Frequency/Phase Movement Analysis by Orthogonal Demodulation

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Abstract

In mixed signal system-on-a-chip (SOC) testing, there are lots of opportunities to measure phase and frequency change of signals. For instance, in a PLL circuitry, the lock up time is one of most typical test items. In a read/write channel device for hard disc drives, small pulse shifts for write pre-compensation are tested. Clock jitter is often evaluated in data transmission and storage devices. Since this kind of measurement needs a time stamp measurement in general, a time interval analyzer (TIA) or a time measurement instrument is applied to do the tests. If the input frequency range of the available TIA is not high enough for the test signal, real-time digitizers having higher sampling rates than the TIA do precise time stamp measurement. The digitized data is processed with the orthogonal demodulation method, and its instantaneous phase/frequency trend is extracted. Swept frequencies, phase-shifted clocks, shifted pulses, shifted edges and clock jitter are analyzed.

1. Introduction

There are many kinds of mixed signal SOC devices found in digital consumer appliances, such as optical disc drives (ODD), set top boxes (STB), xDSL modems and hard disc drives (HDD).

Almost all mixed signal SOC's contain phase-locked-loop (PLL) circuits inside. From the testing point of view, a PLL has various important factors such as the frequency capture range, the lock up time and its stability. In order to test these items, SOC testers are required to have precise time interval measurement functionality, which can be achieved by time stamp measurement.

In hard disc drives there is a read/write channel device located right after the magnetic head amplifier. New technology high-density magnetic recording systems need some tweaks of the pulse positioning that is called write pre-compensation to decrease the peak shift caused by the magnetic inter-symbol interference. Small quantity of pulse or edge shift so that the precise time measurement is indispensable.

Even in the digital consumer mixed signal SOC, there are often high-speed data transmission scheme integrated as a peripheral interface. In high-speed data transmission system, valid time window is extremely narrow, so that jitter of the clock is one of the key parameters to be tested. Jitter measurement requires time stamp functionality in its time measurement instrument.

But the items mentioned above require consecutive time interval measurement functionality. Especially the frequency domain analysis of time data array is required, so that the real-time time stamp measurement is necessary.

At first, an operation principle of a typical time interval analyzer (TIA) is described below in Figure 1. The target waveform is sampled at a certain threshold level, generating a series of time data, which is called a time stamp array. In order to sample the rising edges or falling edges of the waveform without missing and do the frequency domain analysis, the instrument must have the ability to sample it at more than twice of the maximum frequency of the test signal.

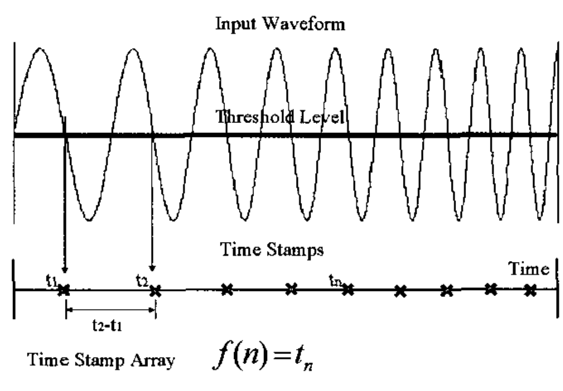

Figure 1 Time Stamp Measurement

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TIAs that can record time stamps in real time are available in the market, but their sampling frequency is not always sufficient for recent high-speed clock systems. Currently, the maximum sampling rate of such a TIA is around 80MHz[1], meaning the test signal frequency that can be analyzed is up to 40MHz. PLL circuitry whose frequency is more than 40MHz cannot be tested with that TIA. In this situation, test engineers need to find alternative instruments that can do time stamp measurements. A possible option is a real-time digitizer, which records the whole waveform of the target clock signal. The waveform carries all information of the test signal, if it is correctly used in the test. When the signal is appropriately processed, all the time related information can be extracted out of the waveform data.

One of the most typical analysis methods is the discrete Fourier transform (DFT), and its high-speed version, the fast Fourier transform (FFT). This is the most powerful tool in the DSP based testing. Probably 99% of the mixed signal tests apply FFT. It generates a frequency spectrum from a time domain waveform, so that you can read out the precise frequency information during the unit test period. It is static information, and the dynamic aspect of the signal cannot be shown in the DFT/FFT result. What you need, for example in a PLL measurement, is the dynamic time information or instantaneous frequency/phase trend information. The FFT/DFT is not well suited for this purpose.

2. Frequency/Phase Movement Analysis

2.1 Clock Signal

The digitized waveform data contains all the time domain information of the signal that can be extracted with appropriate signal processing. In order to analyze how parameters in the signal are changing, you need to observe instantaneous values of the signal, not average values in the unit test period. In general, the clock signal $g(t)$ is described as a fundamental sinusoidal wave and its series of harmonics as follows.

$$g(t) = A_k \sum_{k=1}^{n} \cos(\omega_k t + \varphi_k) \tag{1}$$

The essential characteristics of the clock signal are contained in the fundamental frequency, and the harmonics contribute only subtly to the wave shape. Given this assumption, the clock can be simply shown as a single sinusoidal waveform as below.

$$g(t) \cong A_1 \cdot \cos(\omega_1 t + \varphi_1) = A \cdot \cos(\omega t + \varphi) \tag{2}$$

where ω is the frequency component, and φ is the phase component. In order to analyze the frequency change or phase change of the clock, both components should be analyzed with respect to time t.

2.2 Instantaneous Phase Analysis

In order to analyze changing frequency or phase, at first, you need to extract the instantaneous phase $\omega t+\varphi$ part of the equation (2). If you assume the *sin*( ) part of the signal has the same phase, then you can calculate the instantaneous phase of the signal using the math function *arctan*( ).

You can derive the *sin*( ) part from the *cos*( ) part using the Hilbert transform. This is introduced as an analytic signal method by Yamaguchi, et al. [2] It is a very sophisticated method to generate the imaginary part of the signal by applying the Hilbert transform in the time domain or with applying the FFT/IFFT (inverse FFT) in the frequency domain. However, the design of good Hilbert transform filter is difficult to do, and the FFT requires the number of data points be restricted to 2^n combinations only, making the FFT and IFFT operations cumbersome. In post processing, since the unwrapped instantaneous phase data is distributed in an extremely large range, it is not so easy to fit a good regression line in order to find the reference phase estimation.

2.3 Orthogonal Demodulation Method

Another new method can be employed to derive the instantaneous phase of the test signal. It is an application of orthogonal demodulation. The sampling frequency of the digitizer and the number of data points are known. Then you can create accurate reference signals with using mathematical *cos*( ) and *sin*( ) operations. They are data arrays of a reference frequency ω_r. When the test signal $g(t)$ in the equation (2) are multiplied by the reference signals, that process is described below.

$$\begin{aligned} g(t)\cdot\cos(\omega_r t) &= A\cdot\cos(\omega t+\varphi)\cdot\cos(\omega_r t) \\ &= \frac{A}{2}\{(\cos(\omega-\omega_r)t+\varphi)+(\cos(\omega+\omega_r)t+\varphi)\} \end{aligned} \tag{3}$$

$$\begin{aligned} g(t)\cdot(-\sin(\omega_r t)) &= A\cdot\cos(\omega t+\varphi)\cdot(-\sin(\omega_r t)) \\ &= \frac{A}{2}\{(\sin(\omega-\omega_r)t+\varphi)-(\sin(\omega+\omega_r)t+\varphi)\} \end{aligned} \tag{4}$$

As you can see in equations (3) and (4), after the multiplication, the signals are split into two frequency components respectively. One is the lower frequency component of (ω-ω_r), and the other is the higher frequency component of (ω+ω_r). If ω_r is close to ω, the (ω-ω_r) component is located around the DC area, while the (ω+ω_r) component is located in a frequency area nearly two times higher than that of the test signal. Then if you apply an appropriate low pass filtering to the signals, you can separate the lower components from the other as in the equation (5) and (6). The cut-off frequency of the low pass filter (LPF) is approximately ω in this case.

$$\frac{A}{2}\cos((\omega-\omega_r)t+\varphi)=x \tag{5}$$

$$\frac{A}{2}\sin((\omega-\omega_r)t+\varphi)=y \tag{6}$$

Now that you have *sin*( ) and *cos*( ) components of the frequency $(\omega\text{-}\omega_r)t+\varphi$, you can calculate the instantaneous phase of the lower signal component as below.

$$\arctan\left(\frac{y}{x}\right)=(\omega-\omega_r)t+\varphi \tag{7}$$

If ω_r is exactly equal to ω, the first term in the equation (7) disappears, and only the phase component φ remains.

This sequential data processing scheme is shown as a block diagram in Figure 2.

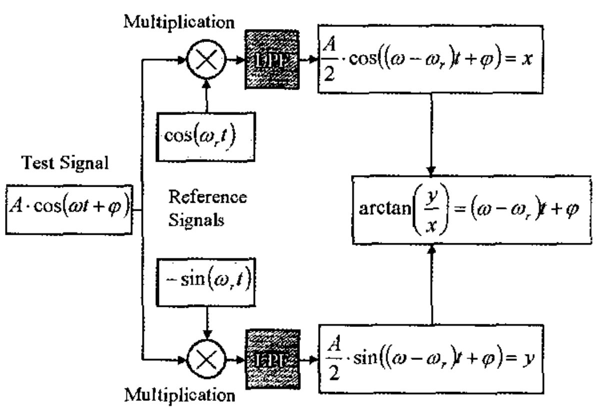

Figure 2 **Mathematical Data Processing Sequence**

The test signal is a series of digitized waveform data. The reference signals of $cos(\omega_r t)$ and $sin(\omega_r t)$ are ideal sinusoidal waveform data, which are generated by using built-in math functions. They can previously be calculated and stored in appropriate arrays for test throughput. The reference signal frequency is up to the test engineer. The multiplication is a simple math operation. The LPF operation is a convolution math operation of the multiplied data and the filter impulse response. The extracted lower frequency components, *cos*( ) and *sin*( ) go to the *arctan*(*y/x*) operation that is also a built-in math function. Finally the instantaneous phase of the lower frequency components is derived. This is the basic flow of this signal processing. Each element of the processing is very simple and is a basic operation. Since the operation does not include FFT/IFFT, there is no restriction of 2^n in terms of the number of data points.

Figure 3 shows the image of the spectrum distribution of all components. The test signal is located at frequency ω, and the reference signal ω_r should be placed as close to the target as possible. If the test signal frequency is well known in advance, the reference should be the same frequency for ease of post processing calculation. After the multiplication, the signals are split into low frequency components and high frequency components. There is no component around ω and ω_r. Since the split signals are separated far from each other, it is easy to extract the lower frequency components from the rest of them with a LPF. The filter curve is so simple that the gain in the lower band is one, and usually does not require a steep rolloff to the test signal.

In terms of the LPF, a finite impulse response (FIR) array can previously be calculated and stored in an array too. Then the convolution is applied for the filter signal processing. This operation is usually given as one of the test vendor's provided program instructions for mathematical calculations. *Cos*(), *sin*() and *arctan*() are built-in C functions.

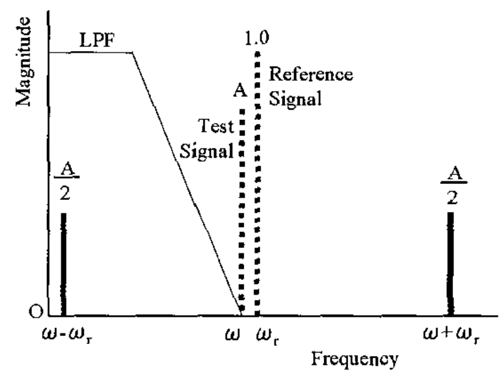

Figure 3 **Spectrum Appearance**

The extracted instantaneous phase component $(\omega\text{-}\omega_r)t+\varphi$ looks like a linear function of time t, if $\omega\text{-}\omega_r$ and φ are constant.

$$p(t)=(\omega-\omega_r)\cdot t+\varphi \tag{4}$$

Now that the term of $\omega\text{-}\omega_r$ is the slope of the line, if ω_r is equal to ω, the line is horizontal and only the phase offset φ remains. If you analyze the ripple of φ, it may be the phase modulation of the signal, or it may be the jitter of the signal.

$$p(t)\,|_{\omega=\omega_r}=\varphi=\varphi(t) \tag{5}$$

If the test signal is changing its frequency, ω must be a function of time t.

$$p(t) = (\omega(t) - \omega_r) \cdot t + \varphi \tag{6}$$

Since ω_r is what the test engineer defined, values of $\omega_r t$ can be calculated with a simple mathematical operation. By adding $\omega_r t$ to $p(t)$, the trend of $\omega(t)$ is extracted as described below.

$$p(t) + \omega_r \cdot t = \omega(t) \cdot t + \varphi \tag{7}$$

Now that this is established as a way to observe the changing frequency trend, it is applicable to many applications, if the signal waveform is sampled with a real-time digitizer. This includes the lock time measurement of PLL, and to the pulse shift measurement in the write pre-compensation test of read/write channel of HDD.

3. Experimental Results

3.1 Changing Frequency Signal Analysis

The first example is a changing frequency signal, which is a typical test case in PLL circuitry. This is a simulation for understanding the calculation procedure. The number of the data array is 4096, which is a good number to compare to an FFT, showing the spectral appearance for comparison. However, the number of 2^n is not required for the method described in this article.

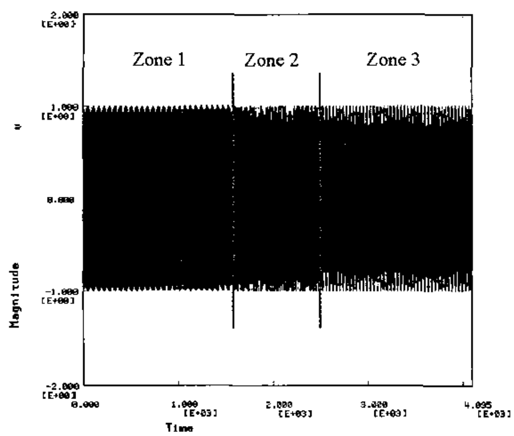

Figure 4 Simulated Waveform of Changing Frequency

Figure 4 shows a waveform whose frequency is changing. The line in the graph is too dense to see it is a sinusoidal waveform. Actually there are three zones in it. Zone 1 has a stable low frequency, which is about 500 cycles. (This is a relative frequency. In this graph, it would be 500 cycles within this observation period.) In Zone 2, the signal frequency is increasing gradually. Zone 3 has a stable final frequency, approximately 1000 cycles. This situation can be seen when a PLL encounters an abrupt change of its input signal frequency.

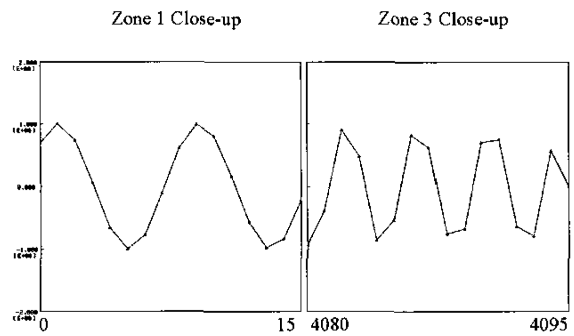

Figure 5 Close-up of the Waveform

The waveform graph looks so dense that the precise waveform cannot be recognized. The first 16 points and also the last 16 points of the waveform are zoomed up in Figure 5.The waveform captured has approximately 8 points per cycle of the sine wave in Zone 1, and approximately 4 points per cycle in Zone 3. Only a few points per cycle can be used for deriving the dynamic phase trend in this method.

An FFT is applied to the waveform to see its spectral appearances, as shown in Figure 6. Windowing is not applied in this operation.

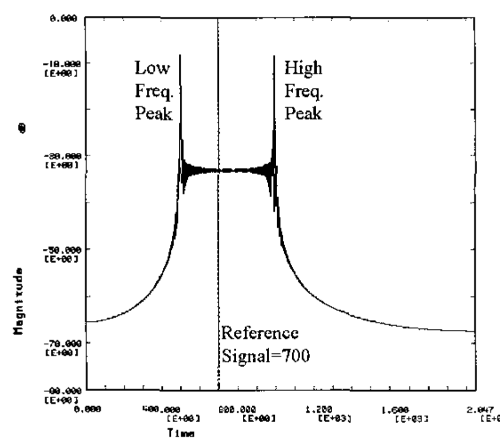

Figure 6 Spectrum of the Changing Frequency Signal

The signal is a kind of frequency-modulated signal so that the spectrum does not appear a simple tone, but spreads around bins 500 to 1000. Frequencies of Zone 1 and Zone 3 in Figure 4 give two significant peaks in Figure 6 respectively. For Zone 2, the frequency is gradually changing from the low peak to the high peak.

The data processing discussed in this article requires reference signals for *cos*( ) and *sin*( ) multiplication. In this particular case, it is good to provide it in the middle of the two peaks. The frequency is set as 700 cycles here. It is shown as a vertical line in Figure 6.

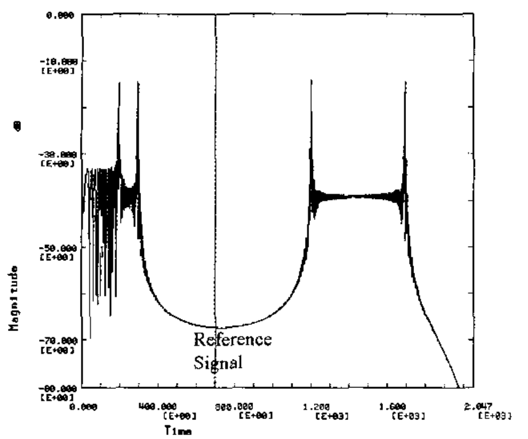

Figure 7 Spectrum of the Reference Multiplied Waveform

For creating *cos*( ) and *sin*( ) data array of 700 cycles of sinusoidal waveform with 4096 points, this math function is usually provided by the test vendor, so that it is easy to generate the data arrays. The test signal is multiplied by the data and the spectrum appears as in Figure 7. You can find two split spectral groups.

In an actual test program, from a test time throughput's point of view, the reference array data should be calculated and stored in memory in advance before going into the test loop.

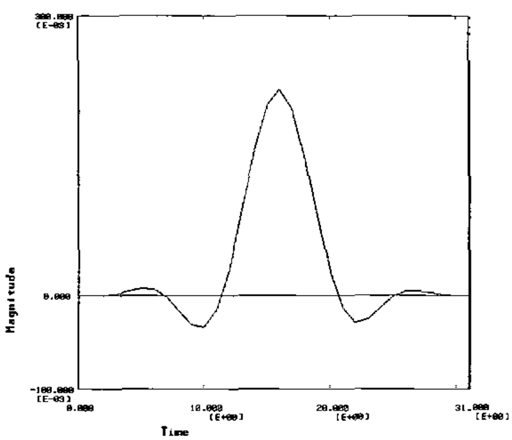

Figure 8 Impulse Response of the LPF

The signal is split into low frequency components and high frequency components. The spectrum will be discriminated by using a digital LPF. Figure 8 shows the finite impulse response (FIR) of the LPF, which is constructed with 32 points of data. It is designed as an ideal rectangular frequency response, and then its impulse response is weighted with Hanning window.

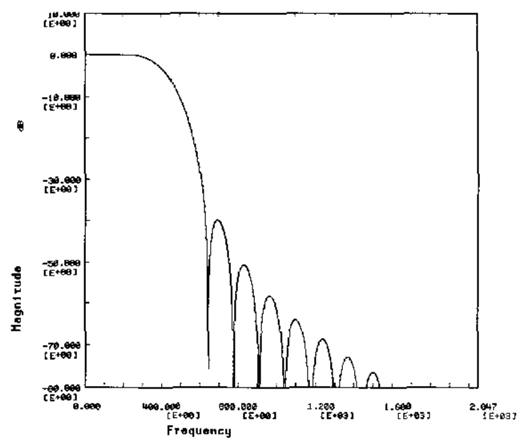

Figure 9 Frequency Response of the LPF

The frequency response of this FIR LPF is shown in Figure 9. It will cut off the high frequency components shown in Figure 7.

Now that the FIR filtering is mathematically processed as a convolution to the signals in Figure 4, the low frequency components are extracted as the waveforms of the *cos*( ) part, and the *sin*( ) part. Then the waveforms are processed with the math function *arctan*(*sin*()/*cos*()), deriving the angle component of them as in Figure 10, in which the angle is wrapped within the $-\pi$ to π region.

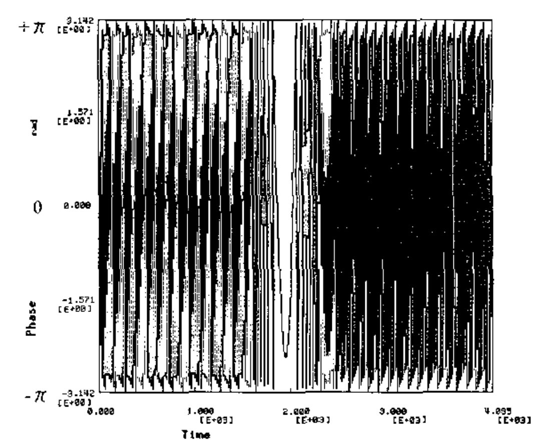

Figure 10 *arctan*( ) Result (Wrapped)

Unwrapping the angle curve, the true phase trend is exposed in Figure 11. Three zones clearly appear in the figure.

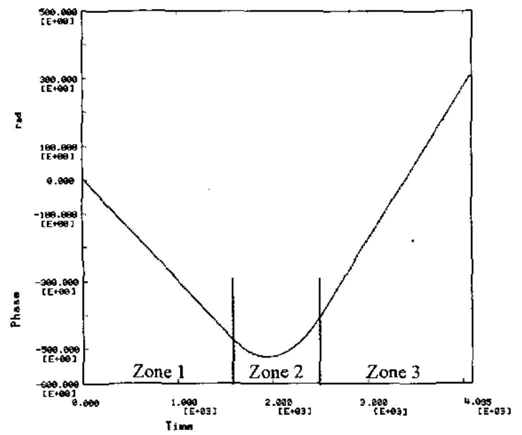

Figure 11 ***arctan*( ) Result (Unwrapped)**

In Zone 1 and Zone 3, the curves are straight lines, but they have the opposite slope to each other. Constant slope means steady frequency. Negative slope means lower frequency than the reference signal, and positive means higher than the reference. In Zone 2, the curve is gradually changing its gradient, meaning the frequency is changing from the low frequency to the high gradually.

The curve in Figure 11 shows the relative phase difference from the reference. When the test signal frequency is exactly equal to the reference, the curve should be a horizontal line. Therefore the phase trend can be converted into the frequency trend as shown in Figure 12.

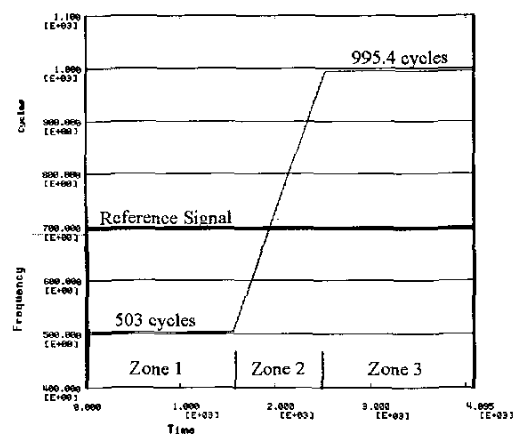

Figure 12 **Derived Frequency Trend**

Three frequency zones appear clearly. Zone 1 is a stable frequency of 503 cycles, Zone 2 is a sweeping area, and Zone 3 is the final stable frequency of 995.4 cycles. The duration of Zone 2 is the lock-up time, if this is a PLL response.

In order to demonstrate the method in the actual tester environment, the Agilent 93000 Mixed Signal SOC Tester is used for the experiment. A digital pin generates a 50MHz clock, which is measured with a real-time digitizer running at 319.75Ms/s. The number of captured data points is 8192.

The first experiment is the case of the on-the-fly clock frequency change generated by a digital pin.

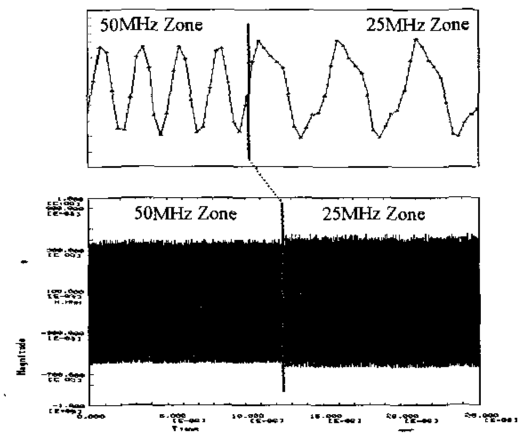

Figure 13 **Digitized Waveform of On-the-fly Clock Change**

Figure 13 shows the waveform that a digital pin generates when a 50MHz clock changes its frequency on-the-fly to 25MHz around the middle of the observation.

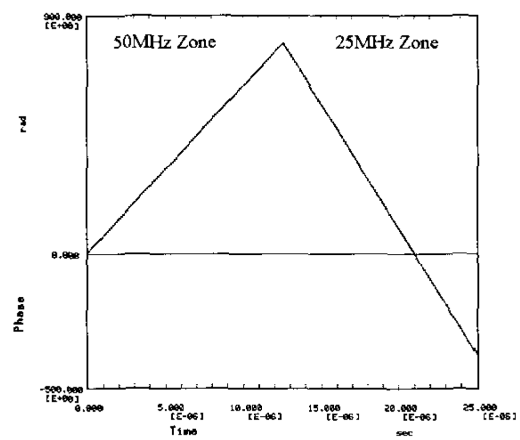

Figure 14 ***arctan*( ) Result (Unwrapped)**

The waveform is processed with the method described above by applying the mathematical reference of 40MHz. The phase trend appears as in Figure 14. The horizontal line is the relative phase of 40MHz. The gradient indicates the relative frequency to the reference.

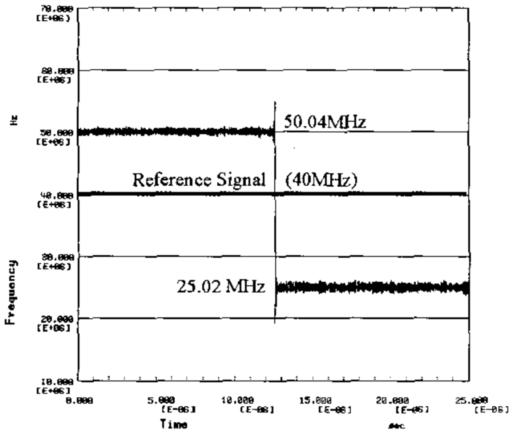

Figure 15 **Derived Frequency Trend**

The trend is interpreted into the frequency as 50.04MHz and 25.02MHz respectively, shown in Figure 15.

3.2 Shifting Clock Phase Analysis

After a PLL tracks to the input clock frequency, it locks up to the phase of the input. When the clock phase fluctuates, the PLL follows it promptly.

In video color composite signal applications, since the phase carries the color information, the differential phase (DP) of the device is one of the important factors.

The experiment in this section is about clock phase shifts. Figure 16 shows a 50MHz clock waveform captured with the digitizer, whose sampling rate is 319.75Msa/s. There are 8192 points of data, and 1281 clock pulses captured in the unit test period. A single clock pulse contains about 6.4 points. The clock stream is divided into four zones. The phase of the clock in each zone delays 1/8 of the unit interval (UI) or 2.5ns in this case one after another.

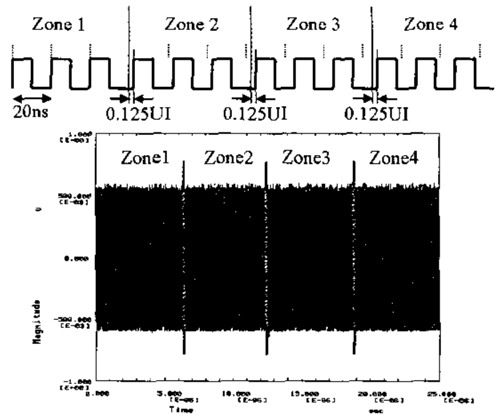

Figure 16 Digitized Waveform of Phase Shifting Clock

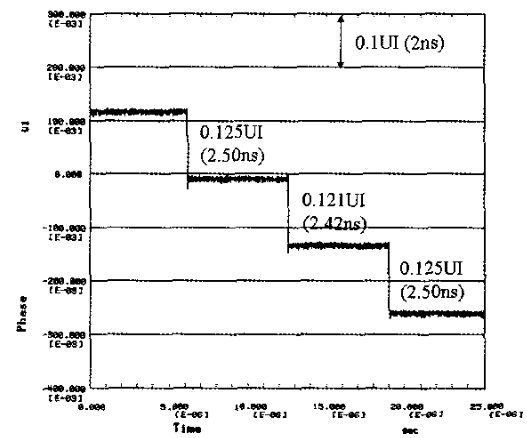

Figure 17 **Derived Phase Trend**

The reference signal used here contains 1281 cycles in the 8192 points. When the test signal waveform is processed with the method described in the article, the phase trend of the signal is derived as shown in Figure 17. It shows that the phase of each zone delays about 2.5ns one after another. In this experiment, eight different edges are used to shift the clock stream on-the-fly. This method can be used to find edge placement errors in a waveform, for example in this waveform edge 3 is off symmetry by 80pSec.

3.3 Single Pulse Shift Analysis

Because of a magnetic effect, pulses read out of magnetic media may slightly shift its location than where they should be.

A single pulse location in the clock stream is discussed in this section with this method.

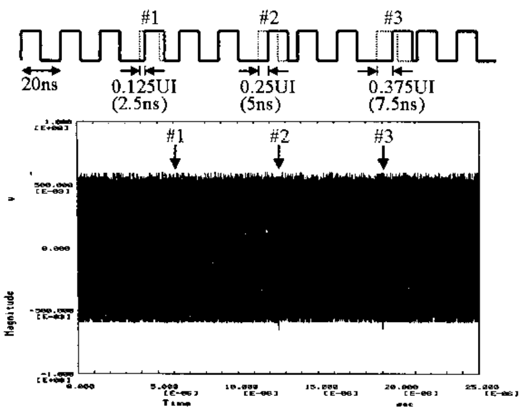

Figure 18 **Digitized Waveform of Pulse Shifted Clock**

The measured test signal waveform is shown in Figure 18. There are 1281 clock pulses captured in 8192 data points. Three marks are indicated in the graph. At each location, a single clock pulse is shifted backward from where it was. The shifts are -0.125UI, -0.25UI and -0.375UI respectively.

The test signal is analyzed with the method described in this article. The phase trend is derived as shown in Figure 19.

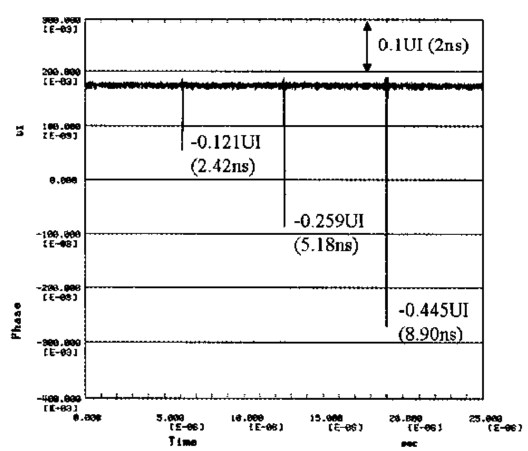

Figure 19 **Derived Phase Trend**

Three significant tics appear at the location where the pulses are shifted. It shows this method can be applied to detect only a single clock pulse movement.

3.4 Single Edge Shift Analysis

The next experiment is a single edge movement instead of a pulse shift as seen the last section. Figure 20 shows the test signal waveform. There are three unusual edges buried in the waveform. The rising edge is moved backward at the locations of the marks in the graph. The shifts are 0.05UI, 0.0625UI and 0.075UI respectively.

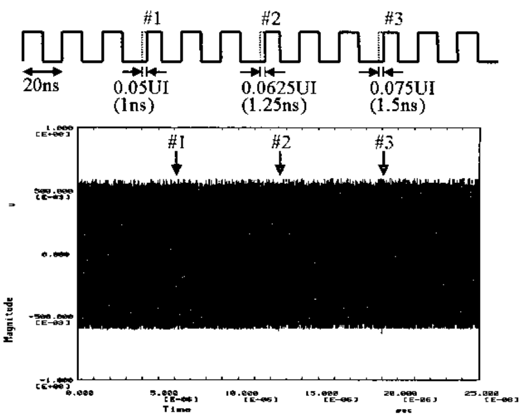

Figure 20 **Digitized Waveform of Edge Shifted Clock**

The waveform is processed with the method described in this article. The phase trend is derived as shown in Figure 21.

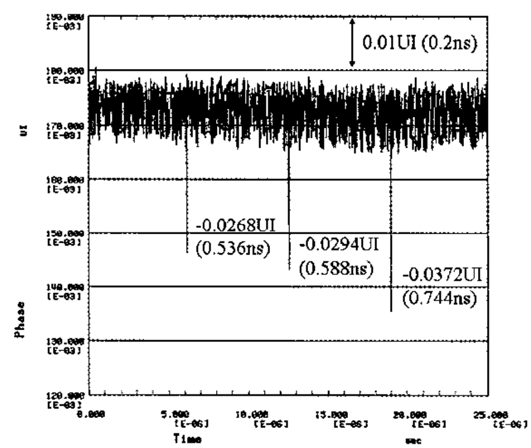

Figure 21 **Derived Phase Trend**

The detected values indicate half of the edge shift, because this method takes the whole cycle of waveform into account. Although there is only one edge shift at each location and also a very small amount of shift, all of them are clearly detected in this signal processing.

3.5 Clock Jitter Analysis

The analysis method described in this article can also be applied to clock jitter measurement. If clock jitter is tested using a TIA, the time stamps captured where the clock signal is sliced with a single threshold level are processed with its appropriate calculation procedure. However, in the method described in this article, the whole waveform of the test signal is analyzed. Jitter comes with the phase fluctuation of the clock. The phase trend analysis can capture all the information contained in the test signal without loss.

A 50MHz clock signal supplied by a pulse generator is measured with the digitizer running at 320Ms/s. Figure 22 shows the waveform. The upper left graph shows the whole waveform captured. 16384 points of data are captured, and there are 2557 cycles of clock pulses contained in it. The upper right graph is a close-up view of the first 31 points. A single clock pulse contains approximately 6 points of data. The lower graph is the reconstructed clock pulse after reshuffling the waveform with respect to its phase order.

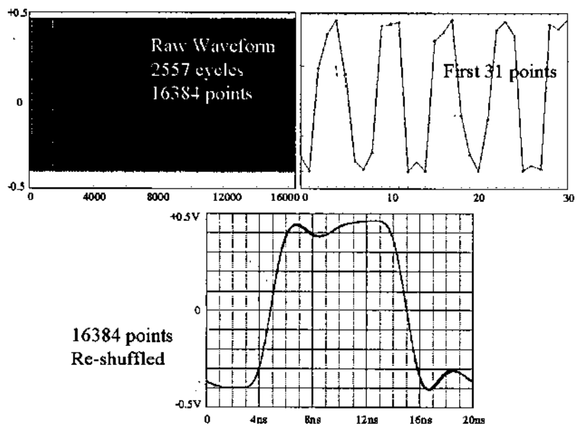

Figure 22 **Digitized Waveform of 50MHz Clock**

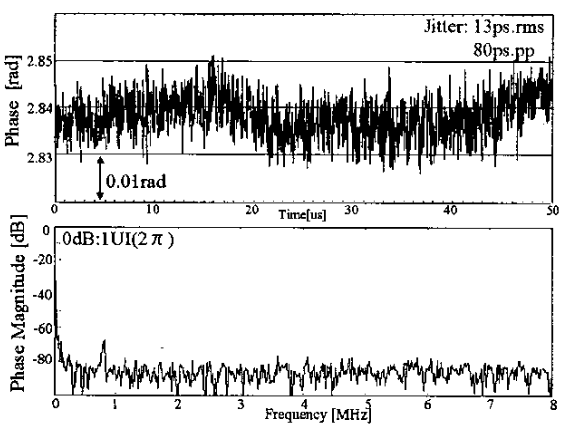

Figure 23 **Derived Jitter Component**

By processing the waveform using the method described in this article, the phase trend is derived as shown in the upper graph of Figure 23. The jitter of this phase trend is calculated as 13ps.rms and 80ps.pp. On top of that, this time axis data is processed with an FFT deriving the spectrum of the jitter as shown in the lower graph of Figure 23. There are low frequency jitter components and some spurious jitter around 800kHz detected in the spectrum display. The test engineer may be able to catch the root cause of the jitter problem with this information. Since this is real time digitizing process, the frequency domain analysis of the phase trend can also be done.

In order to validate the result, the reconstructed clock pulse is analyzed at the rising edge and the falling edge of the trace fluctuation.

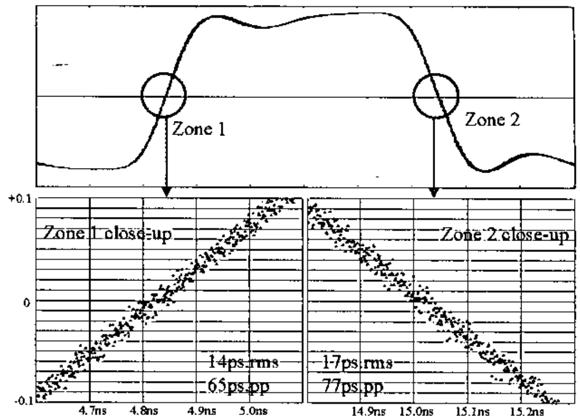

Figure 24 **Rising Edge/Falling Edge Fluctuation**

The upper graph in Figure 24 is the reconstructed waveform, which contains all 16384 points in a single cycle clock waveform. The lower graphs are a close-up view in the vicinity of the zero crossing areas. The scattered data points process and calculate its fluctuation as the jitter of the clock. The results are 14ps.rms and 65ps.pp on the rising edge, and 17ps.rms and 77ps.pp on the falling edge. The test result derived by the orthogonal demodulation method has a good correlation to these values.

This shows an effective example of how the jitter of clock signals can be measured with using this method.

4. Limitations

A critical point of the method is separating the signal into low frequency components and high frequency components. The target signal must be localized in a certain area of the spectral band. If the signal contains a very broadband spectrum, such as what you can see in random bit stream, it cannot be clearly separated into two frequency groups.

The most useful area of this method is analysis of clock or clock-like signals. However, if you can control any target signal as a repetitive one, the limitation would be minimized.

In a mathematical discussion, this method itself could be applicable to any precise measurement. However, in reality, the method fully counts on the performance of the digitizer for data acquisition. Its resolution and sampling clock jitter will define the performance limit of this method. Further investigation will be necessary from this point of view.

5. Discussion

This method is introduced as a substitute of a TIA whose time stamp functionality is not enough for the target signal frequency. It is efficient for test engineers that a tester resource can be used effectively for multiple purposes. The number of tester resources can then be minimized in the test application as much as possible for throughput, DUT board design and tester price.

Beside that, sometimes the TIA is not an easy instrument to use appropriately, because it usually has no mechanism to see the actual input waveform itself making it hard for test engineers to debug the test condition to get a reasonable result. To the contrary, a real-time digitizer can show the waveform so that it is very easy to confirm the validity of the digitized test signal.

The method described in this article is a purely mathematical one. The upper limit of the applicable test signal frequency depends on the maximum sampling rate of the real-time digitizer available.

6. Conclusions

There are lots of testing opportunities to measure changing frequency trends, changing phase trends, edge/pulse placement shift of signals, and jitter of clocks in mixed signal SOC testing. For those tests, usually the first selection of tester resources is the time measurement unit or the time interval analyzer. In order to measure the trend of the changing time factors, you need to capture a real-time series of time stamps to analyze the spectral aspect of the time domain parameters. A signal whose frequency exceeds the sampling specifications of a TIA cannot be measured with that TIA.

High-speed real-time digitizers are usually available and popular in mixed signal SOC testers. Test engineers can fully utilize digitizers to measure the dynamic frequency/phase trends effectively and accurately.

In this paper, the precise procedure of the measurement and calculation by using orthogonal demodulation is described. It can be achieved with very simple mathematical operations, such as *sin*, *cos*, *arctan*, multiplications and convolutions.

Many experimental results are shown for understanding the effectiveness of the method. This method is especially useful for measurement of the lock-up time and the stability of PLL circuitry, differential phase of video composite signals, the edge or pulse placement shift in read/write channel devices for HDD, and the jitter of a clock. Besides that the method can be used to find the edge placement errors in a waveform of the digital system.

Since the whole waveform is captured with the digitizer, the waveform can be examined directly, therefore it is very easy for test engineers to debug the test conditions as to whether they captured valid and correct edges in the waveform or not.

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8. Reference

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