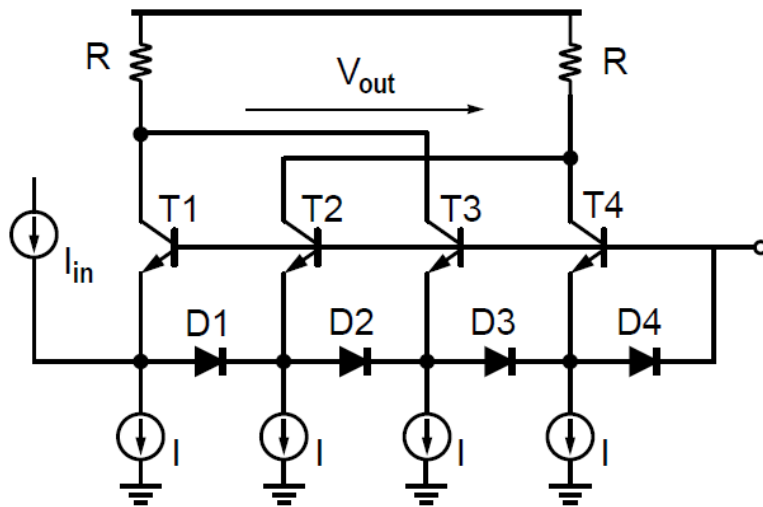
(a) [Smith, IRE 1956]



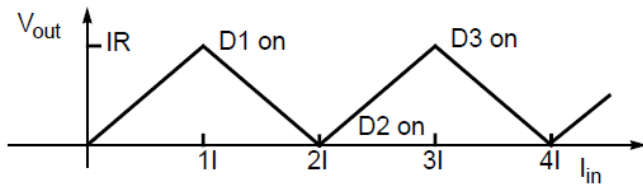
(a) [van de Grift, JSSC 1987]

- In (a), the diodes select the *min* of the two voltages  $A \pm 2v_n$  to realize a V-shape folding output:  $v_{n+1} = A - |2v_n|$ .
- In (b), the output emitter followers select the *max* among the input V-shape foldings and stitch them by alternating their polarity.

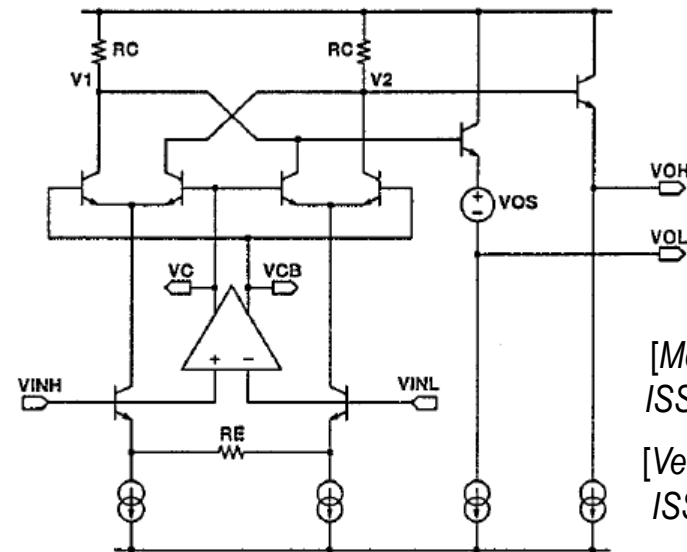
# Folding by Rectifier Self-Switching



[van de Plassche, JSSC 1979]

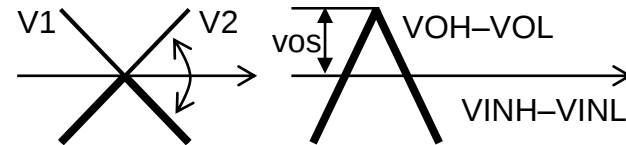


(a)



[Moreland, ISSC 1995]

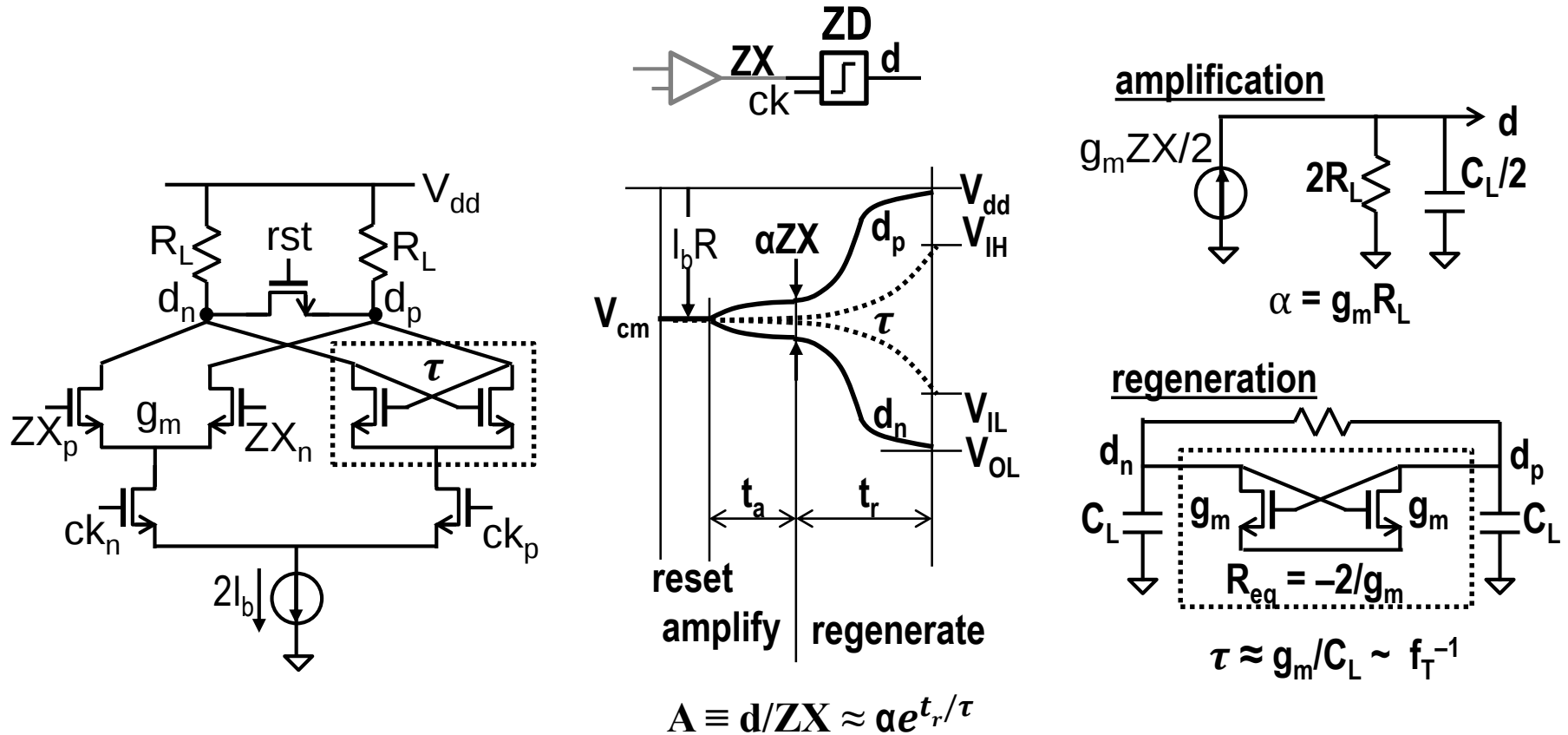
[Verbruggen, ISSC 2008]



(b)

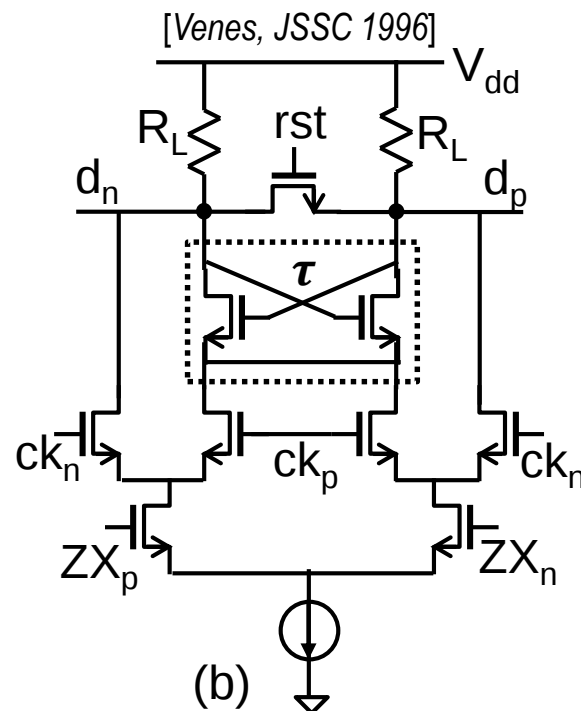
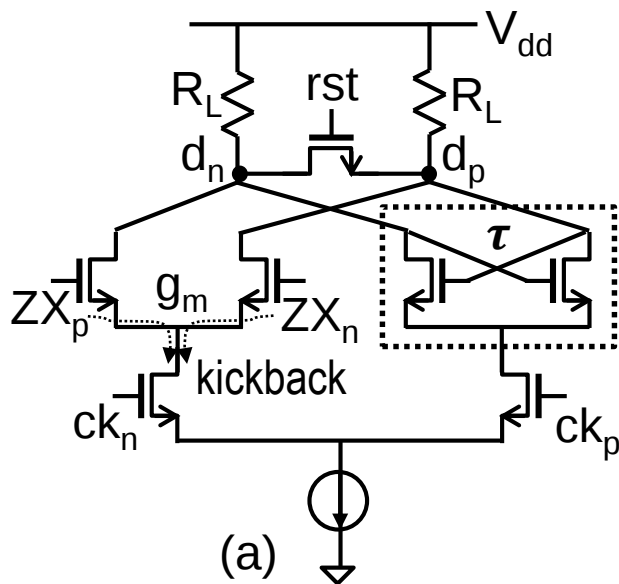
- At  $I_{in} \geq kI$ ,  $T_k$  shuts off,  $D_k$  turns on,  $I_{in}-kI$  switches side by  $T(k+1)$ .
- At (b) output, input signal multiplies its polarity by self-switching.
- ZX is detected by an op-amp instead of a regenerative latch (ZD).

# Static CML Latches for ZX Detection



- ZD uses latch regeneration (positive feedback) to maximize the gain.
- Two-phase operation: tracking/amplification & sampling/regeneration.
- Regeneration speed or time constant ( $\tau$ ) is limited by technology ( $f_T$ ).

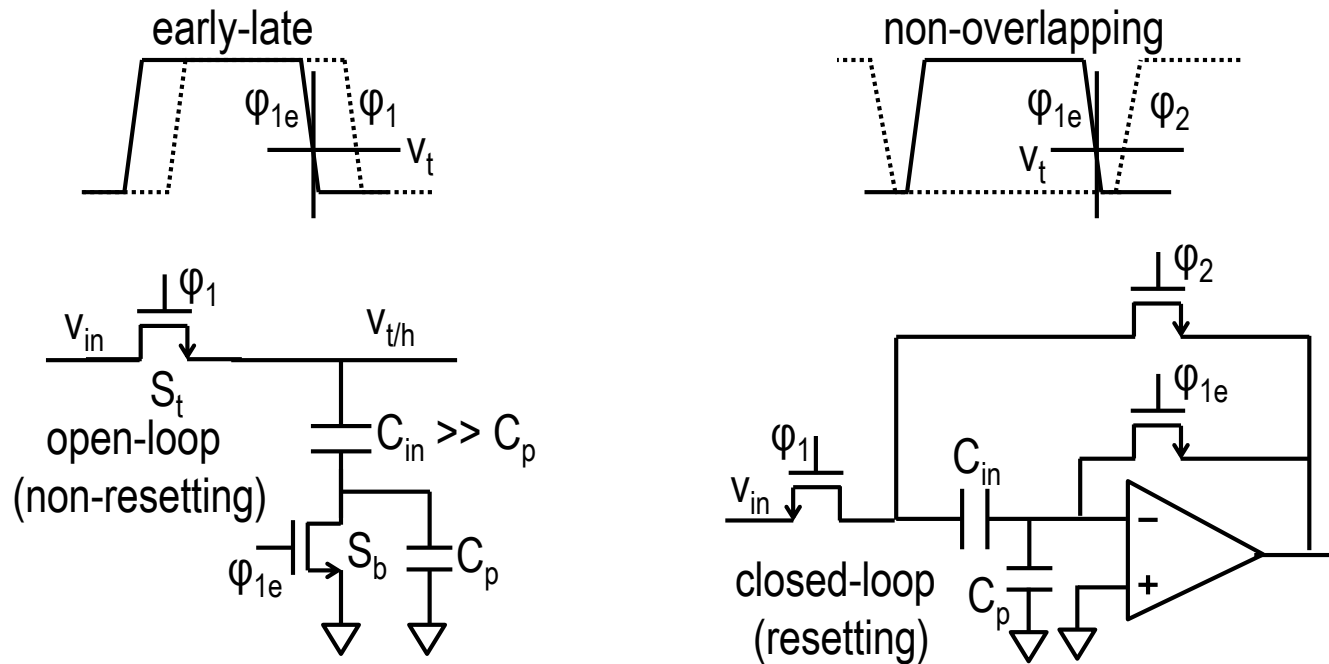
# Static CML Latch Improvements



- Steady bias current and output CM avoids dynamic offset.
- Kickback is in (a) but not in (b) with balanced clocks isolating output.
- Input feedthrough is also blocked by the stacked clock diffpairs in (b).
- A reset pulse (rst) before amplification helps to erase the hysteresis.

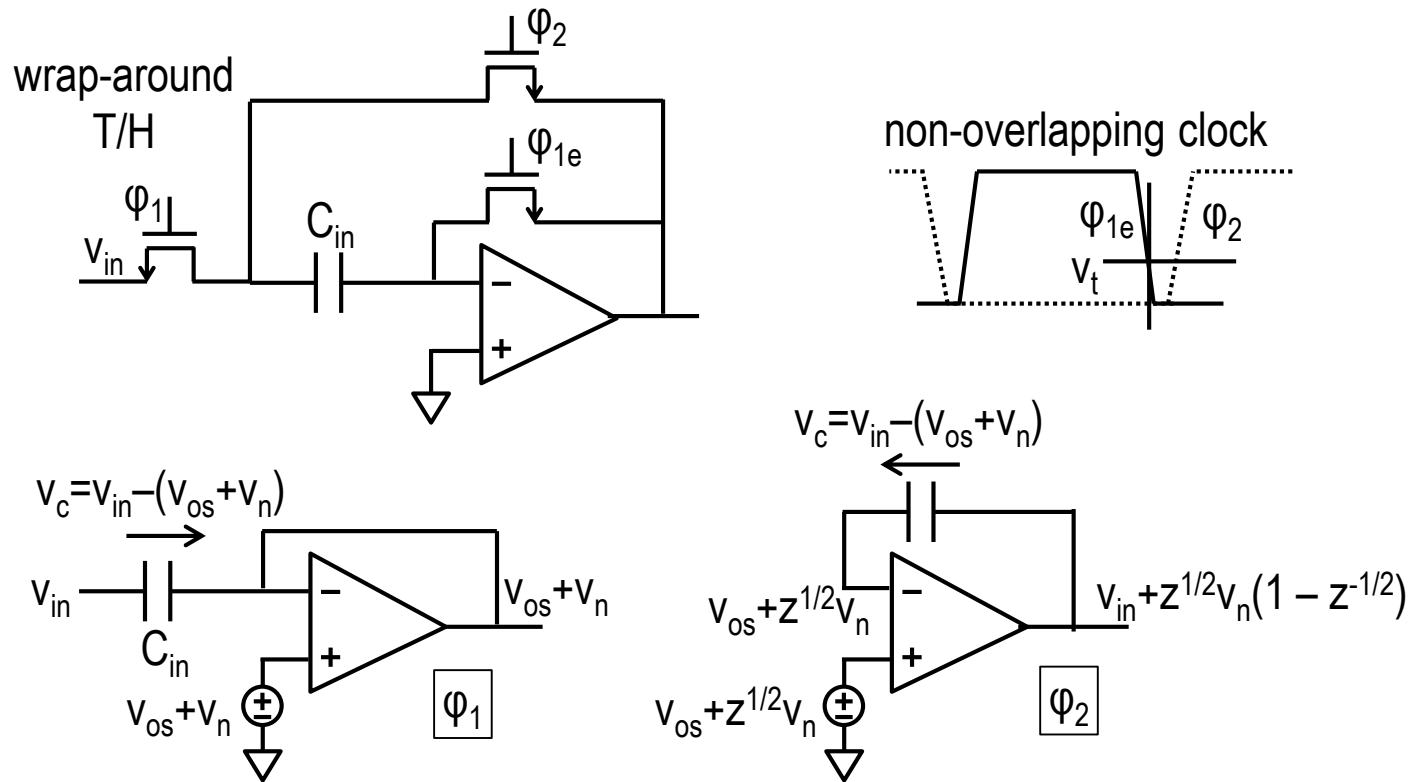
# Imperfections and Corrections

# T/H Bottom-Plate Sampling



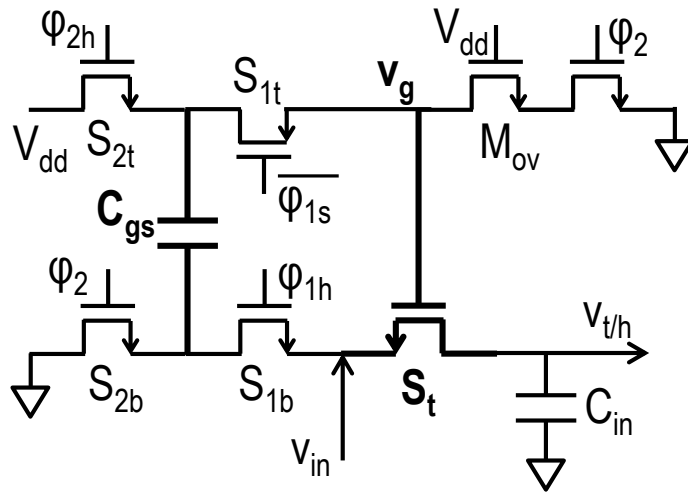
- Bottom plate switch  $S_b$  opens before top-plate switch  $S_t$ .
- Switch  $S_b$  determines the sampling time and aperture.
  - Imperfections are signal independent.
- Closed-loop virtual ground removes parasitic ( $C_p$ ) effect.
  - Eliminate error from charge injection/feedthrough.

# Offset Auto-Zeroing and Correlated Double Sampling

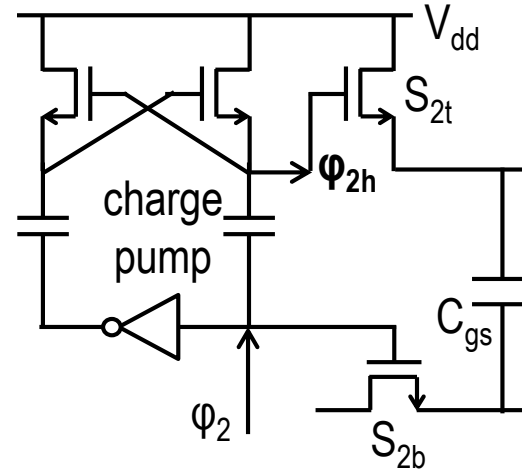


- Offset is sampled on  $C_{in}$  and cancelled in next phase.
- Op-amp input referred noise is filtered by  $(1-z^{-1})$ .
- Auto-zeroing is based on correlated double sampling (CDS) as slow varying op-amp offset is sampled twice in each cycle.

# Input Switch Gate Bootstrapping

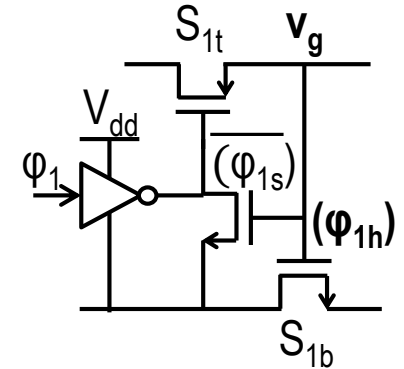


(a)  $S_t$  bootstrapped with  $C_{gs}$



(b) high-V clock  $\phi_{2h}$

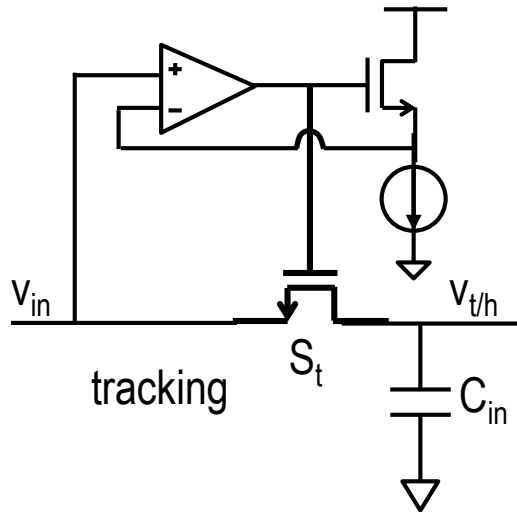
[Abo, JSSC 1999]



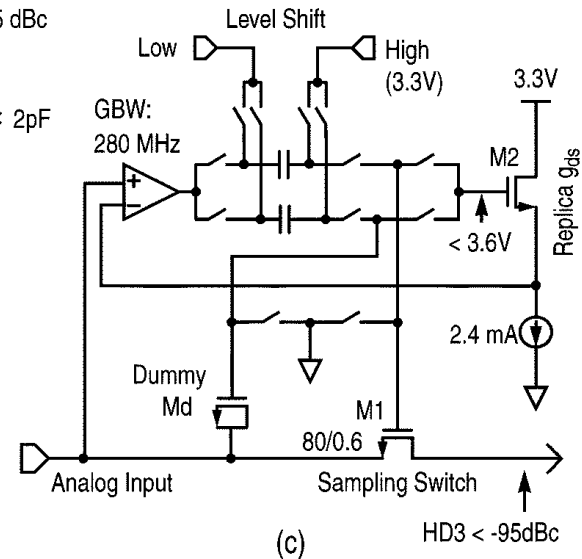
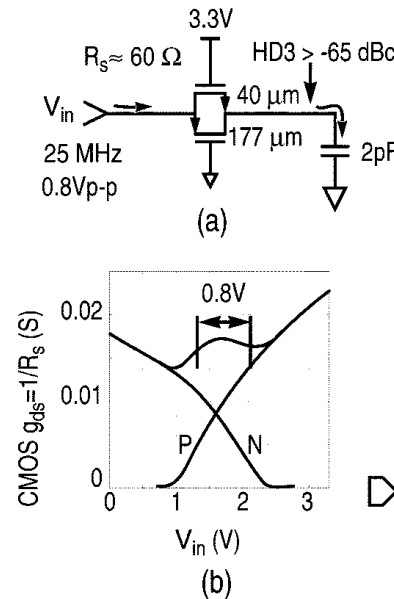
(c) high-V clock  $\phi_{1h}$

- Input switch gate is bootstrapped with  $v_{gs} = V_{dd}$  free of OV.
- $S_t$  on-resistance  $R_{sw}$  is linearized except for body effect.
- Potential reliability issue with node voltage  $v_g$ ,  $\phi_{2h} > V_{dd}$ .

# Closed-Loop Switch Gate Bootstrapping



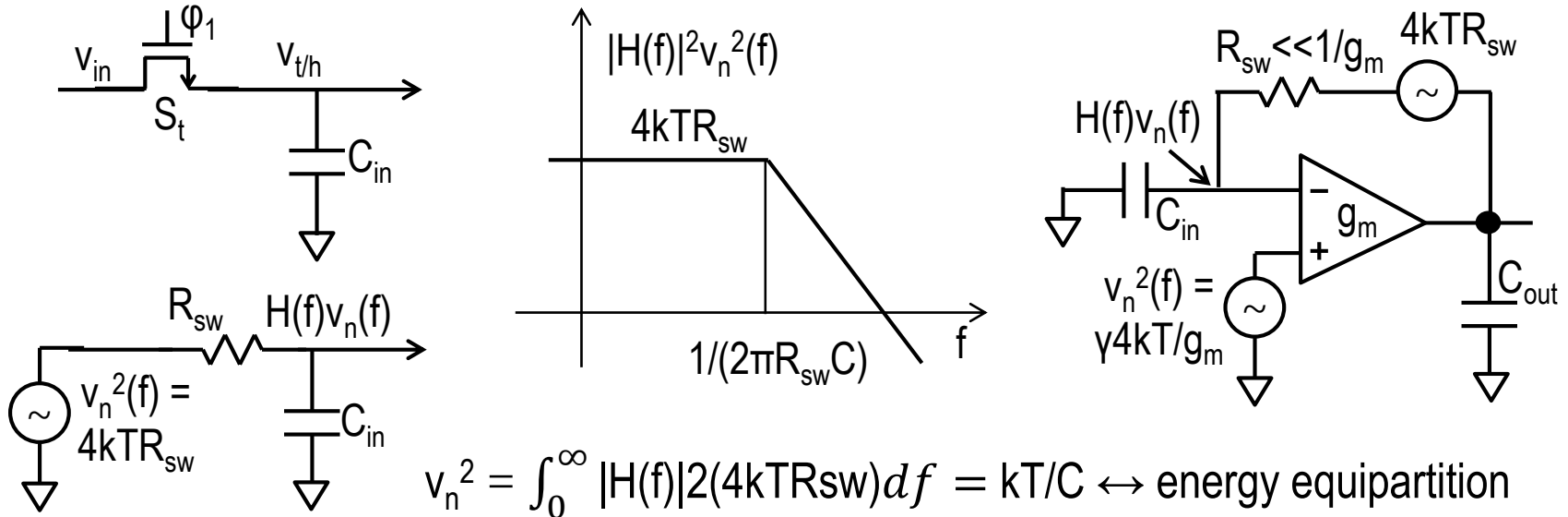
(a) closed-loop bootstrapping



[*Pan, JSSC 2000*]

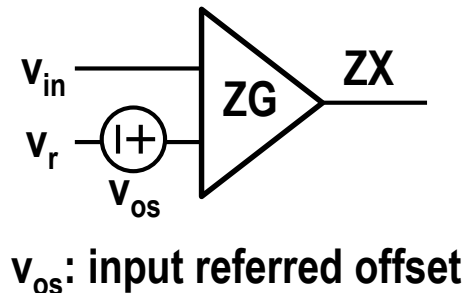
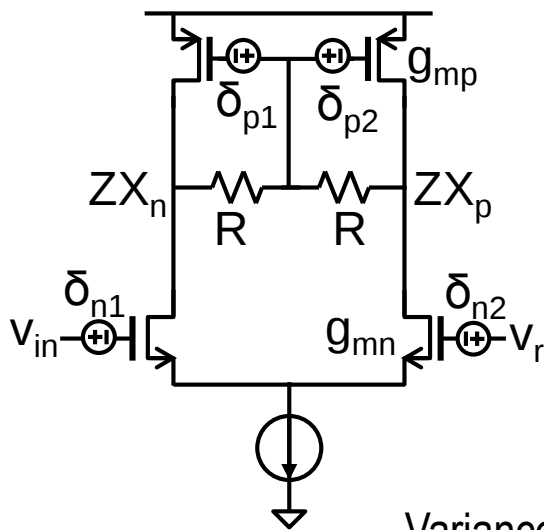
- Super-linear switch with body effect compensated
- Less reliability concern with lower overvoltage
- Too much power for the op-amp driving the gate

# KT/C Noise from Switch and Op-amp



- Effects of  $R_{sw}$  (or  $g_m$ ) on noise BW and level cancel each other.
- Each sampling (NOT each switch) incurs  $KT/C$  noise across  $C_{in}$ .
- Noise power inversely proportional to  $C$  and therefore power.
- Large signal is preferred to reduce  $C$  and current quadratically.
- Autozeroing  $(1-z^{-1})$  is not effective on  $KT/C$  thermal noise.

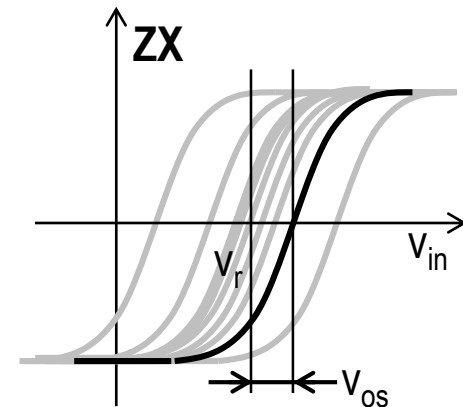
# Preamp Input Referred Offset: ZX Error



$V_{os}$ : input referred offset

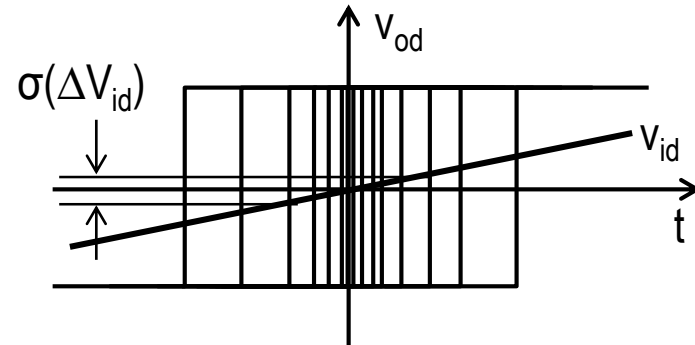
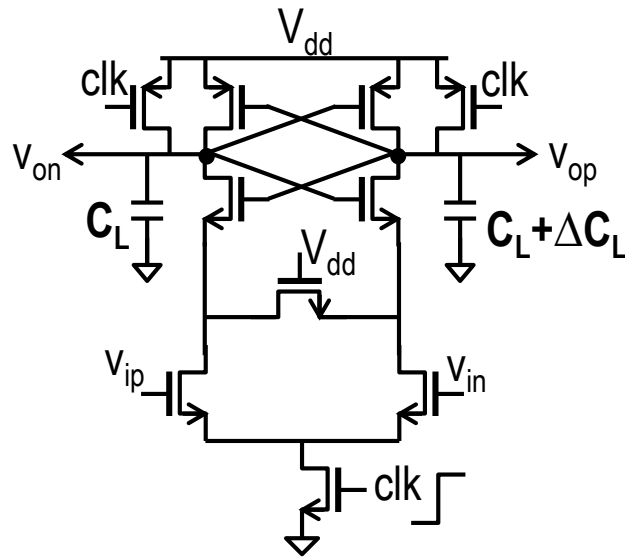
$$V_{os} = \delta_{n1} - \delta_{n2} + (\delta_{p2} - \delta_{p1})(g_{mp}/g_{mn})$$

$$\text{Variance: } (\sigma_{os})^2 = 2(\sigma_{vt\_n})^2 + 2(\sigma_{vt\_p})^2(g_{mp}/g_{mn})^2$$



- Preamp offset arises from mismatch in MOSFET threshold  $V_t$ , carrier mobility, size, output load, etc.
- Static preamp offset is mitigated by auto-zeroing, sizing, averaging/spatial filtering, calibration, etc.

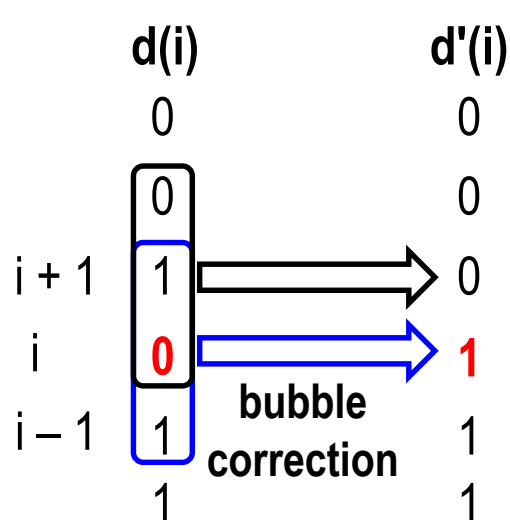
# Latch Dynamic Offsets and Simulations



Transient Monte Carlo simulations for evaluation of input referred offset  $\sigma(\Delta V_{id})$

- Latch switching activities cause dynamic offsets [Kim, TCAS1 2009]
  - Due to CM-to-DM conversion (e.g.  $\Delta V_{out} = (\Delta C_L / C_L) \Delta V_{cm}$ )
  - Due to kick back to input stages such as reference
  - Due to clock/signal feedthrough and noise coupling
  - Sensitive to supply, clock/signal, temperature, parasitic variations
  - Hard to be corrected, usually mitigated with preamps
- Dynamic offsets are simulated with clock running and input swept

# Flash-ADC Bubble Error Correction



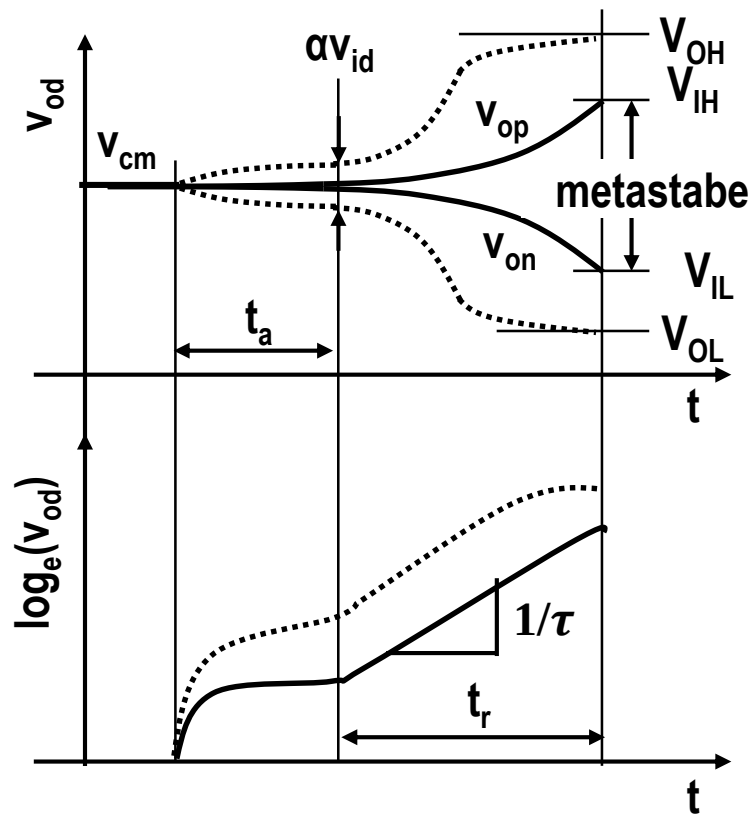
majority vote:  
 $d'(i) = 1$  when 2 out of 3  
 immediate neighbors are 1

$$d'(i) = d(i-1) * d(i) + d(i) * d(i+1) + d(i-1) * d(i+1)$$

[Mangelsdorf, JSSC 1990]

- Adjacent comparator thresholds may cross each other, causing bubbles in the thermometer code and sparkle error
- This may happen in high speed ADCs due to excessive comparator offset and clock skew among adjacent comparators.
- Bubbles are detected and corrected by looking at more than two adjacent bits as is done in the majority vote scheme.

# Latch Regeneration and Metastability

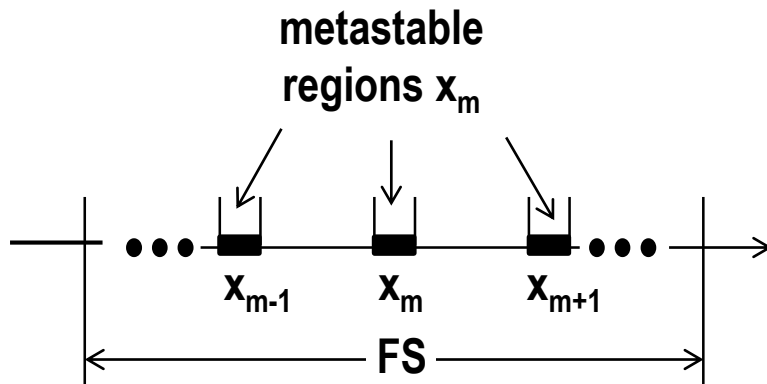


$$A = \alpha e^{t_r/\tau}$$

$$P_m(ZD) \approx (\alpha e^{t_r/\tau})^{-1}$$

- Small signal regeneration is linear operation and can be modelled with a negative resistance and negative time-constant  $-\tau$ .
- Time constant  $\tau$  can be obtained from the slope of  $\log_e(v_{od}(t))$ .
- Metastability occurs when latch output  $v_{od}$  falls within  $[V_{IL}, V_{IH}]$ .
- Metastability Probability  $P_m(ZD) = ((V_{IH}-V_{IL})/A)/FS \approx (\alpha e^{t/\tau})^{-1}$ , not affected by noise. [Figueiredo, JSSC 2013]

# ADC Metastability and Mitigation



## ADC Metastability Probability

$$P_m(\text{ADC}) = (2^n x_m) / \text{FS} \approx 2^n P_m(\text{ZD})$$

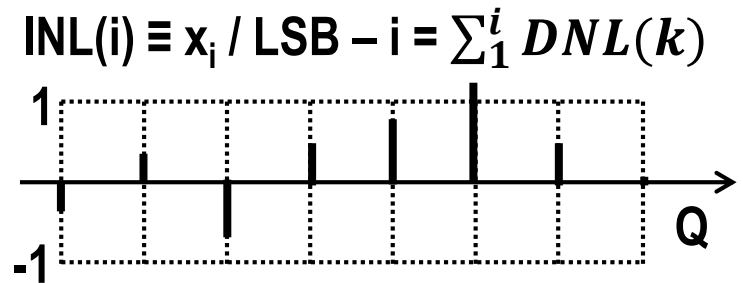
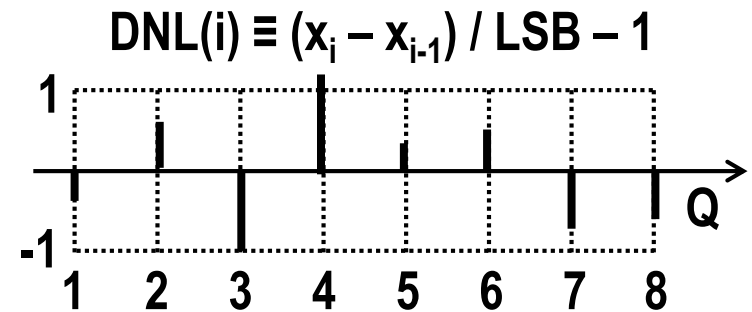
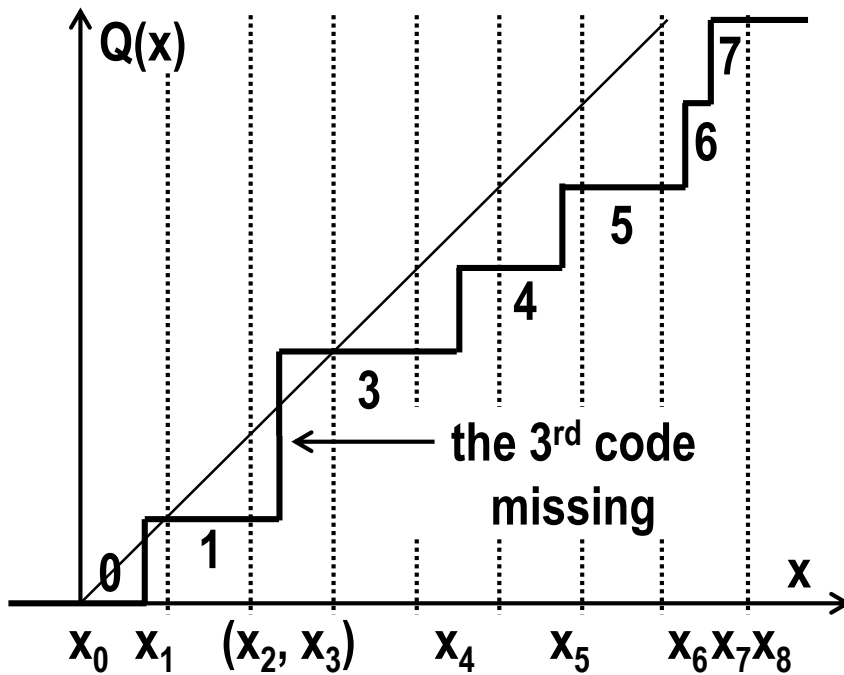
## Comparator Metastability Probability

$$P_m(\text{ZD}) = e^{-t_r/\tau}/A$$

- Metastability is a fundamental ADC issue that can not be eradicated.
- Noise may break a metastable event but does not alter the probability.
- Metastability causes large sparkle errors even at very high S/N levels.
- Metastability and its effect on ADC output error can be mitigated.
  - Increasing (effectively) the regeneration time  $t_r$ 
    - Cascading latches or DFFs before the encoding logic
    - Time interleaving the ADCs or the comparators
    - Asynchronous clocking [Chen, ISSC 2006] for need based regeneration time.
  - Reducing the metastability impact on encoding output [Portmann, JSSC 1996]
  - Reducing regeneration time constant in fast technologies

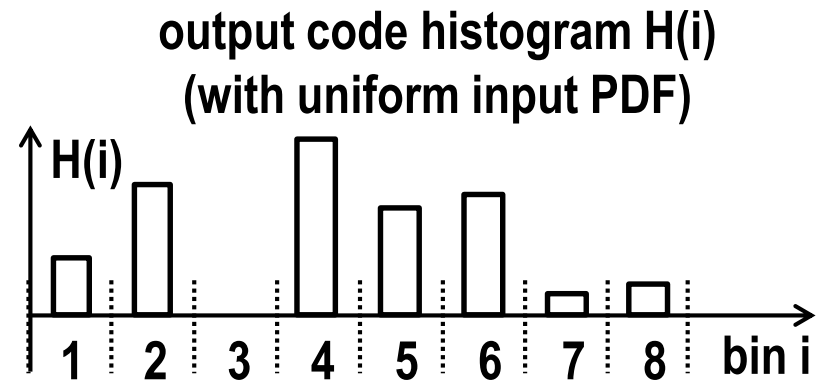
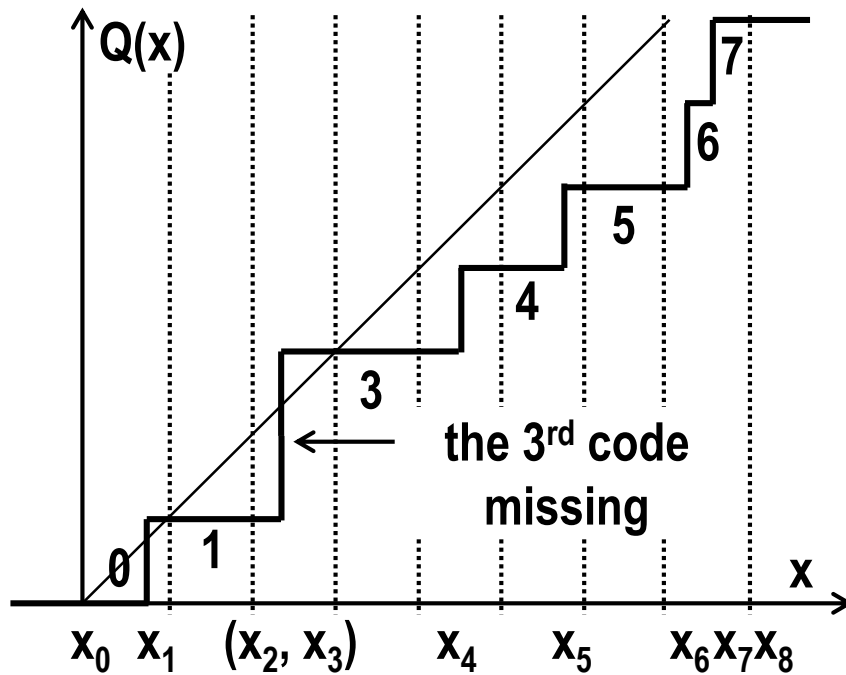
# Metrics and Measurements

# Integral & Differential Nonlinearity (INL & DNL)



- Gain and offset errors are extracted by linear fitting and excluded from INL and DNL.
- INL and DNL both capture ADC nonlinearity as quantization threshold ( $x_i$ ) errors in LSB.
- INL highlights low-order distortion from analog front-end such as input/ref. buffer, T/H, etc.
- DNL stresses high-order quantization errors from comparator offsets, RA inaccuracy, etc.
- INL and DNL do not capture signal dependent threshold errors (e.g. latch (ZD) hysteresis.)
- INL and DNL can be measured by static DC sweep or by dynamic code density test.

# Output Code Density Test for DNL and INL



$$DNL(i) = (H(i) / \sum H(i)) / P(i) - 1$$

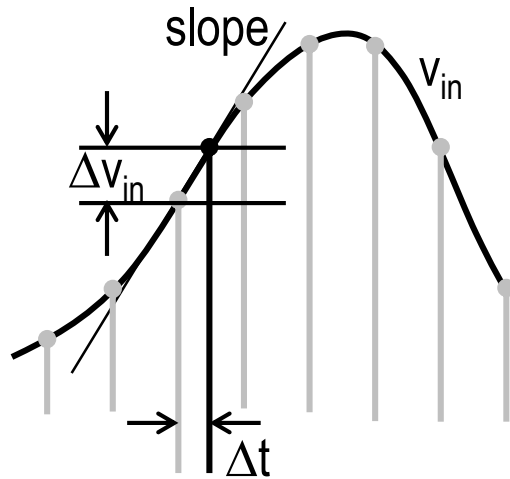
For uniform input PDF,  $P(i) = 2^{-n}$ .

For sinewave input, see the ref:

[Doernberg, JSSC 1984]

- Freq. of code occurrence is proportional to bin size  $R_i$  given uniform input PDF.
- FS sinewave asynchronous to  $f_s$  with well-defined  $P(i)$  is used as the test signal.
- $H(i)$  averages out noise, resulting in accurate DNL/INL, gain, offset measurement.
- Frequency response and dependence on conversion rate are also captured.

# Sampling Jitter Effect on ADC Performance



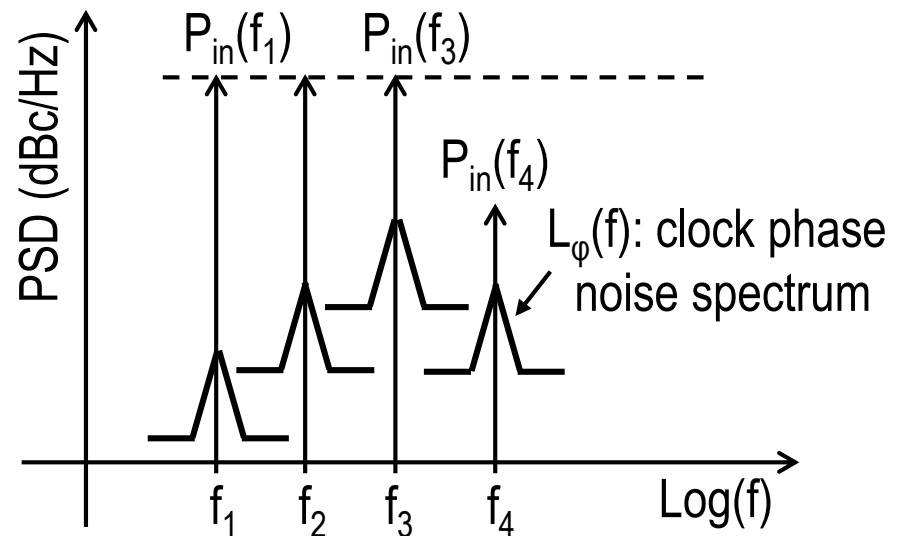
$$\Delta v_{in} \approx \frac{dv_{in}(t)}{dt} \Delta t \sim 2\pi f_{in} A \Delta t < \text{LSB}$$

$$\text{Jitter (rms)} \sigma(\Delta t) < T_{in}/2^n$$

$$\text{FT}(\Delta v_{in}) \approx j\omega V_{in}(\omega) \star \Delta\phi(\omega)/\omega_s$$

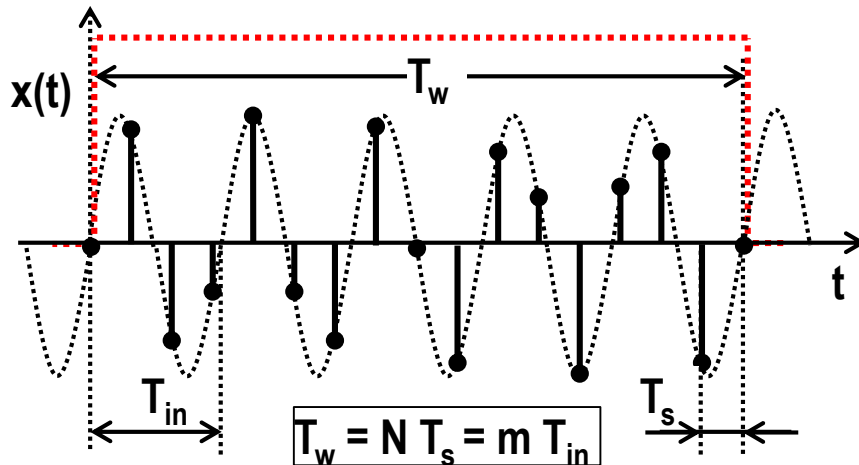
$\Delta\phi(\omega)$ : clock phase noise;  $\omega_s$ : clock freq.

$$P_{\text{jitter}}(f) \approx (f_k^2/f_s^2)L_\phi(f - f_k)P_{in}(f_k)$$

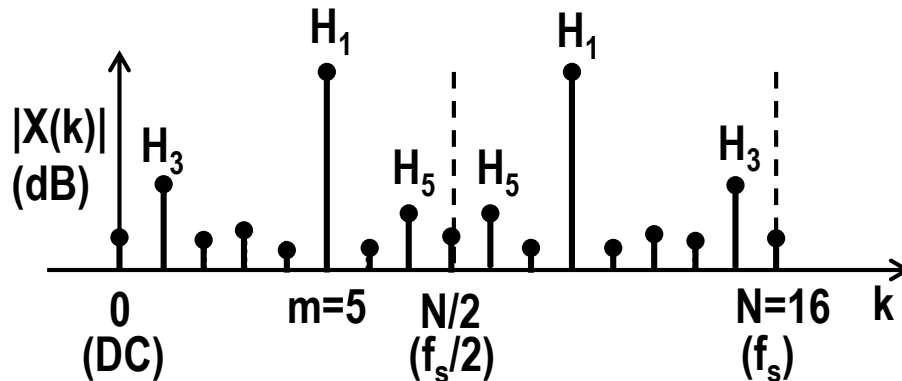


- Noise due to clock jitter scales with sampled signal frequency and amplitude.
- Jitter spec can be orders of magnitude relaxed for wideband and large PAPR.
- Jitter slower than system tracking BW or observation time can be excluded.

# Frequency Domain Characterization Using FFT



↕ FFT: DFT with  $N = 2^n$



- Single-tone test:  $|H_1|^2 \sim P_{FS}$

$$SNDR \equiv |H_1|^2 / (\sum_{k \neq m}^{N/2} |X(k)|^2)$$

$$SFDR \equiv |H_1|^2 / \max_{k \neq m} |X(k)|^2$$

$$THD \equiv \sum_2^I |H_i|^2 / |H_1|^2$$

(e.g.  $I = 15$ )

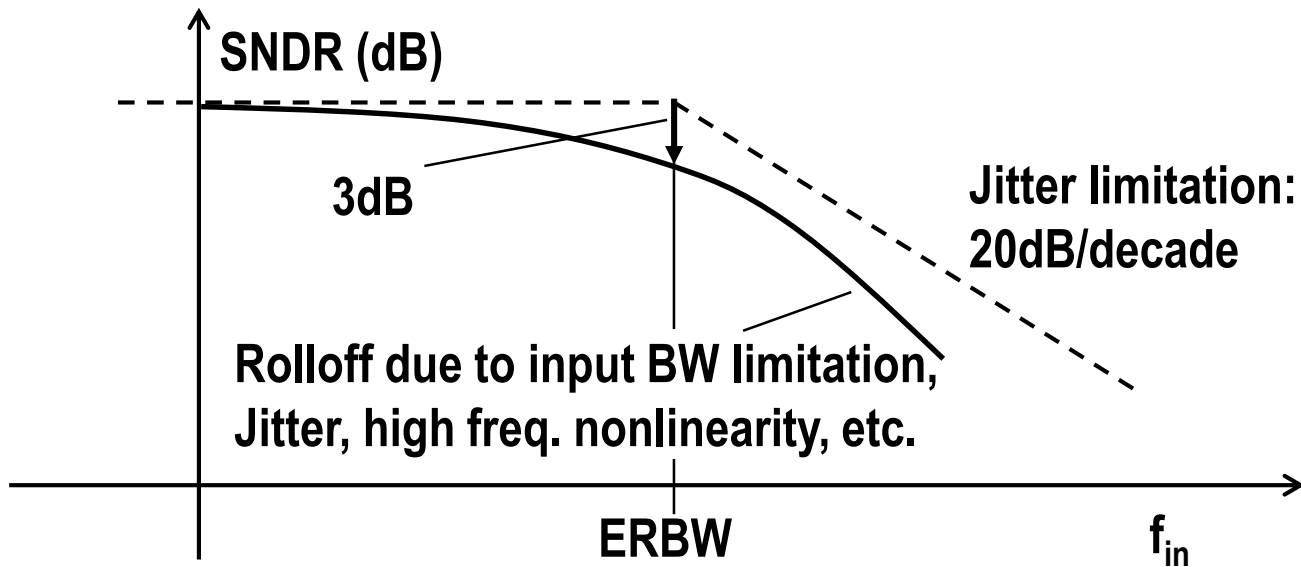
$$SNR \equiv |H_1|^2 / (\sum_1^{N/2} |X(k)|^2 - \sum_2^I |H_i|^2)$$

- Multi-tone test:

$$F_{inj} = m_j f_{bin}; j = 1, 2, \dots, M$$

$$|H_{jj}|^2 = P_{FS} / M^2$$

# Effective Resolution Bandwidth (ERBW)



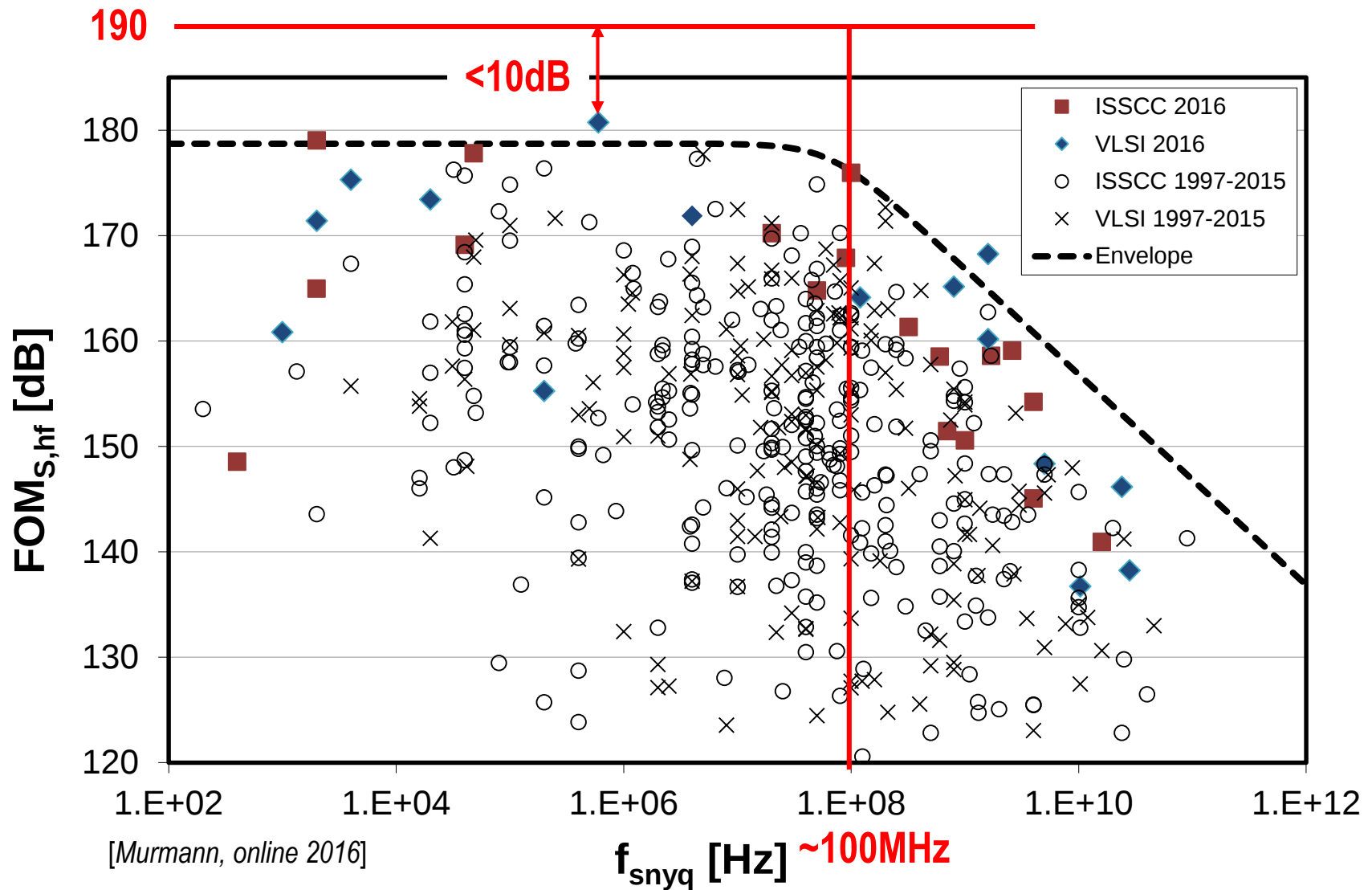
$$\text{ERBW} \equiv f_{in} @ \text{ENOB} = \text{ENOB}(\text{dc}) - 0.5$$

- An  $n$ -bit ADC is supposed to have  $\text{ENOB}(\text{dc}) > \text{NOB} - 0.5$ .
- An  $n$ -bit Nyquist ADC is supposed to have  $\text{ERBW} > f_s / 2$ .
- Metrics for FS single-tone test near Nyquist tend to over-specify ADCs.
  - Input signal freq. usually occupies the lower portion of the Nyquist band
  - Input signal amplitude rms is usually much smaller than the full scale

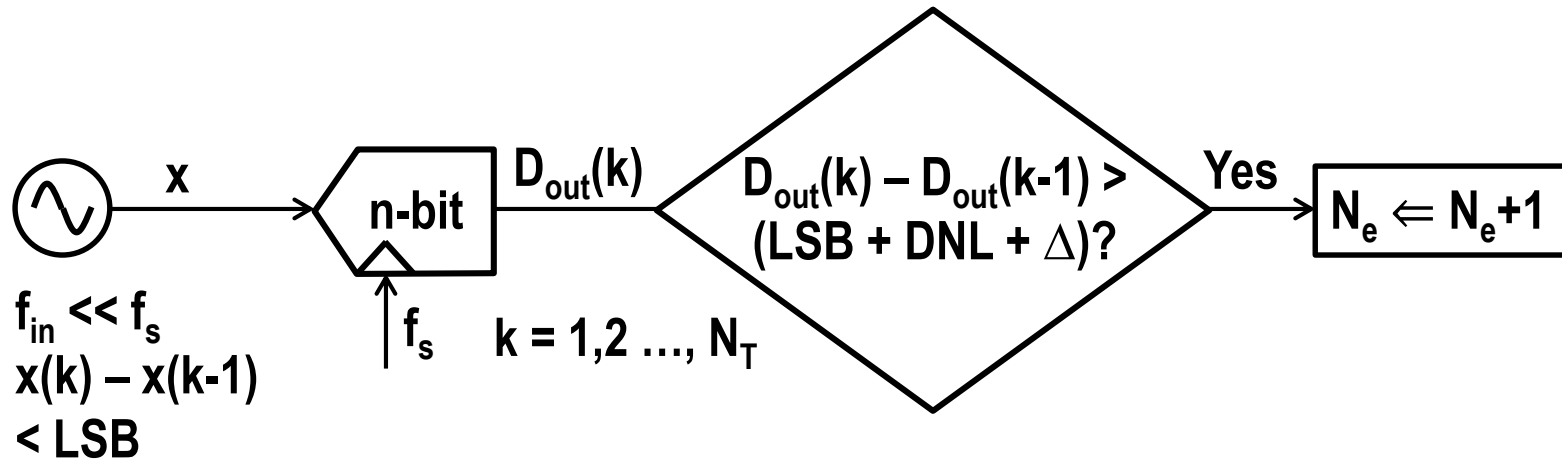
# ADC Figures of Merit (FoM)

- Walden (ISSCC)  $\text{FoM}_w = P / (2^{\text{ENOB}} f_s)$ 
  - For low resolution ( $n < 10\text{b}$ ) full-flash ADCs where ENOB dominates the total number of comparators and total power.
- Schreier  $\text{FoM}_s = (2^{\text{ENOB} \cdot 2} \text{BW}) / P \approx \text{SNDR} + 10\log(\text{BW}/P)$  (dB)
  - For high resolution ( $n \geq 10\text{b}$ ) sw-cap based ADCs, ENOB determines  $KT/C$ , and the capacitor  $C$  dominates the total power.
  - For  $P = V_{\text{FS}}^2 C f_s = V_{\text{FS}}^2 f_s kT/12 / \text{LSB}^2 = 2^{\text{ENOB} \cdot 2} 12kT f_s$ ,  $\text{FoM} = (24kT)^{-1} \approx 190(\text{dB})$ .
- Many FoMs emphasize different aspects for different applications.
- Many factors affect the FoM
  - ADC definition (e.g. buffers/PGA/AAF),  $f_t$ , trimming, jitter, digital, etc.
- Many publications and surveys are there on FoMs
  - For example: <http://converterpassion.wordpress.com/>
  - B. Murmann, "ADC Performance Survey 1997-2016," [Online]. Available: <http://web.stanford.edu/~murmman/adcsurvey.html>.

# Survey Results: Schreier FoM vs. Fs



# ADC Meta-Stability Test

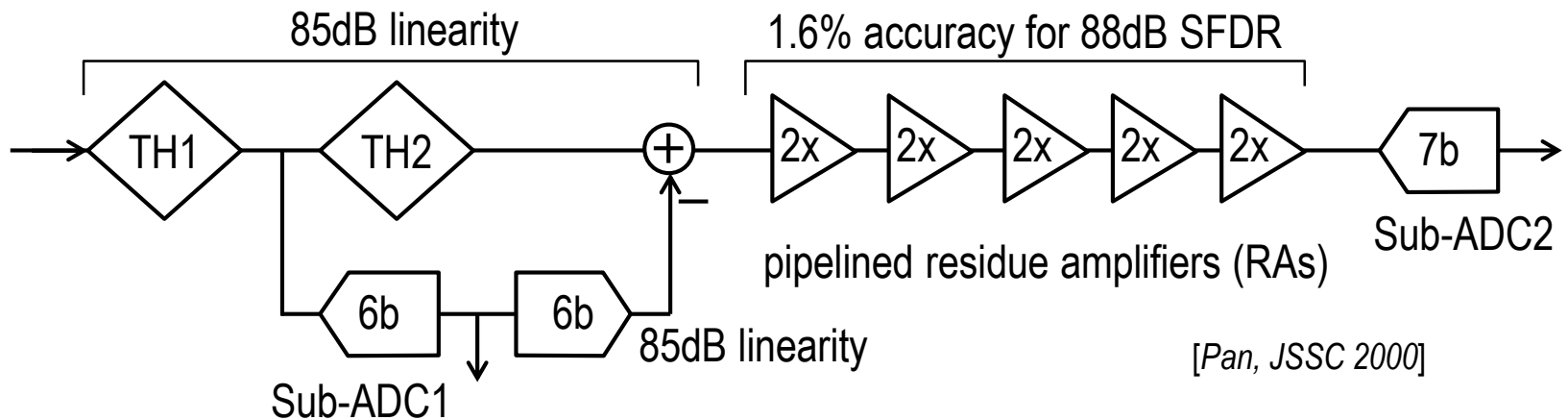


$$P_m(\text{ADC}) \approx N_e / N_T$$

- Meta-stability is hard to catch in transient simulations but can be easily measured on- or off-chip.
- Meta-stability probability  $P_m$  can be accurately estimated in simulation from regeneration transient slope.

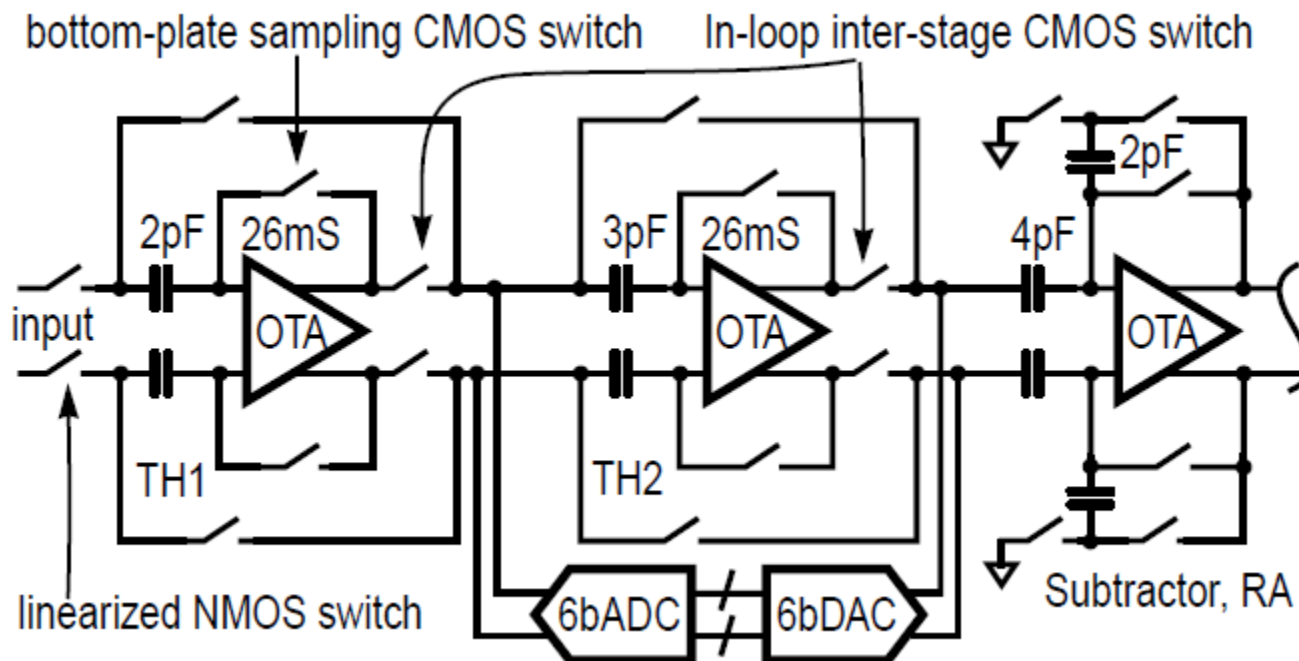
# Evolution and Trends

# A 3.3V 50MS/s 12b ADC with 85dB SFDR in 0.6 $\mu$ m CMOS



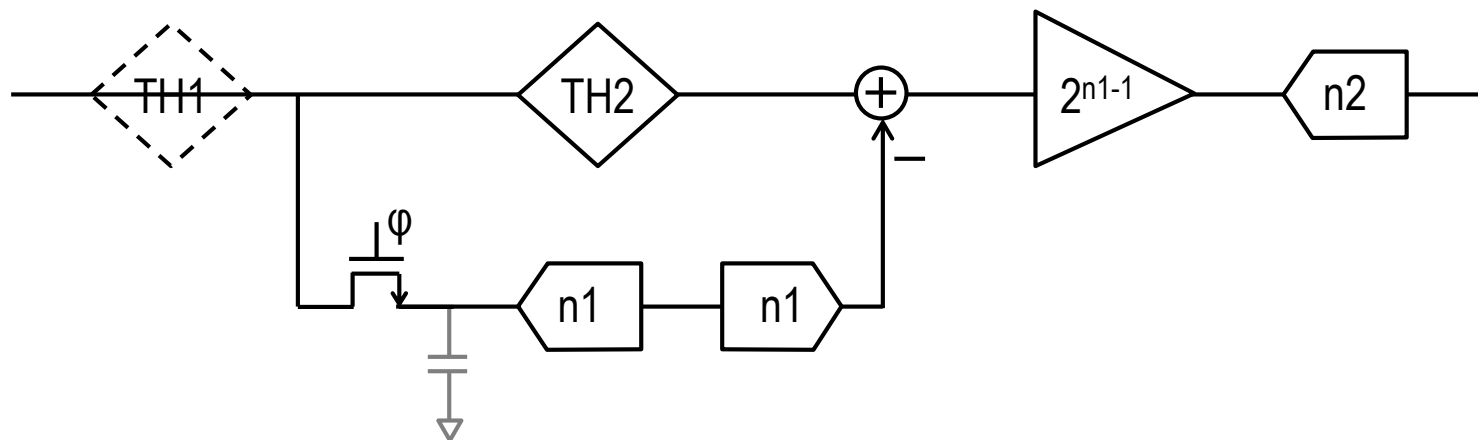
- This ADC is intended for low power base-station IF sampling.
- Pipelined 2-step conversion achieves 50MS/s and 85dB SFDR in 0.6 $\mu$ m CMOS with  $V_{dd}$  lowered to 3.3V from standard 5V.
- 6b first step removes SFDR bottleneck imposed by the residue gain inaccuracy; T/H and DAC linearity determine the SFDR.
- Pipelined RAs avoid tradeoff between amplifier gain and BW.
- Signal Folding reduces the 6b sub-ADC and the DAC complexity.
- Optimal spatial filtering reduces input cap of each sub-ADC by 7X.

# Input T/Hs and MDAC



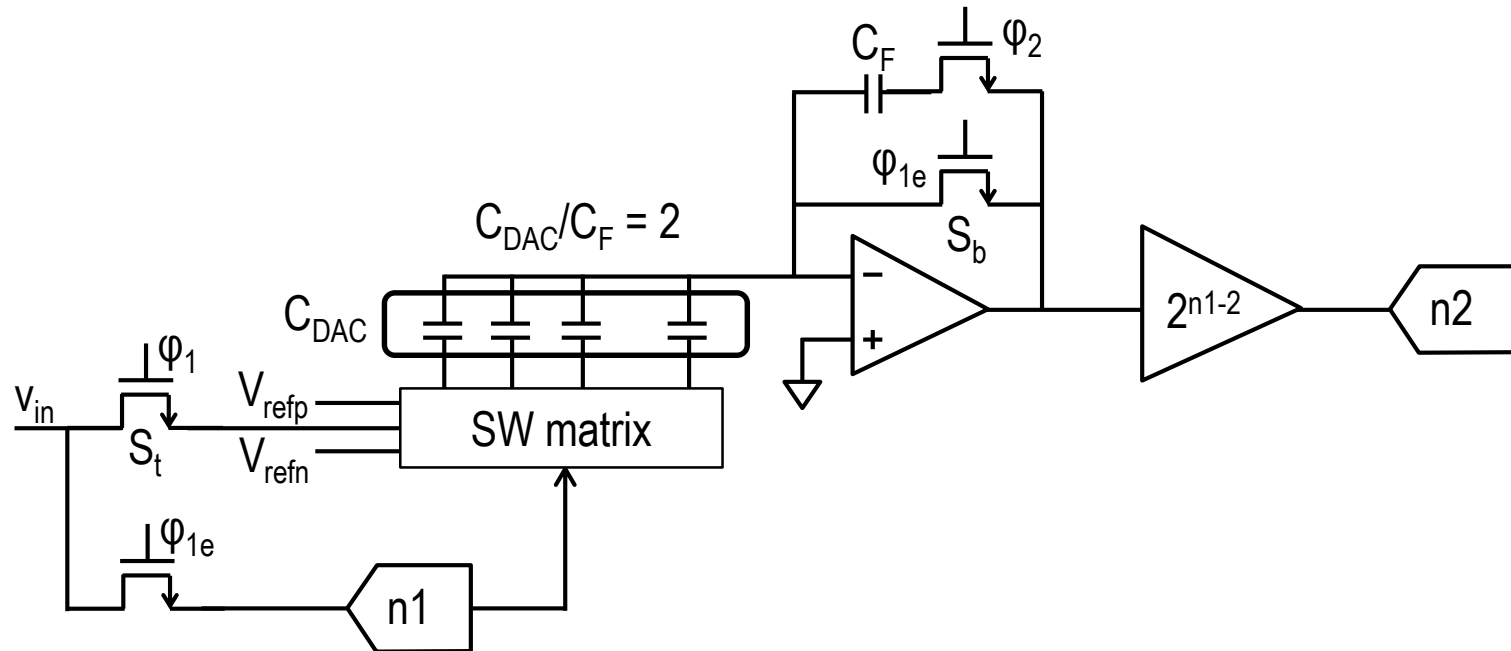
- Input switches are linearized with closed-loop bootstrapping for >85dB SFDR.
- Wrap-around T/Hs with bottom plate sampling for max. BW and min. errors.
- TH2 removes the ADC speed bottleneck associated with sub-ADC latency.
- In-loop inter-stage switch minimizes effect of finite switch BW on ADC speed.

# Evolution with Technology: Remove 1<sup>st</sup> T/H Amplifier



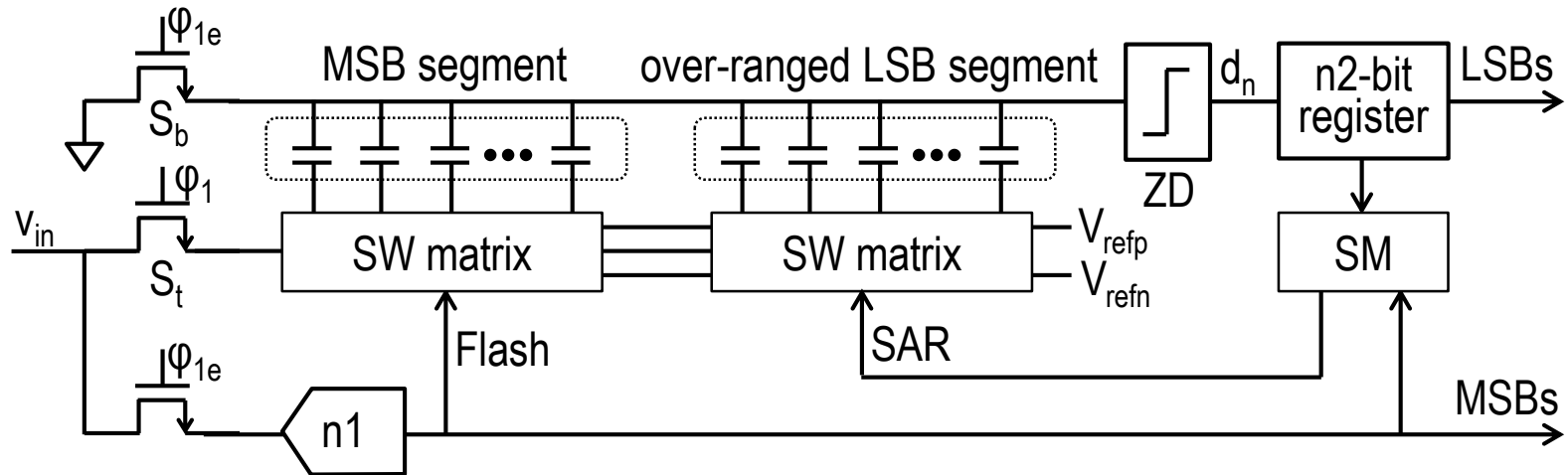
- A separate T/H driving the  $n1$ -bit sub-ADC needs only  $n1$ -bit accuracy and can use simple open-loop switch.
- Timing between the two signal paths needs to be within  $n1$ -bit, more feasible in an advanced technology.

## Evolution with Technology: Remove 2<sup>nd</sup> T/H Amplifier



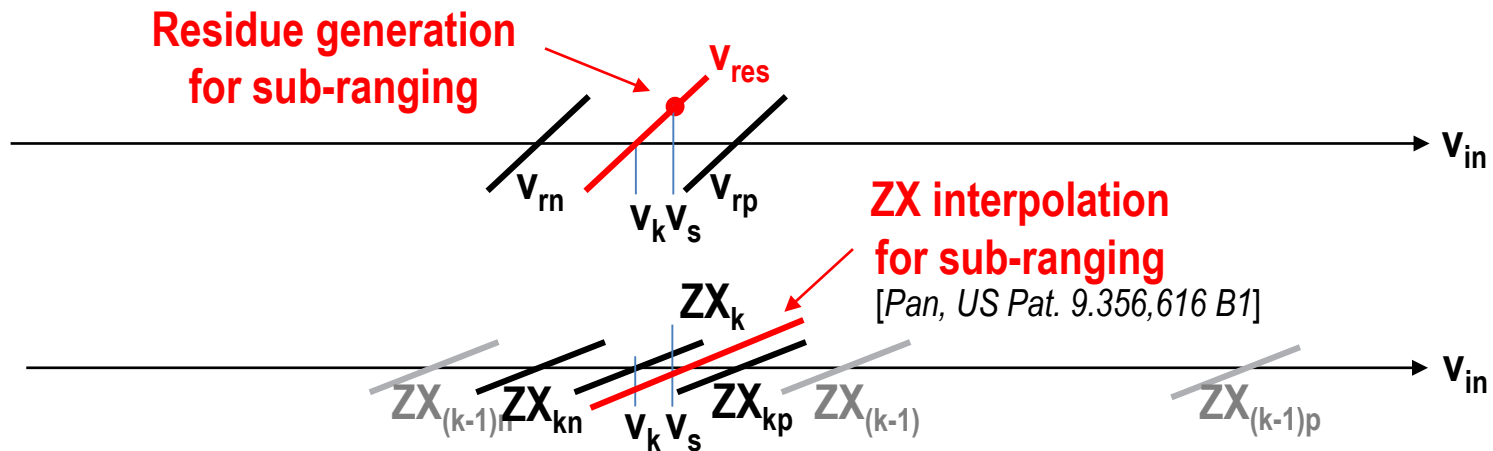
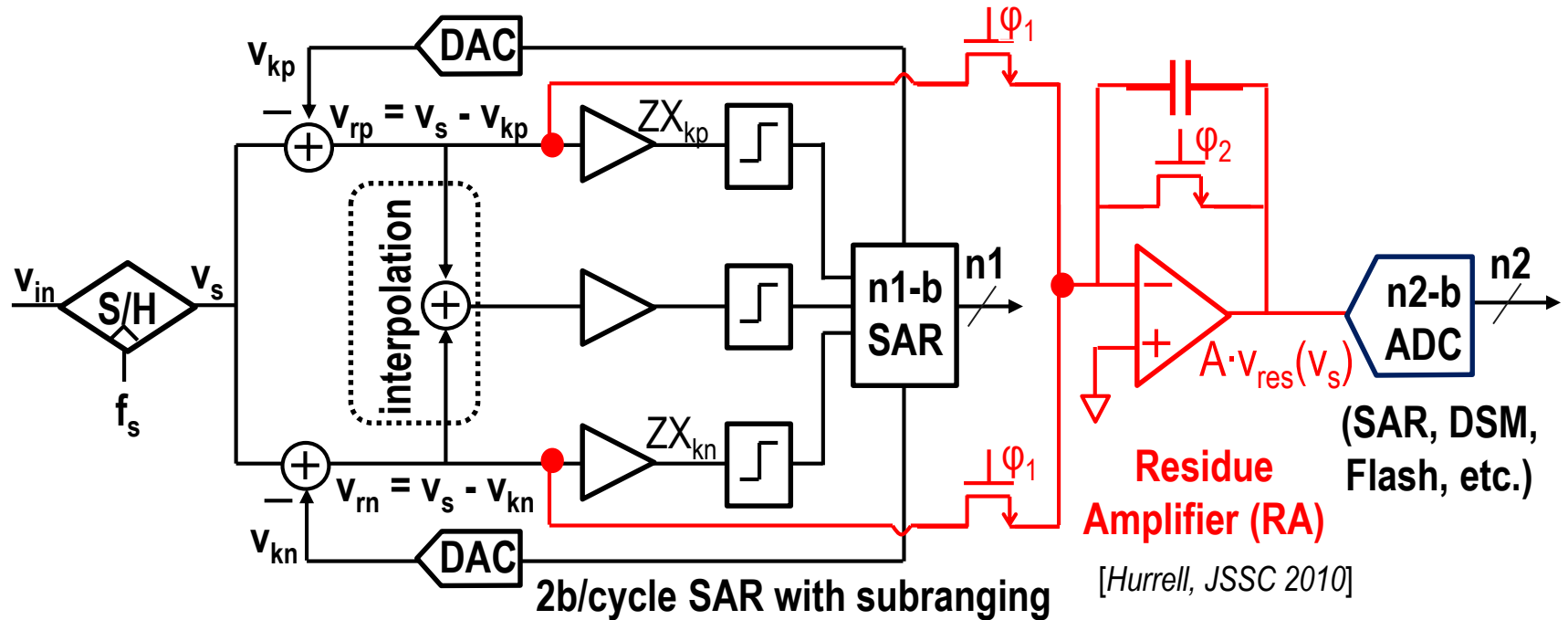
- Sub-ADC latency is reduced with fast comparator decision in advanced technology, obviating the need for the 2<sup>nd</sup> T/H.
- The reconstruction DAC caps are directly driven by the input, eliminating the T/Hs and the associated  $KT/C$  noises [Lee, JSSC 2011]..

## Evolution with Technology: Remove the RAs



- The 2<sup>nd</sup> sub-ADC is implemented in a SAR architecture that does not need residue amplification for using only one comparator for the entire conversion.
- The SAR in an advanced process can achieve the required conversion rate.
- The 1<sup>st</sup> sub-ADC output directly control the MSB segment of the SAR DAC.
- The SAR LSB segment is over-ranged using radix  $< 2$  for the DAC array.
- The 2-step pipeline ADC evolves to a sub-ranged SAR ADC [Lin, VLSI 2010] ...
- Technology has a drastic impact on ADC architecture evolutions and FoMs.

# Pipelined Sub-ranging of Multi-bit/cycle SAR ADC



# Closing Remarks

- Quantization is a search process ranging from the most efficient serial binary search to the fastest brute-force parallel search.
- Parallel search is reduced in complexity by signal folding but at the cost of delicate analog signal processing (e.g. MDAC).
- Serial binary search combined with comparator reuse results in the most efficient SAR ADCs at the expense of conversion rate.
- SAR ADC speed can be increased by time-interleaving as many quantizers as needed, with the mismatch error digitally calibrated.
- Digital calibration and error correction overheads diminish in advanced CMOS processes, leading to SAR ADC popularity.
- SAR ADC is enhanced with sub-ranging for resolution, pipelining for speed, optimal switching for power efficiency, etc.

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