Analysis of Direct Action Fuzzy PID Controller Structures

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***Abstract*— The majority of the research work on fuzzy PID controllers focuses on the conventional two-input PI or PD type controller proposed by Mamdani [1]. However, fuzzy PID controller design is still a complex task due to the involvement of a large number of parameters in defining the fuzzy rule base. This paper investigates different fuzzy PID controller structures, including the Mamdani-type controller. By expressing the fuzzy rules in different forms, each PID structure is distinctly identified. For purpose of analysis, a linear-like fuzzy controller is defined. A simple analytical procedure is developed to deduce the closed form solution for a three-input fuzzy inference. This solution is used to identify the fuzzy PID action of each structure type in the dissociated form. The solution for single-input–single-output nonlinear fuzzy inferences illustrates the effect of nonlinearity tuning. The design of a fuzzy PID controller is then treated as a two-level tuning problem. The first level tunes the nonlinear PID gains and the second level tunes the linear gains, including scale factors of fuzzy variables. By assigning a minimum number of rules to each type, the linear and nonlinear gains are deduced and explicitly presented. The tuning characteristics of different fuzzy PID structures are evaluated with respect to their functional behaviors. The rule decoupled and one-input rule structures proposed in this paper provide greater flexibility and better functional properties than the conventional fuzzy PID structures.**

***Index Terms*—Apparent linear gains, apparent nonlinear gains, fuzzy control, linear-like fuzzy, PID structures, two-level tuning.**

I. INTRODUCTION

OVER THE past two decades, the field of fuzzy controller applications has broadened to include many industrial control applications, and significant research work has supported the development of fuzzy controllers. In 1974, Mamdani [1] pioneered the investigation of the feasibility of using compositional rule of inference that has been proposed by Zadeh [2], for controlling a dynamic plant. A year later, Mamdani and Assilian [3] developed the first fuzzy logic controller (FLC), and it successfully implemented to control a laboratory steam engine plant. In a strict sense, the first fuzzy controller shown in [3] was equivalent to two-input fuzzy PI (or PI-like) controllers where error and error change, were used as the inputs for the inference. Mamdani's pioneering work also introduced the most common and robust fuzzy reasoning method, called Zadeh–Mamdani min–max gravity reasoning. Also, a significant number of in-depth theoretical and analytical investigations related to this structure have been reported in [4]–[8]. Takagi and Sugeno [9] introduced a different linguistic description of the output fuzzy sets, and a numerical optimization approach to design fuzzy controller structures.

There are several types of control systems that use FLC as an essential system component. The majority of applications during the past two decades belong to the class of fuzzy PID controllers. These fuzzy controllers can be further classified into three types: the direct action (DA) type, the gain scheduling (GS) type and a combination of DA and GS types. The majority of fuzzy PID applications belong to the DA type; here the fuzzy PID controller is placed within the feedback control loop, and computes the PID actions through fuzzy inference. In GS type controllers, fuzzy inference is used to compute the individual PID gains and the inference is either error driven self-tuning [10] or performance-based supervisory tuning [11]. In addition to the common Mamdani-type PI structure, several other structures using one- or three-input controllers have been reported. For comparison, a few selected error driven fuzzy PID applications are listed in Table I. It is clear from this literature review that the majority of these applications belong to the class of two-input fuzzy PID type structures. The majority of other related fuzzy PID references, which have not been included in this table, fall into the category of two-input Mamdani-type PID structures. In our recent work [38], a one-input fuzzy PID structure was used to control several first- and second-order plant models. The one-input FLC with fewer rules has not been commonly used for simultaneously deriving the three fuzzy PID actions. Based on this literature review, we can argue that different fuzzy PID structures are possible in the context of knowledge representation, and that they should be evaluated with respect to their functional behaviors. Therefore, in this paper we intend to deduce and evaluate different fuzzy PID structures, including the commonly available fuzzy PID controllers. Since the DA type fuzzy PID is the most commonly used, our study is restricted to those controllers only.

The linear PID controllers are easy to implement, and sufficient tuning rules are available to cover wider range of process specifications. Moreover, the available PID tuning heuristics are easy to understand and implement for practical control problems. Fuzzy controllers generally provide the nonlinear transfer elements for nonlinear control [39]. The system of if-then rules in the fuzzy knowledge base system

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TABLE I
DIFFERENT FUZZY PID STRUCTURES IN THE LITERATURE, e-ERROR, Δe-CHANGE OF ERROR, $\Delta^2 e$-RATE OF CHANGE OF ERROR, y-PLANT RESPONSE, Δy-CHANGE OF PLANT RESPONSE, + GS TYPE, * COMBINED DA AND GS TYPES. OTHERS DA TYPE

Input Conditions	Type	References
Three-input FLCs:		
$e, \Delta e, \Delta^2 e$	PID	[12]*,[13], [14]
e, Δe, $\int edt$	PID	[15]
Two-input FLCs:		
e, Δe	PI	[3]-[7],[16]-[25]
e, Δe	PD	[26]-[30]
e, Δe	PID	[31], [32], [33]
e, Δe	PID+	[10], [34]
Two two-input FLCs		
(e, Δe)+y, Δy	PI+D	[35]
Two+one-input FLCs		
e, Δe +e	PD+I	[36], [37]
one-input FLCs		
e	P	[16]
e	I	[16]
e	PI	[38]
Δe	P	[20]

is transformed into this nonlinear transfer elements. As a result the FLC has been successfully implemented in the past to for many linear and nonlinear processes [9], [11], [18], [36]. The natural representation of control knowledge through fuzzy paradigms allows the control action to be either linear or nonlinear and provides improved control in comparison with a conventional PID controller using linear control policy. The final tuning of fuzzy controllers, however, is still a difficult task. Many off-line techniques have been developed in the past for deciding the nonlinear transfer elements of the fuzzy controllers. As an example, cell-to-cell mapping [30], training algorithms using input/output data [40], and genetic search algorithms [38], [41] are capable of generating the optimum or near optimum solutions to the fuzzy systems in a high dimensional space, but at the cost of extensive computer simulations and time. Although the genetic algorithms are quite powerful in handling a large number of variables, the number of iteration cycles and the accuracy definitions (or resolution) allows one to reach only a near optimum rather than global optimum. Due to the complexity of the nonlinear control surface that is generated by conventional two-input fuzzy controllers, identifying and solving a large number of tuning parameters by an analytical means is extremely difficult. In this paper we propose simple fuzzy PID controller structures for reducing the dimensionality in designs. Functional behaviors of these fuzzy controllers are evaluated to show the main drawbacks of the conventional two-input fuzzy PID controllers.

In this paper we describe three contributions. First, new fuzzy PID structures are identified in addition to commonly available conventional fuzzy PID structures. These include a one-input–three-outputs fuzzy controller using error mapping for generating individual fuzzy PID actions. Second, a new analytical procedure is presented for the general three-input LLFLC inference based on min–max gravity reasoning. Two and one-input simplifications are included to cover a spectrum of fuzzy PID structures. Third, the apparent nonlinear and apparent linear PID gain analysis is presented for identifying the two-level tuning of fuzzy PID structures. Therefore the work in this paper is arranged as follows.

1) Fuzzy PID elements are proposed and then six different fuzzy PID structures, including commonly available structures, are constructed.
2) Closed form solutions for the outputs of fuzzy PID elements are deduced based on a linear-like fuzzy logic controller (LLFLC). Also the output of a SISO nonlinear like fuzzy controller with three rules is deduced.
3) Using the closed-form expressions, apparent nonlinear and apparent linear fuzzy PID gains are deduced while considering two-levels of tuning.
4) The structures are evaluated in terms of two-levels of tuning. Nonlinear tuning is evaluated with respect to the functional behaviors of structures.

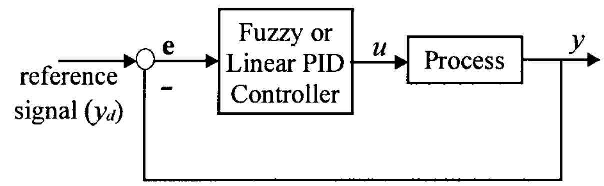

Fig. 1. Cascade type feedback PID controlled system.

II. FUZZY PID STRUCTURAL ELEMENTS

The linear PID controllers can be classified into different categories with respect to the positioning of the three terms in the closed-loop control system. In computer controlled single-input single-output (SISO) plant systems, the cascade-form PID controller is commonly used. Therefore in this study we restrict our classification to cascade type PID controllers as shown in Fig. 1. Other forms [35]–[37] can be obtained by extending the fundamental principle we propose in this study.

Considering a linear PID controller in Fig. 1, the controller signal at any given time instance n with a sampling time T_s can be expressed in two forms; (1) shows the output in the absolute form, while (2) shows it in the incremental form

$$u_{\text{PID}}(n) = K_P e(n) + K_I T_s \sum_{q=0}^{n} e(q) + (K_D/T_s)\Delta e(n) \tag{1}$$

$$\Delta u_{\text{PID}}(n) = K_P \Delta e(n) + K_I T_s e(n) + (K_D/T_s)\Delta^2 e(n)$$

and

$$u_{\text{PID}}(n) = u_{\text{PID}}(n-1) + \Delta u_{\text{PID}}(n). \tag{2}$$

The terms K_P, K_I, and K_D stand for proportional, integral, and derivative gains, respectively. The error state variables

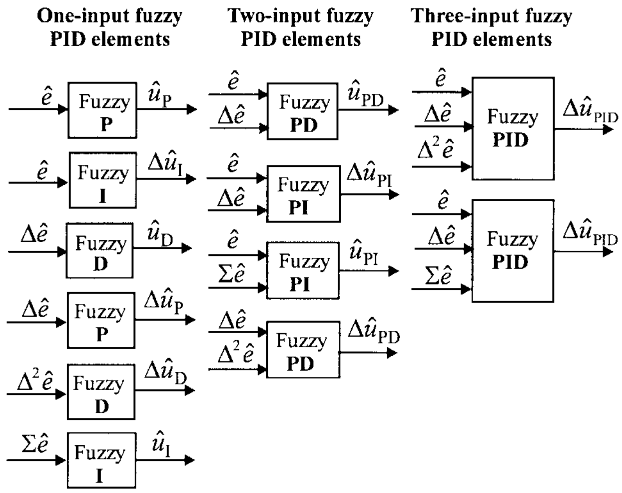

Fig. 2. Fuzzy PID structural elements.

are defined as: error $e(n) = y(n) - y_d(n)$, error change $\Delta e(n) = e(n) - e(n-1)$, rate of error change $\Delta^2 e(n) = \Delta e(n) - \Delta e(n-1)$, and sum-of-error $\Sigma e(n) = \sum_{q=0}^{n} e(q)$, with $y(n)$ being the feedback response signal, and $y_d(n)$ the desired response or reference input at the nth sampling instant. In a fuzzy PID controller, the error terms in (1) or (2) are expressed in a linguistic form and the fuzzy rules are used to infer a fuzzy control action. Since the linguistic expressions are qualitative, the linguistic variables are usually confined to an arbitrary space or universe of discourse. The error state variables defined above are the four basic inputs to any fuzzy PID type controller. In defining general fuzzy rule bases, the input error variables are therefore transformed to the normalized regions. Such normalization is quite useful for representing the fuzzy outputs in a unique fashion. For convenience, we sometimes drop the time instant notation n from all the control variables. The scale factors (S) for error variables are defined to obtain the normalized error terms as

$$\hat{e} = S_e e, \quad \Delta\hat{e} = S_{ce}\Delta e, \quad \Delta^2\hat{e} = S_{rce}\Delta^2 e, \quad \Sigma\hat{e} = S_{se}\Sigma e \tag{3}$$

where $\hat{e}$, $\Delta\hat{e}$, $\Delta^2\hat{e}$, and $\Sigma\hat{e}$, are the normalized error variables corresponding to the error terms e, Δe, $\Delta^2 e$, and Σe, respectively. The defuzzified output after the fuzzy reasoning is represented by $\hat{u}$.

Assume the error elements in (1) and (2) are fuzzy variables. Then by observing (1) and (2), fuzzy rules can be expressed to generate absolute and incremental fuzzy PID signals. By using three or two variables the coupled rules for three-input or two-input control elements are formed. In the case of one-input elements, the rules are defined for individual control actions in a PID signal. Thus, basic structural elements are identified as shown in Fig. 2; here each element shown in a block is described by fuzzy rules of the form "**If** (*input* 1 **and** *input* 2 $\cdots$) **then** (*output*)." In the case of two-input configurations, only PD and PI controller elements are considered. A subscript with the normalized output variable $\hat{u}$ is used for identifying the corresponding action in a fuzzy PID controller.

In deriving a practical fuzzy PID structure the following remarks are made.

Remark 1: It is difficult to formulate control rules with the input variable sum-of-error Σe, as its steady-state value is unknown for most control problems. As an example, the load disturbances at the plant input, dead weights and friction in drive systems are always unknown. Therefore it is difficult to identify membership values and their locations in the universe of discourse for defining control rules corresponding to steady-state conditions. It is possible to use this variable only if *a priori* knowledge about the steady state conditions is available [15].

Remark 2: For any fuzzy PID controller, the error (e) is considered the necessary input for deriving any PID structure. The error input provides the nonlinear proportional actions through the fuzzy inference. For any system to drive from a dead state, proportional control is the basic action required from the three-term PID controller. For example, in case of a steady offset in the system response, or in case of a time-delay process, the magnitude of all error derivatives becomes negligible. In those circumstances the steady error is the only available information that can provide a finite control action to divert the output from a dead situation.

III. Fuzzy PID Structures

By taking different combinations of the fuzzy PID structural elements defined in the previous section, we can now construct fuzzy controllers to represent PID actions in a nonlinear form. Based on Remarks 1 and 2, some of the structural elements can be considered to be "bad" and can be eliminated in building a fuzzy PID structure. Therefore in this systematic investigation we evaluate six types of controllers and compare their performance. In 1975, Zadeh published a three-part paper [42] describing the fundamentals of fuzzy logic principles for using in decision-making systems. Zadeh has included many definitions and concepts to generalize the broader perspectives of *humanistic* systems. The FLC systems uses some of those concepts for describing the knowledge base.

Define the linguistic variables that correspond to the input scaled variables $\hat{e}$, $\Delta\hat{e}$, and $\Delta^2\hat{e}$ as $\{E_i\}$, $\{\Delta E_j\}$, and $\{\Delta^2 E_k\}$, respectively. The indices i, j, and k represent the linguistic values or fuzzy states of the input fuzzy variables and their ranges are $i = 0, 1, 2, \cdots, N_1 - 1$, $j = 0, 1, 2, \cdots, N_2 - 1$, and $k = 0, 1, 2, \cdots, N_3 - 1$, where N_1, N_2, and N_3 denote the total numbers of fuzzy states assigned for each of the fuzzy variables. Let the de-normalizing scale factor S_u is given by the relation $u = S_u\hat{u}$ where u is the final controller output. Assign linguistic variables for the controller output as $\{U_m\}$ for absolute output signal $\hat{u}$, or $\{\Delta U_m\}$ for incremental signal $\Delta\hat{u}$, The index $m = 0, 1, 2, \cdots, M-1$. The value M denotes the total number of fuzzy states defined for the output fuzzy variable. For each element used in the following structures, the nonlinear function $f(.)$ is used to denote the nonlinear mapping between inputs and output.

Type I—Three-Input FLC Structure with Coupled Rules: It is practically difficult to assign linguistic values or terms for the input $\Sigma\hat{e}$ as explained in Remark 1. Therefore with a three-input configuration the fuzzy PID controllers are unable to produce an absolute signal. Hence the possible inputs are $\hat{e}$, $\Delta\hat{e}$, and $\Delta^2\hat{e}$, corresponding to an incremental type fuzzy PID

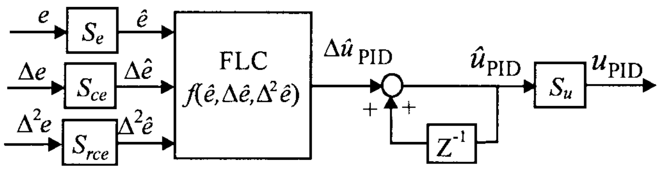

Fig. 3. Three-input fuzzy PID (Type I).

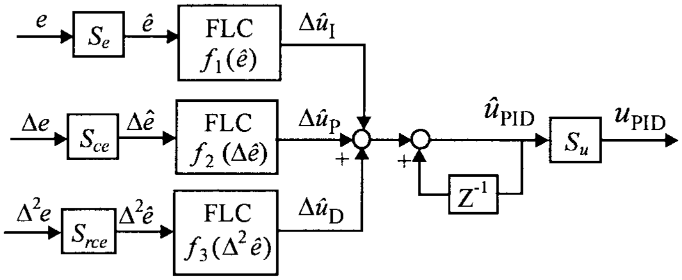

Fig. 4. Three-input fuzzy PID (Type II).

controller. Using the rule base notation of [11], Type-I fuzzy PID structure can be expressed by

$$\underset{i,j,k}{\mathrm{ELSE}}\begin{bmatrix}\text{IF } \hat{e} \text{ IS } E_i \text{ AND } \Delta\hat{e} \text{ IS } \Delta E_j \text{ AND } \Delta^2\hat{e} \\ \text{IS } \Delta^2 E_k \text{ THEN } \Delta\hat{u}_{\mathrm{PID}} \text{ IS } \Delta U_{m,\,\mathrm{PID}}\end{bmatrix}. \tag{4}$$

The final PID control output is produced after taking the cumulative sum of the FLC output as shown in Fig. 3. The total number of rules required for a complete description of the normalized space is $N_1 \times N_2 \times N_3$. The final controller output can be expressed by

$$u_{\mathrm{PID}}(n) = S_u \sum_{q=0}^{n} \Delta\hat{u}_{\mathrm{PID}}(q). \tag{5}$$

Type II—Three-Input FLC Structure with Decoupled Rules: The idea of knowledge based decoupling has been used in [11] and [43] to formulate a simple set of rules for GS type fuzzy controllers where the performance based inference is used for fuzzy tuning of conventional PID controllers. When this idea is extended for DA type fuzzy PID applications, we can select three one-input structural elements corresponding to decoupled rules of Type I for generating the fuzzy incremental control signals. Each incremental PID control action is now represented by a separate set of rules. The knowledge base is expressed by three rules sets

$$\left.\begin{aligned}&\underset{i}{\mathrm{ELSE}}\,[\text{IF } \hat{e} \text{ IS } E_i \text{ THEN } \Delta\hat{u}_I \text{ IS } \Delta U_{m_1,\,I}]\\ &\underset{j}{\mathrm{ELSE}}\,[\text{IF } \Delta\hat{e} \text{ IS } \Delta E_j \text{ THEN } \Delta\hat{u}_P \text{ IS } \Delta U_{m_2,\,P}]\\ &\underset{k}{\mathrm{ELSE}}\,[\text{IF } \Delta^2\hat{e} \text{ IS } \Delta^2 E_k \text{ THEN } \Delta\hat{u}_D \text{ IS } \Delta U_{m_3,\,D}]\end{aligned}\right\}. \tag{6}$$

The inference of each rule base is independent and the output constitutes three separate nonlinear functions. The total number of rules required is $N_1 + N_2 + N_3$. The fuzzy PID structure is shown in Fig. 4. The final control action is given by

$$u_{\mathrm{PID}}(n) = S_u \sum_{q=0}^{n} (\Delta\hat{u}_P(q) + \Delta\hat{u}_I(q) + \Delta\hat{u}_D(q)). \tag{7}$$

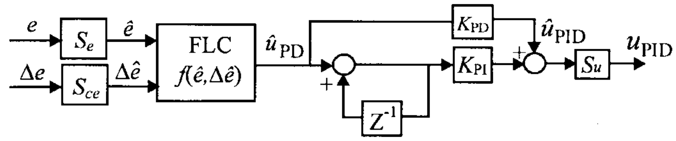

Fig. 5. Two-input fuzzy PID (Type III).

Type III—Two-Input FLC Structure with Coupled Rules: By observing the two-input control elements shown in Fig. 2 we select the elements having the inputs ($\hat{e}$, $\Delta\hat{e}$) as the useful PID elements for fuzzy control. They are corresponding to the incremental PI or absolute PD signals. The other two-input control elements shown in the Fig. 2 are eliminated according to the Remarks 1 and 2. By combining both PI and PD actions as shown in Fig. 5, a two-input fuzzy PID controller can be formed. The rule base structure is identical to Mamdani-type fuzzy PI controller. The basic rule base of this conventional type is given by

$$\underset{i,j}{\mathrm{ELSE}}\begin{bmatrix}\text{IF } \hat{e} \text{ IS } E_i \text{ AND } \Delta\hat{e} \text{ IS } \Delta E_j \\ \text{THEN } \Delta\hat{u}_{\mathrm{PD}} \text{ IS } U_{m,\,\mathrm{PD}}\end{bmatrix}. \tag{8}$$

The total number of rules required in this case is equal to $N_1 \times N_2$. With additional gains K_{PD} and K_{PI} the final PID control signal shown in Fig. 5 is given by

$$u_{\mathrm{PID}}(n) = S_u\left[K_{\mathrm{PI}} \sum_{q=0}^{n} \Delta\hat{u}_{\mathrm{PI}}(q) + K_{\mathrm{PD}}\hat{u}_{\mathrm{PD}}(n)\right], \quad \text{where } \Delta\hat{u}_{\mathrm{PI}}(n) = \hat{u}_{\mathrm{PD}}(n). \tag{9}$$

Type IV—Two-Input FLC Structure with Decoupled Rules: The decoupled structure corresponding to the two-input coupled structure is described next. When the rules are decoupled from the two-input fuzzy PD element, the individual P and D actions can be generated by two one-input elements described by the inputs $\hat{e}$ and $\Delta\hat{e}$, respectively. The two rule bases corresponding to the two one-input control elements are given by

$$\left.\begin{aligned}&\underset{i}{\mathrm{ELSE}}\,[\text{IF } \hat{e} \text{ IS } E_i \text{ THEN } \hat{u}_P \text{ IS } U_{m_1,\,P}]\\ &\underset{j}{\mathrm{ELSE}}\,[\text{IF } \Delta\hat{e} \text{ IS } \Delta E_j \text{ THEN } \hat{u}_D \text{ IS } U_{m_2,\,D}]\end{aligned}\right\}. \tag{10}$$

From the one-input elements we can infer that $\hat{u}_P \equiv \Delta\hat{u}_I$ and therefore by taking the cumulative sum of the fuzzy proportional action, the fuzzy PID structure is derived as shown in Fig. 6. The total number of rules required in this case is equal to $N_1 + N_2$. With additional gains K_P, K_I, and K_D, the final PID controller action is given by

$$u_{\mathrm{PID}}(n) = S_u\left[K_P\hat{u}_P(n) + K_I \sum_{q=0}^{n} \hat{u}_P(q) + K_D\hat{u}_D(n)\right]. \tag{11}$$

Type V—One-Input FLC Structure with Single Rule-Base: The error signal is the essential and fundamental control component in PID control (Remark 1). Therefore by using the input variable $\hat{e}$, a one-input fuzzy PID control system is formed. This is simply the nonlinear mapping of error into

Type	K_{Pa}	K_{Ia}	K_{Da}	Tuning variables
I	$\frac{S_u S_{ce} d_3}{a_{ce}}$	$\frac{S_u S_e d_3}{a_e T_s}$	$\frac{S_u S_{rce} d_3 T_s}{a_{rce}}$	S_u, S_{ce}, S_{rce}
II	$S_u S_{ce}$	$\frac{S_u S_e}{T_s}$	$S_u S_{rce} T_s$	S_u, S_{ce}, S_{rce}
III	$d_2 S_u \left[\frac{S_{ce} K_{\mathrm{PI}}}{a_{ce}} + \frac{S_e K_{\mathrm{PD}}}{a_e} \right]$	$\frac{S_u S_e d_2 K_{\mathrm{PI}}}{a_e T_s}$	$\frac{S_u S_{ce} d_2 K_{\mathrm{Pd}} T_s}{a_{ce}}$	$S_u, S_{ce}, K_{\mathrm{PD}}$ $(K_{\mathrm{PI}} = 1)$
IV	$S_u S_e K_{\mathrm{P}}$	$\frac{S_u S_e K_{\mathrm{I}}}{T_s}$	$S_u S_{ce} K_{\mathrm{D}} T_s$	$S_u, S_{ce}, K_{\mathrm{I}}$ $(K_{\mathrm{P}} = K_{\mathrm{D}} = 1)$
V	$S_u S_e K_{\mathrm{P}}$	$\frac{S_u S_e K_{\mathrm{I}}}{T_s}$	$S_u S_e K_{\mathrm{D}} T_s$	$K_{\mathrm{P}}, K_{\mathrm{I}}, K_{\mathrm{D}}$ $(S_u = 1)$
VI	$S_u S_e K_{\mathrm{P}}$	$\frac{S_u S_e K_{\mathrm{I}}}{T_s}$	$S_u S_e K_{\mathrm{D}} T_s$	$K_{\mathrm{P}}, K_{\mathrm{I}}, K_{\mathrm{D}}$ $(S_u = 1)$

Fig. 6. Two-input fuzzy PID (Type IV).

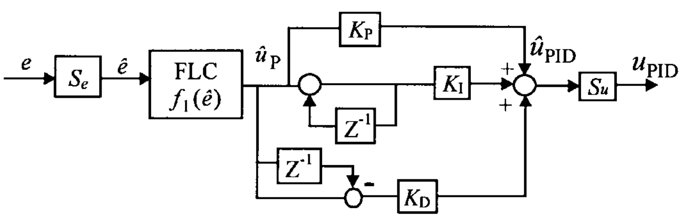

Fig. 7. One-input fuzzy PID (Type V).

fuzzy proportional action. The rule base of the one-input fuzzy proportional control element is given by

$$\underset{i}{\text{ELSE}}\,[\text{IF } \hat{e} \text{ IS } E_i \text{ THEN } \hat{u}_P \text{ IS } U_{m,P}]. \tag{12}$$

Similar to the previous case, we can infer from one-input elements $\hat{u}_P \equiv \Delta\hat{u}_I$ and by assuming the analogy between the proportional and derivative actions as, $\hat{u}_D(n) \equiv \hat{u}_P(n) - \hat{u}_P(n-1)$, the fuzzy PID structure is derived as shown in Fig. 7. This is the simplest fuzzy PID structure requiring only N_1 rules. With additional gains K_P, K_I, and K_D, the final control action is given by

$$u_{\mathrm{PID}}(n) = S_u \left[K_P \hat{u}_P(n) + K_I \sum_{q=0}^{n} \hat{u}_P(q) + K_D(\hat{u}_P(n) - \hat{u}_P(n-1)) \right]. \tag{13}$$

Type VI—One-Input FLC Structure with Three Rule-Bases: In this structure, three separate rule bases, using only error as the input variable, are used for generating three separate fuzzy proportional actions. The knowledge base parameters can be independently chosen or tuned to produce different nonlinearity for the individual PID actions. With respect to

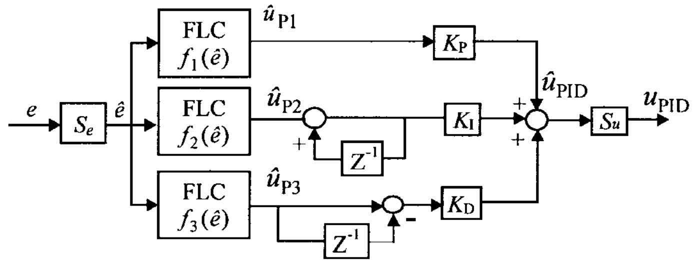

Fig. 8. One-input fuzzy PID (Type VI).

the input error variable, three rule bases are defined as

$$\left.\begin{array}{l} \underset{i}{\text{ELSE}}\,[\text{IF } \hat{e} \text{ IS } E_{1,i} \text{ THEN } \hat{u}_P \text{ IS } U_{m_1,P1}] \\ \underset{j}{\text{ELSE}}\,[\text{IF } \hat{e} \text{ IS } E_{1,j} \text{ THEN } \hat{u}_P \text{ IS } U_{m_2,P2}] \\ \underset{k}{\text{ELSE}}\,[\text{IF } \hat{e} \text{ IS } E_{1,k} \text{ THEN } \hat{u}_P \text{ IS } U_{m_3,P3}] \end{array}\right\}. \tag{14}$$

An additional integer suffix is used to separate the three proportional fuzzy rule bases. Using the same basic principle as used in the Type V controller, the integral and derivative actions are now generated using different nonlinear proportional sources as shown in Fig. 8. The total number of rules required in this case is $N_1 + N_2 + N_3$. Using three additional gains K_P, K_I, and K_D, the final control action is given by

$$u_{\mathrm{PID}}(n) = S_u \left[K_P \hat{u}_{P1}(n) + K_I \sum_{q=0}^{n} \hat{u}_{P2}(q) + K_D(\hat{u}_{P3}(n) - \hat{u}_{P3}(n-1)) \right]. \tag{15}$$

When the three rule bases are identical to the each other (identical knowledge base parameters), the structure would be same as the Type-V structure. Therefore, this is the most general form of the one-input fuzzy PID structure.

IV. Inference Analysis for Linear-Like and Nonlinear-Like Fuzzy Logic Controllers

The purpose of this analysis is to provide an analytical base for the evaluation of the above fuzzy controller structures and also to identify a tuning basis for those controllers. In order to simplify the problem, we define two classes of controllers: A general three-input linear-like fuzzy logic controller (LLFLC) and one-input nonlinear fuzzy controller. The output of each is derived using the standard Zadeh–Mamdani's (Z–M) min–max gravity reasoning method.

In this analysis, we show a new solution procedure for generating the output of three-input LLFLC. The two- and one-input solutions are obtained as special cases of the general solution. The main difficulty in a three-input Z–M based fuzzy inference analysis is the visualization of the output space with respect to three error state variables. Compared to a conventional two-input analytical procedure [5], [35], the three-input Z–M inference may require a minimum of 48 different equations to represent the controller output. In order to reduce this complexity of having multi-expressions in the solution, a transformation technique is provided and the general solution is expressed with only two different nonlinear terms. In addition to the above we have used the standard center of area (COA) defuzzification method rather than center of heights (COH) [39] or center average defuzzification [40] that was used in [5], [6], and [35]. The COH method is a convenient way to obtain output solution with least number of expressions. However, the COH method ignores the effect of *fuzziness* [11] associated with the output linguistic variables and is equivalent to taking fuzzy singleton functions. As an example, the COH method ignores the width of the support set or the partitioning of the output membership functions during the defuzzifications. As a result the COH produces less nonlinearity than the COA method, particularly for one-input fuzzy inferences. On the other hand COH is better for obtaining piece-wise linearity. For high degree of nonlinearity, it requires a large number of rules. This particular characteristics has been exploited to obtain the nonlinear function approximations [40], but at the expense of larger number of rules. However the COG method is difficult to analyze for a highly nonlinear rule bases. The nonlinear like analysis we perform in the latter part of this paper (Section IV-D) clearly demonstrates the benefits of COG method.

A. Definition—Linear-Like Fuzzy Logic Controller

Let the three error inputs in any order be defined as $\mathbf{e} = \{e_1, e_2, e_3\}^T$. After scaling the three error inputs, let the normalized input error vector at any time instant be given by $\hat{\mathbf{e}} = \{\hat{e}_1, \hat{e}_2, \hat{e}_3\}^T$. For simplicity we assume symmetrical triangular membership functions for each control variable. It is important to note that any symmetrical shaped membership function can be used for deriving the output of the LLFLC. The universe of discourse of each variable is uniformly partitioned and the membership functions are placed with a 50% overlap. The variables are defined by the following specifications.

1) The universe of discourse of each input variable is defined to be within the range $[-1, 1]$ as shown in Fig. 9(a). The total numbers of linguistic variables used for $\hat{e}_1$, $\hat{e}_2$, and $\hat{e}_3$ are N_1, N_2, and N_3, respectively, and the corresponding distances between two adjacent memberships are given by

$$\begin{aligned} a_1 &= 2/(N_1 - 1),\\ a_2 &= 2/(N_2 - 1),\\ a_3 &= 2/(N_3 - 1). \end{aligned} \tag{16}$$

2) The output linguistic variables are defined within the universe of discourse of $[-(1+d), (1+d)]$, where d is the distance between two adjacent output membership functions as shown in Fig. 9(b). Total number of membership functions defined for the output variable $\hat{u}$ is equal to $(N_1 + N_2 + N_3 - 2)$.

3) Using $(N_1 \times N_2 \times N_3)$ rules, the rule base is defined as

$$\underset{i,j,k}{\text{ELSE}} \left[\begin{array}{c} \text{IF } \hat{e}_1 \text{ IS } E_{1,i} \text{ AND } \hat{e}_2 \text{ IS } E_{2,j} \\ \text{AND } \hat{e}_3 \text{ IS } E_{3,k} \text{ THEN } \hat{u} \text{ IS } U_{i+j+k} \end{array} \right]. \tag{17}$$

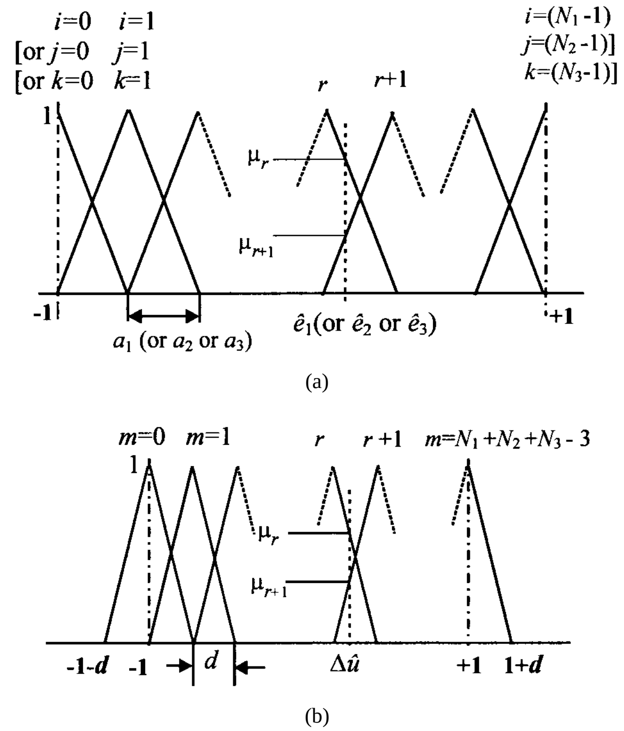

Fig. 9. Membership distributions of fuzzy variables.

Assigning $d = d_3$ for three-input LLFLC, the output modal spacing can be expressed as

$$d_3 = 2/(N_1 + N_2 + N_3 - 3)$$

or

$$1/d_3 = 1/a_1 + 1/a_2 + 1/a_3. \tag{18}$$

B. LLFLC Output Solution Procedure

1) Three-Input LLFLC Output Solution: The general solution to the three-input LLFLC is provided with the following seven steps and the derivation of the nonlinear term is shown in the Appendix.

Step 1*:* Define input variables and their associated scale parameters.

$\hat{e}_1 = S_1 e_1$, $\hat{e}_2 = S_2 e_2$, and $\hat{e}_3 = S_1 e_3$ where, S_1, S_2, and S_3 are the scale factors. For the purpose of defuzzification, define the error saturation limits as $\hat{e}_w = \max(-1, \min(1, S_w e_w))$. The index $w = 1, 2, 3$ and $\hat{e}_w \in [-1, +1]$. The values for a_1, a_2, a_3, and d_3 are obtained from (18).

Step 2*:* Define an input index vector and reference error inputs. Input index vector is defined as

$$\mathbf{i_a}(w) = \{i_a, j_a, k_a\}^T \tag{19}$$

where i_a, j_a, and k_a are the nearest integers given by, $i_a = \text{round}((1 + \hat{e}_1)/a_1))$, $j_a = \text{round}((1 + \hat{e}_2)/a_2))$, and $k_a = \text{round}((1 + \hat{e}_3)/a_3))$. Reference input variables are given by

$$\begin{aligned} \hat{e}_{1,i} &= -1 + i_a a_1\\ \hat{e}_{2,j} &= -1 + j_a a_2\\ \hat{e}_{3,k} &= -1 + k_a a_3. \end{aligned} \tag{20}$$

TABLE II
NONLINEAR TERM FOR THE THREE-INPUT LLFLC OUTPUT

Sign			Modify		Nonlinear Term
m_1	m_2	m_3	m_1	i_n	β_3
+	+	+	m_1	i_n	α_1
-	+	+	$1-\|m_1\|$	i_n-1	α_1
+	-	+	m_1	i_n	α_2
-	-	+	$1-\|m_1\|$	i_n-1	α_2
+	+	-	$-1+\|m_1\|$	i_n+1	$-\alpha_2$
-	+	-	m_1	i_n	$-\alpha_2$
+	-	-	$-1+\|m_1\|$	i_n+1	$-\alpha_1$
-	-	-	m_1	i_n	$-\alpha_1$

Step 3*:* Define the incremental input vectors. Normalized incremental input vector and Normalized absolute incremental input vector are respectively given by

$$\delta\hat{\mathbf{x}} = \{\delta x_{1,i}/a_1,\ \delta x_{2,j}/a_2,\ \delta x_{3,k}/a_3\}^T \tag{21}$$

$$\delta\hat{\mathbf{x}}_a = \{|\delta x_{1,i}|/a_1,\ |\delta x_{2,j}|/a_2,\ |\delta x_{3,k}|/a_3\}^T. \tag{22}$$

The incremental values are

$$\begin{aligned} \delta x_{1,i} &= (\hat{e}_1 - \hat{e}_{1,i}) \\ \delta x_{2,j} &= (\hat{e}_2 - \hat{e}_{2,j}) \\ \delta x_{3,k} &= (\hat{e}_3 - \hat{e}_{3,k}) \end{aligned} \tag{23}$$

$$(\delta x_{1,i}/a_1),\ (\delta x_{2,j}/a_2),\ (\delta x_{3,k}/a_3) \in [-0.5,\ 0.5].$$

Step 4*:* Input transformation.

1) Compute the transformed absolute incremental inputs and identify the corresponding incremental input vector positions w_1, w_2, and w_3

$$\left.\begin{aligned} |m_1| &= \max(\delta\hat{\mathbf{x}}_a) = \delta\hat{\mathbf{x}}_a(w_1) \\ |m_3| &= \min(\delta\hat{\mathbf{x}}_a) = \delta\hat{\mathbf{x}}_a(w_3) \\ |m_2| &= \delta\hat{\mathbf{x}}_a(6 - w_1 - w_3) = \delta\hat{\mathbf{x}}_a(w_2) \end{aligned}\right\}. \tag{24}$$

2) Compute the transformed true incremental inputs

$$m_1 = \delta\hat{\mathbf{x}}(w_1), \qquad m_2 = \delta\hat{\mathbf{x}}(w_2), \qquad m_3 = \delta\hat{\mathbf{x}}(w_3). \tag{25}$$

3) Redefine the transformed index vector

$$i_n = \mathbf{i_a}(w_1), \qquad j_n = \mathbf{i_a}(w_2), \qquad k_n = \mathbf{i_a}(w_3) \tag{26}$$

If $m_1 = m_2 = m_3$ then $w_1 = 1$, $w_2 = 2$, $w_3 = 3$.

Step 5*:* Assign the nonlinear output term. The values of m_1 and i_n may be modified depending on the signs of the three values computed in Steps 4-2) and shown in Table II. Using the modified terms in the Table II, the nonlinear term β_3 is deduced. The values are given by

$$\left.\begin{aligned} \alpha_1 &= \frac{1}{2}\left(\frac{7|m_3| + 5|m_2| + 3|m_1| - m_3^2 - m_2^2 - m_1^2}{1 + |m_3| + |m_2| + |m_1| - m_1^2}\right) \\ \alpha_2 &= \frac{1}{2}\left(\frac{5|m_3| - 3|m_2| + 3|m_1| - m_3^2 + m_2^2 - m_1^2}{1 + |m_3| + |m_2| + |m_1| - m_1^2}\right) \end{aligned}\right\}. \tag{27}$$

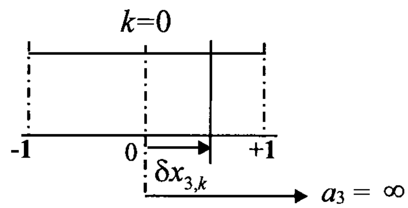

Fig. 10. Input fuzzy variable with a single fuzzy set.

Step 6*:* Reassign the modified index values to the input index vector

$$i_a = \text{modified}(i_n), \qquad j_a = j_n, \qquad k_a = k_n.$$

Step 7*:* Compute the LLFLC output $\hat{u}$

$$\hat{u} = \hat{u}_{i+j+k} + d_3\beta_3 \tag{28}$$

where the reference modal position is

$$\hat{u}_{i+j+k} = -1 + (i_a + j_a + k_a)d_3.$$

Using Table II and (20), (23), and (28), the general LLFLC output $\hat{u}$ can be decomposed into two parts; a linear controller output ($\hat{u}_{NL3}$) and a nonlinear controller output ($\hat{u}_{NL3}$). The linear controller is defined as the equivalent linear controller (ELC) of the LLFLC system

$$\left.\begin{aligned} \hat{u} &= \hat{u}_{L3} + \hat{u}_{NL3} \\ \hat{u}_{L3} &= (\hat{e}_1/a_1 + \hat{e}_2/a_2 + \hat{e}_3/a_3)\, d_3 \\ \hat{u}_{NL3} &= (\beta_3 - \delta x_{1,i}/a_1 - \delta x_{2,j}/a_2 - \delta x_{3,k}/a_3)\, d_3 \end{aligned}\right\}. \tag{29}$$

2) Two-Input LLFLC Output Solution: When only two inputs are considered, the third variable can have only a single fuzzy set "*Any*" for any crisp input value. Therefore the total number of fuzzy sets is equal to one and we assign this for the redundant input variable. Let us assume this variable is $\hat{e}_3$ and $N_3 = 1$. From (15), $a_3 = 2/(N_3 - 1) = \infty$. The triangular membership function defined for the single linguistic variable will now have an infinite long support set as shown in Fig. 10. The fuzzy membership function will be a horizontal line with a unit grade of membership height. The modal position of the single fuzzy set becomes $\hat{e}_{3,k} = 0$ with $k_a = 0$. Also, any normalized incremental input value measured from this modal position becomes $\lim_{a_3 \to 0}(\delta x_{3,k}/a_3) = 0$. Thus for any input conditions the $\min(\delta\hat{\mathbf{x}}_\mathbf{a}) = 0$ which implies $m_3 = 0$. The two-input rule base for generating the LLFLC surface can now be described by $(N_1 \times N_2)$ linear rules and is obtained by simplifying the three-input rule base in (17) as

$$\operatorname*{ELSE}_{i,j}\,[\text{IF } \hat{e}_1 \text{ IS } E_{1,i} \text{ AND } \hat{e}_2 \text{ IS } E_{2,j} \text{ THEN } \hat{u} \text{ IS } U_{i+j}]. \tag{30}$$

The modal spacing of output membership functions ($d = d_2$) is given by, $1/d_2 = 1/a_1 + 1/a_2$. Since we now have only two input variables, the eight cases in Table II reduce to four cases and α_2 is eliminated. For a two-input fuzzy controller, Steps 1–7 are used while equating one of the input variables to zero. Taking the special case for α_1 when $m_3 = 0$ we can obtain the

TABLE III
NONLINEAR TERM FOR THE TWO-INPUT LLFLC OUTPUT

Sign		Modify		Nonlinear Term
m_1	m_2	m_1	i_n	β_2
+	+	m_1	i_n	θ
-	+	$1-\lvert m_1\rvert$	i_n-1	θ
+	-	$-1+\lvert m_1\rvert$	i_n+1	$-\theta$
-	-	m_1	i_n	$-\theta$

corresponding nonlinear term (β_2) as shown in Table III. With the modified terms the nonlinear term is deduced by θ, where

$$\theta = (\alpha_1)_{m_3=0} = \frac{1}{2}\left(\frac{5|m_2| + 3|m_1| - m_2^2 - m_1^2}{1 + |m_2| + |m_1| - m_1^2}\right). \quad (31)$$

The LLFLC output is given by

$$\hat{u} = \hat{u}_{i+j} + d_2\beta_2, \qquad \text{where } \hat{u}_{i+j} = -1 + (i_a + j_a)d_2. \quad (32)$$

Similar to the three-input case, the general output expression for the two-input LLFLC output can be obtained as the sum of linear ($\hat{u}_{L2}$) and nonlinear ($\hat{u}_{NL2}$) controller outputs

$$\left.\begin{aligned} &\hat{u} = \hat{u}_{L2} + \hat{u}_{NL2} \\ &\hat{u}_{L2} = (\hat{e}_1/a_1 + \hat{e}_2/a_2)\, d_2 \\ &\hat{u}_{NL2} = (\beta_2 - \delta x_{1,i}/a_1 - \delta x_{2,j}/a_2)\, d_2 \end{aligned}\right\}. \quad (33)$$

3) One-Input LLFLC Output Solution: Similar to the two-input case, the second and third variables can now be assigned single fuzzy sets. Therefore both a_3, $a_2 \to \infty$ and the system simplifies to a one-dimensional problem. The corresponding LLFLC rule base structure can be represented by, N_1 rules as

$$\underset{i}{\text{ELSE}}\,[\text{IF } \hat{e}_1 \text{ IS } E_{1,i} \text{ THEN } \hat{u} \text{ IS } U_i]. \quad (34)$$

Allowing $k_a = j_a = 0$ and $(\delta x_{3,k}/a_3) = (\delta x_{2,j}/a_2) = 0$ for any (e_2, e_3) we can take the special case for α_1 when $m_3 = m_2 = 0$. The corresponding nonlinear term β_1 and its values are shown in Table IV. The term ϕ is given by

$$\phi = (\alpha_1)_{m_3=m_2=0} = \frac{1}{2}\left(\frac{3|m_1| - m_1^2}{1 + |m_1| - m_1^2}\right). \quad (35)$$

The one-input LLFLC output is given by

$$\hat{u} = \hat{u}_i + \beta_1 d_1 \quad (36)$$

where for a SISO LLFLC system $d = d_1 = a_1$ and $\hat{u}_i = -1 + i_a d_1$. Similar to the two cases above, the general solution for one-input LLFLC output can be expressed as the sum of linear ($\hat{u}_{L1}$) and nonlinear ($\hat{u}_{NL1}$) controller outputs, and is given by

$$\left.\begin{aligned} &\hat{u} = \hat{u}_{L1} + \hat{u}_{NL1} \\ &\hat{u}_{L1} = \hat{e}_1 \\ &\hat{u}_{NL1} = (\beta_1 - \delta x_{1,i}/a_1)\, d_1 \end{aligned}\right\}. \quad (37)$$

TABLE IV
NONLINEAR TERM FOR THE ONE-INPUT LLFLC OUTPUT

Sign	Nonlinear Term
m_1	β_1
+	ϕ
-	$-\phi$

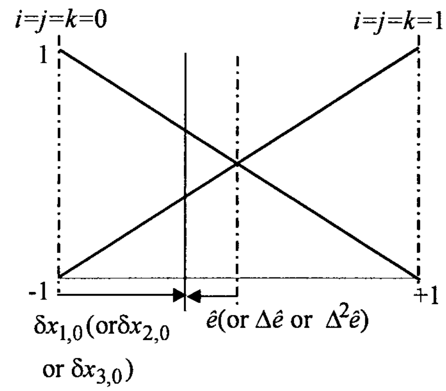

Fig. 11. Two uniformly distributed memberships for input variables of the simplest LLFLC.

C. Output Solution of a Simplest Type LLFLC

We define a fuzzy controller with least number of rules [5] to obtain concise expressions relating the error variables for the purpose of analyzing fuzzy PID gains in the next section. In this simplest LLFLC structure, each input is assigned two uniformly distributed membership functions as shown in Fig. 11, where $a_1 = a_2 = a_3 = 2$. Since the index values i, j, and k now have only two values, 0 and 1, we first consider the positive incremental inputs measured from the 0 index positions. For any given input error vector $\{\hat{e}, \Delta\hat{e}, \Delta^2\hat{e}\}^T$ the incremental values are

$$\begin{aligned} \delta x_{1,0} &= (1 + \hat{e})/2 \\ \delta x_{2,0} &= (1 + \Delta\hat{e})/2 \\ \delta x_{3,0} &= (1 + \Delta^2\hat{e})/2. \end{aligned}$$

Considering the PID structural elements in Fig. 2, the LLFLC outputs are deduced.

1) For Three-Input Elements: As it was shown in Section IV-B1 and Table II, the value of the nonlinear term changes with respect to the relative difference between the normalized input variables. In order to express the outputs in terms of the actual input terms (without transformation) and also to aid the PID gain analysis, a single case is considered. Assume $(\delta x_{1,0}/a_1) > (\delta x_{2,0}/a_2) > (\delta x_{3,0}/a_3) \geq 0$. Using the general solution in (29) we can show

$$\Delta\hat{u}_{\text{PID}} = \frac{1}{3}\left(\frac{4\hat{e} - 2\hat{e}^2 + 2\Delta\hat{e} - (\Delta\hat{e})^2 + 6\Delta^2\hat{e} - (\Delta^2\hat{e})^2}{9 - \hat{e}^2 - 2\Delta\hat{e} + 2\Delta^2\hat{e}}\right). \quad (38)$$

The (38) is rewritten in the dissociated form as

$$\Delta\hat{u}_{\text{PID}} = \frac{(2-\Delta\hat{e})\Delta\hat{e}}{3P} + \frac{(2-\hat{e})2\hat{e}}{3P} + \frac{(6-\Delta^2\hat{e})\Delta^2\hat{e}}{3P}$$
$$\text{where } P = 9 - \hat{e}^2 - 2\Delta\hat{e} + 2\Delta^2\hat{e}.$$

Assuming the dissociated form $\Delta\hat{u}_{\text{PID}} = \Delta\hat{u}_P^{\text{PID}} + \Delta\hat{u}_I^{\text{PID}} + \Delta\hat{u}_D^{\text{PID}}$ we define

$$\begin{aligned}\Delta\hat{u}_P^{\text{PID}} &= \frac{(2-\Delta\hat{e})\Delta\hat{e}}{3P}\\ \Delta\hat{u}_I^{\text{PID}} &= \frac{(2-\hat{e})2\hat{e}}{3P}\\ \Delta\hat{u}_D^{\text{PID}} &= \frac{(6-\Delta^2\hat{e})\Delta^2\hat{e}}{3P}.\end{aligned} \tag{39}$$

The superscript PID is used to show the inference source.

2) For Two-Input Elements: Considering the case, $(\delta x_{1,0}/a_1) > (\delta x_{2,0}/a_2) \geq 0$. and using the two-input solution in (33), we can show

$$\hat{u}_{\text{PD}} = \Delta\hat{u}_{\text{PI}} = \frac{1}{2}\left(\frac{4\hat{e}+\hat{e}^2+4\Delta\hat{e}-(\Delta\hat{e})^2}{7+2\Delta\hat{e}-\hat{e}^2}\right). \tag{40}$$

Equation (40) is rewritten in the dissociated form as

$$\hat{u}_{\text{PD}} = \Delta\hat{u}_{\text{PI}} = \frac{\hat{e}(4+\hat{e})}{2Q} + \frac{\Delta\hat{e}(4-\Delta e)}{2Q},$$
$$\text{where } Q = 7 + 2\Delta\hat{e} - \hat{e}^2.$$

Assuming the dissociated form $\Delta\hat{u}_{\text{PD}} = \hat{u}_P^{\text{PD}} + \hat{u}_D^{\text{PD}}$ and $\hat{u}_{\text{PI}} = \Delta\hat{u}_P^{\text{PI}} + \Delta\hat{u}_I^{\text{PI}}$ we define

$$\hat{u}_P^{\text{PD}} = \Delta\hat{u}_I^{\text{PI}} = \hat{e}(4+\hat{e})/(2Q),$$

and

$$\hat{u}_D^{\text{PD}} = \Delta\hat{u}_P^{\text{PI}} = \Delta\hat{e}(4-\Delta e)/(2Q). \tag{41}$$

The superscript PI or PD is used to show the inference source.

3) For One-Input Elements: Considering the one-input LLFLC solution in (37), the fuzzy outputs for one-input control elements can be expressed by

$$\left.\begin{aligned}\hat{u}_P = \Delta\hat{u}_I &= 4\hat{e}/(5-\hat{e}^2)\\ \hat{u}_D = \Delta\hat{u}_P &= 4\Delta\hat{e}/(5-(\Delta\hat{e})^2)\\ \Delta\hat{u}_D &= 4\Delta^2\hat{e}/(5-(\Delta^2\hat{e})^2)\end{aligned}\right\}. \tag{42}$$

D. Output Solution of a Simplest One-Input Nonlinear Like Fuzzy Controller

For this simple analysis, we consider a three-rule fuzzy controller [38] with triangular membership functions for both antecedent and consequent variables. All membership functions are assumed to be triangular and symmetrical. The input membership functions are assumed to be uniformly distributed over the input universe of discourse with a 50% overlap to satisfy the rule completeness [39] during the inference. The output membership positions (support sets) are varied while keeping a symmetrical partitioning of the output universe of discourse about zero. Fig. 12 shows the membership distribution where the fuzzy variables are labeled with “NB,” “ZE,” and “PB” to represent “negative big,” “zero,” and “positive big,” respectively. The knowledge base design parameters are then reduced to terms s_1 and s_2 as shown in the figure. For the derivation, assume the one-input fuzzy proportional element where normalized input and output crisp variables are $\hat{e}$ and $\hat{u}_{Pn}$, respectively. The additional suffix “n” is used to identify the nonlinear like controller. The simplest three linear rules, R1–R3, for a one-input PID controller element can be then represented by

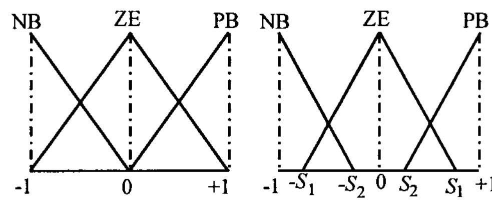

Fig. 12. Membership distributions for the nonlinear like SISO fuzzy controller.

$$\left.\begin{aligned}&\text{R1: If } \hat{e} \text{ is NB then } \hat{u}_{Pn} \text{ is NB}\\ &\text{R2: If } \hat{e} \text{ is ZE then } \hat{u}_{Pn} \text{ is ZE}\\ &\text{R3: If } \hat{e} \text{ is PB then } \hat{u}_{Pn} \text{ is PB}\end{aligned}\right\}. \tag{43}$$

All the variables are normalized into the range [−1, 1]. In order to reduce the complexity of the solution, the following constraint is imposed for the membership variables.

Range for $s_1 \in (0, 1]$, for keeping the ZE fuzzy set triangular about zero.

Range for $s_2 \in [-s_1, 1)$, for obtaining unique expressions for the fuzzy output. The solution has two main cases to be considered; nonoverlapping memberships or overlapping memberships, as shown in Fig. 13. The derivation is based on the Z–M inference and COA defuzzification as described in the Appendix. As the input membership functions are uniformly distributed (Fig. 12), for any given input value the inference always fires two rules simultaneously, except when $\hat{e} = 0$ or $\hat{e} = \pm 1$. The same error saturation limits given in the step 1 of the LLFLC output solution procedure (Section IV-B1) is imposed. During the fuzzy inference, the inferred fuzzy set (output) takes different shapes depending on the input value. The gray areas of Fig. 13 show these shapes. As a result the overlapping case has three different equations corresponding to three different ranges of the input. These ranges are determined by the membership height corresponding to the intersection of the consequent membership functions and the relative difference between the two membership variables as shown in Fig. 13. The common membership height s_d at this intersection is given by

$$s_d = (s_1 - s_2)/(1 + s_1 - s_2). \tag{44}$$

The final expressions for the defuzzified output are obtained by taking the center of the gray areas shown in Fig. 13. The cases when $s_2 < 0$ constitutes similar relationships to those given below. Let $\hat{e}_a = |\hat{e}|$ and $z_2 = 1 - s_2$.

Case I—Nonoverlapping: $s_1 \leq s_2$

$$\hat{u}_{Pn} = \frac{\hat{e}z_2}{3}\left[\frac{3(1+s_2) - 3s_2\hat{e}_a - z_2\hat{e}_a^2}{2s_1 + 2z_2\hat{e}_a - (2s_1+z_2)\hat{e}_a^2}\right]. \tag{45a}$$

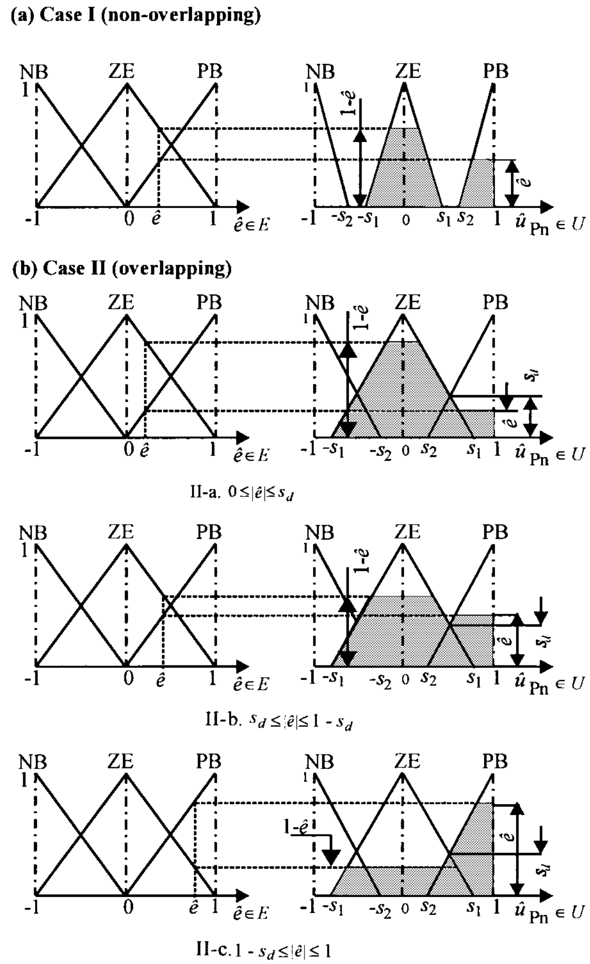

Fig. 13. Fuzzy outputs (shaded areas) corresponding to different input conditions.

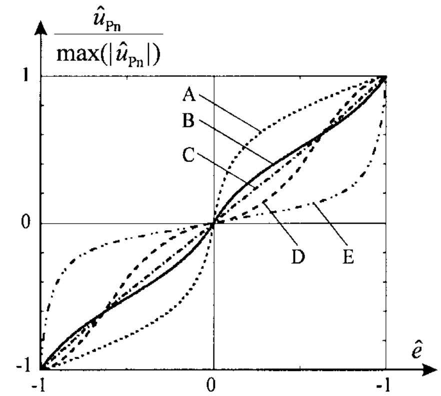

Fig. 14. Effect of membership parameters on nonlinear fuzzy proportional action, A: $s_1 = 0.1, s_2 = 0$, B: $s_1 = 0.3, s_2 = 0.3$, C: $s_1 = 0.566, s_2 = -0.2$, D: $s_1 = 0.95, s_2 = 0.9$, and E: $s_1 = 0.8, s_2 = 0.8$.

Case II—Overlapping: $s_1 > s_2$ AND

a) $[(s_1 - s_2) \le 1$ AND $0 \le \hat{e}_a < s_d]$ OR $[(s_1 - s_2) \ge 1$ AND $0 \le \hat{e}_a < 0.5]$

$$\hat{u}_{Pn} = \frac{\hat{e}}{3}\left[\frac{3(1-s_1^2) + s_1^2(3\hat{e}_a - \hat{e}_a^2)}{2s_1 + 2(1-s_1)\hat{e}_a - s_1\hat{e}_a^2}\right]. \tag{45b}$$

b) $[(s_1 - s_2) \le 1$ AND $s_d \le \hat{e}_a < (1 - s_d)]$

$$\hat{u}_{Pn} = \frac{\hat{e}}{3\hat{e}_a}\left[\frac{\begin{array}{c}(s_1 - s_2)(3s_d s_1 + 2s_d z_2 - 4s_1 + s_2)s_d \\ +\hat{e}_a z_2(3 + 3s_2 - 3\hat{e}_a s_2 - z_2\hat{e}_a^2)\end{array}}{2s_1 - (s_1 - s_2)s_d + 2z_2\hat{e}_a - (2s_1 + z_2)\hat{e}_a^2}\right]. \tag{45c}$$

c) $[(s_1 - s_2) \le 1$ AND $(1 - s_d) \le \hat{e}_a < 1]$ OR $[(s_1 - s_2) \ge 1$ AND $0.5 \le \hat{e}_a \le 1]$

$$\hat{u}_{Pn} = \frac{\hat{e}}{3\hat{e}_a}\left[\frac{\begin{array}{c}(3s_2 + z_2^2 - s_1^2) - 3s_2^2\hat{e}_a + 3z_2^2\hat{e}_a^2 \\ +(s_1^2 - 2z_2^2)\hat{e}_a^3\end{array}}{(1 + s_1 + s_2) - 2s_2\hat{e}_a - s_1\hat{e}_a^2}\right]. \tag{45d}$$

Equations (45a)–(45d) allow the nonlinearity of the proportional actions to be changed using the membership parameters s_1 and s_2. Different curves corresponding to the values of s_1 and $s2$ are shown in Fig. 14. Curve C shows the most approximate linear function of this fuzzy system.

V. Fuzzy PID Gain Analysis

Any fuzzy PID structure together with its fuzzy knowledge base usually results in nonlinear PID actions. In many fuzzy adaptive controllers, the nonlinearity has been adjusted online via modifying rules [13], [17] or membership functions [18]. Computational or numerical search techniques [9], [30], [38], [40], [41] are commonly used to produce optimum nonlinear controllers using fuzzy paradigms. An attempt has been made to investigate this nonlinear behavior by identifying the nonlinear proportional action in [38]. In this part we derive the fuzzy PID gains for each structure in order to identify the two-levels of tuning [33]. The first level of tuning relates to the *normalized nonlinear characteristics* and is usually obtained by changing the knowledge base parameters of the fuzzy system (rules, membership functions or support sets). We define apparent nonlinear gain (ANG) terms for all structures to identify the first tuning level. The second level of tuning is related to scale factors and other gain parameters used in constructing the fuzzy PID system. These parameters provide desired magnifications to the control surface in the directions of state axes. Therefore the second tuning level determines the *overall characteristics* of the controller. For this purpose we define PID apparent linear gain (ALG) terms.

A. Apparent Nonlinear Gains

Better understanding of nonlinear tuning in various fuzzy structures requires an analysis of FLC's in terms of the variables including the knowledge base parameters. This is a complex task due to high dimensionality in FLC systems and moreover the higher dimensionality may leads to nontransparency of the fuzzy output. In practical fuzzy controller designs, design experience usually reduces the dimensionality due to the availability of *a priori* knowledge. This is also a

TABLE V
ANG TERMS OF DIFFERENT FUZZY PID STRUCTURES

Type	ANG-Proportional $\hat{K}_{\mathrm{Pa}}(n)$	ANG-Integral $\hat{K}_{\mathrm{Ia}}(n)$	ANG - Derivative $\hat{K}_{\mathrm{Da}}(n)$
I	$\frac{1}{\hat{e}(n)}\sum_{q=0}^{n}\left(\frac{(2-\Delta\hat{e}(q))\Delta\hat{e}(q)}{3P(q)}\right)$	$\frac{1}{\sum_{q=0}^{n}\hat{e}(q)}\sum_{q=0}^{n}\left(\frac{2(2-\hat{e}(q))\hat{e}(q)}{3P(q)}\right)$	$\frac{1}{\Delta\hat{e}(n)}\sum_{q=0}^{n}\left(\frac{(6-\Delta^2\hat{e}(q))\Delta^2\hat{e}(q)}{3P(q)}\right)$
II	$\frac{1}{\hat{e}(n)}\sum_{q=0}^{n}\left(\frac{4\Delta\hat{e}(q)}{5-(\Delta\hat{e}(q))^2}\right)$	$\frac{1}{\sum_{q=0}^{n}\hat{e}(q)}\sum_{q=0}^{n}\left(\frac{4\hat{e}(q)}{5-(\hat{e}(q))^2}\right)$	$\frac{1}{\Delta\hat{e}(n)}\sum_{q=0}^{n}\left(\frac{4\Delta^2\hat{e}(q)}{5-(\Delta^2\hat{e}(q))^2}\right)$
III	$\hat{K}_{\mathrm{Pa}}^{\mathrm{PD}}=\frac{(4+\hat{e}(n))}{2Q(n)}$ $\hat{K}_{\mathrm{Pa}}^{\mathrm{PI}}=\frac{1}{\hat{e}(n)}\sum_{q=0}^{n}\left(\frac{(4-\Delta\hat{e}(q))\Delta\hat{e}(q)}{2Q(q)}\right)$	$\frac{1}{\sum_{q=0}^{n}\hat{e}(q)}\sum_{q=0}^{n}\left(\frac{(4+\hat{e}(q))\hat{e}(q)}{2Q(q)}\right)$	$\frac{(4-\Delta\hat{e}(n))}{2Q(n)}$
IV	$\frac{4}{5-\hat{e}(n)^2}$	$\frac{1}{\sum_{q=0}^{n}\hat{e}(q)}\sum_{q=0}^{n}\left(\frac{4\hat{e}(q)}{5-\hat{e}(q)^2}\right)$	$\frac{4}{5-(\Delta\hat{e}(n))^2}$
V	$\frac{4}{5-\hat{e}(n)^2}$	$\frac{1}{\sum_{q=0}^{n}\hat{e}(q)}\sum_{q=0}^{n}\left(\frac{4\hat{e}(q)}{5-\hat{e}(q)^2}\right)$	$\frac{4(5+\hat{e}(n)^2)}{5-\hat{e}(n)^2}$
VI	$\frac{\hat{u}_{\mathrm{Pn}}(\hat{e}(n),\mathbf{x}_1)}{\hat{e}(n)}$	$\frac{1}{\sum_{q=0}^{n}\hat{e}(q)}\sum_{q=0}^{n}\hat{u}_{\mathrm{Pn}}(\hat{e}(q),\mathbf{x}_2)$	$\frac{d\hat{u}_{\mathrm{Pn}}(\hat{e}(n),\mathbf{x}_3)}{d\hat{e}(n)}$

question for inner loop controllers where the availability of such controller experience is minimal [11]. In many cases, the nonlinear tuning is carried out arbitrarily by changing rules and membership function parameters, and observing the effect in computer simulations. A generic analysis is extremely difficult, particularly for coupled three-input or two-input rule bases. As we are primarily interested in comparing fuzzy PID structures, a simplest LLFLC rule base structure is assumed for deriving ANG terms of controller structure types I–V. The ANG terms of type VI controller are shown with respect to the nonlinear like fuzzy controller.

The nonlinear PID gains (ANG terms) related to normalized PID actions are defined as

$$\hat{K}_{Pa}(n)=\hat{u}_P(n)/\hat{e}(n),$$
$$\hat{K}_{Ia}(n)=\hat{u}_I(n)\Big/\sum_{q=0}^{n}\hat{e}(q)$$

and

$$\hat{K}_{Da}(n)=\hat{u}_D(n)/\Delta\hat{e}(n) \tag{46}$$

where $\hat{K}_{Pa}$, $\hat{K}_{Ia}$, and $\hat{K}_{Da}$ are the apparent nonlinear proportional, integral, and derivative gains, respectively. The ANG terms obtained for each structure type are listed in the Table V and the steps followed are described as follows.

1) ANG for Type I: Using the dissociated form given in (39), the normalized control action corresponding to (5) can be described by

$$\hat{u}_{\mathrm{PID}}(n)=\sum_{q=0}^{n}\Delta\hat{u}_P^{\mathrm{PID}}(q)+\sum_{q=0}^{n}\Delta\hat{u}_I^{\mathrm{PID}}(q)+\sum_{q=0}^{n}\Delta\hat{u}_D^{\mathrm{PID}}(q). \tag{47}$$

The equivalent form with ANG terms are then arranged as

$$\hat{u}_{\mathrm{PID}}(n)=\hat{K}_{Pa}(n)\hat{e}(n)+\hat{K}_{Ia}(n)\sum_{q=0}^{n}\hat{e}(q)+\hat{K}_{Da}\Delta\hat{e}(n). \tag{48}$$

Substituting the terms in (39) to (47), the ANG terms that correspond to the arrangement in (48) are thus obtained.

2) ANG for Type II: The normalized control action corresponding to (7) can be described by

$$\hat{u}_{\mathrm{PID}}(n)=\sum_{q=0}^{n}\Delta\hat{u}_P(q)+\sum_{q=0}^{n}\Delta\hat{u}_I(q)+\sum_{q=0}^{n}\Delta\hat{u}_D(q). \tag{49}$$

The expression of the ANG terms arrangement for (49) is identical to (48). Substituting one-input element outputs in (42) to (49), the ANG terms that correspond to the arrangement in (48) are thus obtained.

3) ANG for Type III: Using the dissociated form given in (41), the normalized output corresponding to (9) in the dissociate form can be described by

$$\hat{u}_{\mathrm{PID}}(n)=K_{\mathrm{PD}}\left(\hat{u}_P^{\mathrm{PD}}(n)+\hat{u}_D^{\mathrm{PD}}(n)\right)+K_{\mathrm{PI}}\left(\sum_{q=0}^{n}\Delta\hat{u}_P^{\mathrm{PI}}(q)+\sum_{q=0}^{n}\Delta\hat{u}_I^{\mathrm{PI}}(q)\right). \tag{50}$$

The equivalent form with ANG terms is then arranged as

$$\hat{u}_{\rm PID}(n) = K_{\rm PD}\Big(\hat{K}_{Pa}^{\rm PD}(n)\hat{e}(n) + \hat{K}_{Da}(n)\Delta\hat{e}(n)\Big) + K_{\rm PI}\left(\hat{K}_{Pa}^{\rm PI}(n)\hat{e}(n) + \hat{K}_{Ia}(n)\sum_{q=0}^{n}\hat{e}(q)\right). \tag{51}$$

Using the dissociated outputs for two-input element in (41) and substituting to (50), the ANG terms that correspond to the arrangement in (51) are thus obtained.

4) ANG for Type IV: In this decoupled rule structure, the normalized output corresponding to (11) can be expressed by

$$\hat{u}_{\rm PID}(n) = K_P\hat{u}_P(n) + K_I\sum_{q=0}^{n}\hat{u}_P(q) + K_D\hat{u}_D(n). \tag{52}$$

The equivalent form with ANG terms can be arranged as

$$\hat{u}_{\rm PID}(n) = K_P\hat{K}_{Pa}(n)\hat{e}(n) + K_I\hat{K}_{Ia}(n)\sum_{q=0}^{n}\hat{e}(q) + K_D\hat{K}_{Da}(n)\Delta\hat{e}(n). \tag{53}$$

Substituting the one-input element outputs in (42) to (52), the ANG terms that correspond to the arrangement in (53) are thus obtained.

5) ANG for Type V: In this one-input structure, the normalized output corresponding to (13) can be expressed by

$$\hat{u}_{\rm PID}(n) = K_P\hat{u}_P(n) + K_I\sum_{q=0}^{n}\hat{u}_P(q) + K_D(\hat{u}_P(n) - \hat{u}_P(n-1)). \tag{54}$$

The expression of the ANG terms arrangement for (54) is identical to (53). Substituting one-input element output for $\hat{u}_P$ in (42)–(54), the ANG terms that correspond to the arrangement in (53) are thus obtained. For small sampling time intervals the equivalent nonlinear derivative gain has been further simplified while using the relation $\hat{K}_{Da} = d\hat{u}_P(n)/d\hat{e}(n)$.

6) ANG for Type VI: Since type V structure is a special case of type VI, with the simplest LLFLC rule bases both types are identical. A practical high performance fuzzy controller requires the knowledge base to have a nonlinear-like structure. However, for the normalized proportional controller output to be monotonic with respect to error, the rules must be arranged in the linear form, as in (34). In such circumstances, the membership functions are placed nonuniformly to obtain the nonlinear tuning. In order to illustrate this, the solution of the simplest nonlinear like fuzzy controller shown by (45) is used. Let $\mathbf{x} = \{s_1, s_2\}$, be the vector containing nonlinear tuning parameters of the one-input fuzzy knowledge base. Then we can define three separate proportional actions with three different $\mathbf{x}$ terms as

$$\begin{aligned}\hat{u}_{P1} &= \hat{u}_{Pn}(\hat{e}, \mathbf{x}_1)\\ \hat{u}_{P2} &= \hat{u}_{Pn}(\hat{e}, \mathbf{x}_2)\\ \hat{u}_{P3} &= \hat{u}_{Pn}(\hat{e}, \mathbf{x}_3).\end{aligned} \tag{55}$$

The normalized output corresponding to (15) can be expressed as

$$\hat{u}_{\rm PID}(n) = K_P\hat{u}_{P1}(n) + K_I\sum_{q=0}^{n}\hat{u}_{P2}(q) + K_D(\hat{u}_{P3}(n) - \hat{u}_{P3}(n-1)). \tag{56}$$

The expression of the ANG terms arrangement for (56) is identical to (53). Substituting (55) into (56), the ANG terms that correspond to the arrangement in (53) are thus obtained. Similar to the type V, the small sampling time is assumed for obtaining the derivative ANG term.

B. Apparent Linear Gains

The overall tuning of fuzzy controllers is generally achieved by the second-level tuning, where scale factors and other gains are adjusted to obtain the desired or optimum response. In practice this is a trial and error procedure. Some tuning rules for these linear gains are reported in [44] for the two-input PI structure. The use of genetic algorithms to select these gains is described in [38] and [41]. In this analysis, apparent linear PID gains are defined for the fuzzy PID structures. The behavior of those gains is expected to be linearly equivalent to conventional PID gains. In order for the apparent gains to be functional, without loss of generality, we impose the following constraints.

Constraint 1: Assume the universe of discourse of all input variables are uniformly partitioned and the membership functions are placed with 50% overlap support sets. The rules are defined in the linear form. Nonlinearity is allowed by changing positions of output membership functions. Let the uniform input membership spacing be given by a_e, a_{ce}, and a_{rce}, respectively for the inputs e, Δe, and $\Delta^2 e$.

Constraint 2: The defuzzified output value is scaled to the range [−1, 1] by modifying the defuzzified output as; $\hat{u}^* = \hat{u}/|\hat{u}|_{\max}$ where $|\hat{u}|_{\max}$ is the maximum defuzzified output when the normalized error input terms are maximum.

Constraint 3: For set point control problems, the scale factor for error is fixed, i.e., $S_e = 1/e_{\max}$ where $e_{\max}$ is the maximum error signal during the transient. As the set point varies this value also varies.

The Constraint 1 is defined for obtaining rule completeness [39]. Also, this allows one to define a particular controller that would be linearly closest to the nonlinear fuzzy controller output. Alternatively, a linear surface equivalent to an existing nonlinear fuzzy output can be determined by linear regression analysis. Since this work is of a more general nature, this constraint is imposed so that the equivalent representation can be justified. As the ELC is derived from LLFLC and its maximum output is normalized within [−1,1], the Constraint 2 is imposed so that any skewed output shapes are normalized to this compact region. The Constraint 3 provides a standard procedure for determining the scale factor for the error input.

In the following analysis the superscript "l" denotes the equivalent linear actions. After substituting the scale factors

TABLE VI
ALG TERMS OF DIFFERENT FUZZY PID STRUCTURES

Type	K_{Pa}	K_{Ia}	K_{Da}	Tuning variables
I	$\frac{S_u S_{ce} d_3}{a_{ce}}$	$\frac{S_u S_e d_3}{a_e T_s}$	$\frac{S_u S_{rce} d_3 T_s}{a_{rce}}$	S_u, S_{ce}, S_{rce}
II	$S_u S_{ce}$	$\frac{S_u S_e}{T_s}$	$S_u S_{rce} T_s$	S_u, S_{ce}, S_{rce}
III	$d_2 S_u \left[\frac{S_{ce} K_{\mathrm{PI}}}{a_{ce}} + \frac{S_e K_{\mathrm{PD}}}{a_e}\right]$	$\frac{S_u S_{ce} d_2 K_{\mathrm{Pd}} T_s}{a_{ce}}$	$\frac{S_u S_e d_2 K_{\mathrm{PI}}}{a_e T_s}$	$S_u, S_{ce}, K_{\mathrm{PD}}$ $(K_{\mathrm{PI}} = 1)$
IV	$S_u S_e K_{\mathrm{P}}$	$\frac{S_u S_e K_{\mathrm{I}}}{T_s}$	$S_u S_{ce} K_{\mathrm{D}} T_s$	$S_u, S_{ce}, K_{\mathrm{I}}$ $(K_{\mathrm{P}} = K_{\mathrm{D}} = 1)$
V	$S_u S_e K_{\mathrm{P}}$	$\frac{S_u S_e K_{\mathrm{I}}}{T_s}$	$S_u S_e K_{\mathrm{D}} T_s$	$K_{\mathrm{P}}, K_{\mathrm{I}}, K_{\mathrm{D}}$ $(S_u = 1)$
VI	$S_u S_e K_{\mathrm{P}}$	$\frac{S_u S_e K_{\mathrm{I}}}{T_s}$	$S_u S_e K_{\mathrm{D}} T_s$	$K_{\mathrm{P}}, K_{\mathrm{I}}, K_{\mathrm{D}}$ $(S_u = 1)$

and assigning $a_e = a_1$, $a_{ce} = a_2$, and $a_{rce} = a_3$, the ELC outputs shown in (29), (33), and (37) are rewritten as follows:

For three-input elements

$$\Delta\hat{u}^l_{\mathrm{PID}}(n) = \frac{d_3 S_e}{a_e} e(n) + \frac{d_3 S_{ce}}{a_{ce}} \Delta e(n) + \frac{d_3 S_{rce}}{a_{rce}} \Delta^2 e(n). \tag{57}$$

For two-input elements

$$\hat{u}^l_{\mathrm{PD}}(n) = \Delta\hat{u}^l_{\mathrm{PI}}(n) = \frac{d_2 S_e}{a_e} e(n) + \frac{d_2 S_{ce}}{a_{ce}} \Delta e(n). \tag{58}$$

For one-input elements

$$\left.\begin{aligned} \hat{u}^l_P(n) &= \Delta\hat{u}^l_I(n) = S_e e(n) \\ \hat{u}^l_D(n) &= \Delta\hat{u}^l_P(n) = S_{ce} \Delta e(n) \\ \Delta\hat{u}^l_D(n) &= S_{rce} \Delta^2 e(n) \end{aligned}\right\}. \tag{59}$$

In order to define the linear apparent PID gains, the outputs corresponding to each PID structure is written in the following form:

$$u^l_{\mathrm{PID}}(n) = K_{Pa} e(n) + K_{Ia} \sum_{q=0}^{n} e(q) T_s + K_{Da} \Delta e(n)/T_s \tag{60}$$

where K_{Pa}, K_{Ia}, and K_{Da} are the linear apparent PID gains. Substituting the linear outputs in (57)–(59) (or ELC terms) to the output expressions given in (5), (7), (9), (11), (13), and (15), the ALG terms corresponding to each structure can be arranged as (60). The final expressions for ALG terms are shown in Table VI. In order to be analogues with linear PID systems, some linear terms have been assigned a value of unity to simplify the overall tuning a three-term tuning problem. The three corresponding tuning parameters of each structure are shown in the same table.

VI. COMPARISON OF FUZZY PID STRUCTURES

Comparison of the PID structures should address two issues:

1) the adjustment of fuzzy PID gains with respect to two-levels of tuning;
2) the assessment of the effect of these gains in the plant performance and tuning criterion related to PID gains.

The second issue is a process dependent problem and also it relates to the stability properties of each structure. This is a future area of research, and in this section we attempt to provide a comparative assessment of the first issue. It is known that linear PID controllers have independent control of the three control actions in a linear form, and that is accomplished by the tuning of three linear gain terms. Fuzzy control exhibits better performance primarily due to its higher level tuning, or the nonlinear gain tuning. A better fuzzy controller should allow maximum versatility and flexibility in tuning these nonlinear gains to achieve superior performance over linear control. Therefore the functional behaviors are considered with respect to the two levels of tuning.

A. High-Level Tuning

When the FLC system is known, the variations of ANG terms with respect to the error response are also known. In optimal designs this is usually achieved by varying the fuzzy knowledge base parameters, which directly affects the nonlinear characteristics of the control surface or curve with respect to normalized state variables. In the recent developments the nonlinear function approximation properties of fuzzy systems have been exploited to train or approximate highly nonlinear dynamical systems [40], [45]. However, in most cases the nonlinear function that requires for control is unknown. The same is true for fuzzy PID control action. Also the changing plant dynamics or environmental effects are unknown and unpredictable during the control. The fuzzy systems have the capabilities to produce these nonlinear functions either in coupled form [40] or in decoupled form [46]. The tuning heuristics and rules for gain adjustments of linear PID controller are usually available in the decoupled form [11], [43]. As an example, when there is a steady state offset in the response, the tuning is performed to increase the integral gain and the other two gains are kept unaltered [43]. In addition to this linear tuning the fuzzy PID controllers can produce local control by changing the ANG terms. Hence, these tuning rules can be used to approximate the unknown nonlinear functions in a single dimension to produce decoupled and independent tuning for ANG terms. To illustrate this effect more clearly for PID structures, the characteristics of the high-level tuning in coupled and decoupled rule bases are discussed with respect to three functional behaviors, namely, action association, input coupling, and gain dependency.

1) Action Association: The basic difficulty in coupled rule bases is the identification of those nonlinear tuning parameters relating to the nonlinear PID gains. In types I and III structures, the output actions are in the associated form. The *Action Association* refers to the singular nature of the output of the three PID actions. In coupled rule bases it is difficult to dissociate the nonlinear tuning parameters with respect to each control action. The basic dissociation that has been done for the simplest LLFLC structure [see (37) and (39)] is an attempt to identify the individual PID actions in dissociated form. A similar approach has been employed in [6] to identify ANG terms of a simplest fuzzy PI controller using different inference methods. This is quite artificial since the algebraic decomposition of nonlinear terms may not show the true representation of the individual fuzzy PID outputs. Furthermore, when the rules are highly nonlinear and memberships are nonuniform, action identification in a dissociated form will become an extremely difficult mathematical task. The nonlinear PID gains become nontransparent for independent nonlinear tuning. The action association is one of major reasons why no satisfactory in-depth analysis has been done in identifying nonlinear tuning parameters in an explicit form for the most common Mamdani-type two-input fuzzy PID controllers.

2) Input Coupling: In the coupled rule bases we again see input coupling in the ANG terms. In the type I controller, all the gains are highly coupled by all three error terms. The advantage of input coupling is the inclusion of generalized damping [47], which gives each nonlinear gain term the effect of error derivatives. The disadvantage is that the proportional and integral actions are unnecessarily complicated by the effect of damping and this results in a more sluggish response. For example, when a process is responding slowly, the coupled action of error rates tends to produce low equivalent gain for the apparent nonlinear proportional action. This can be numerically verified by comparing the maximum proportional ANG values when all the error derivatives are forced to zero. This is one of the reasons why in [7] the conventional (type III) fuzzy PI structure was unable to perform better than an optimally designed linear PI controller.

3) Gain Dependency: This functional behavior can be seen when one fuzzy action is generated by another fuzzy action as in type III–V structures and can be described mathematically by the following analysis.

a) Dependency between coupled PI and PD controllers: The dependency that exist in the type III controller outputs is given by $\hat{u}_{\mathrm{PI}}(n) = \sum_{q=0}^{n} \hat{u}_{\mathrm{PD}}(q)$. Replacing the normalized terms with ANG terms, the gain dependency can be expressed by

$$\begin{aligned}\hat{K}_{Pa}^{\mathrm{PI}}(n)\hat{e}(n) + \hat{K}_{Ia}(n)\sum_{q=0}^{n}\hat{e}(q) \\ = \sum_{q=0}^{n}\Big(\hat{K}_{Pa}^{\mathrm{PD}}(q)\hat{e}(q) + \hat{K}_{Da}(q)\Delta\hat{e}(q)\Big).\end{aligned} \tag{61}$$

b) Dependency between P and I controllers: The dependency that exists in the types IV and V controller outputs is given by, $\hat{u}_I(n) = \sum_{q=0}^{n} \hat{u}_P(q)$. Substituting the normalized terms with ANG terms the gain dependency can be described by

$$\hat{K}_{Ia}(n)\sum_{q=0}^{n}\hat{e}(q) = \sum_{q=0}^{n}\hat{K}_{Pa}\hat{e}(q). \tag{62}$$

By assuming the continuous form for small sampling intervals, the above expression can be further simplified. The gain dependency can be described by the following nonlinear differential equation

$$\hat{K}_{Ia} + \frac{1}{2}\frac{d\hat{K}_{Ia}}{d\hat{e}}\,\hat{e} - \hat{K}_{Pa} = 0. \tag{63}$$

c) Dependency between P and D controllers: The dependency that exist in the type V controller output is given by $\hat{u}_D(n) = \hat{u}_P(n) - \hat{u}_P(n-1)$. With ANG terms this gain dependency in the type V controller can be expressed by

$$\hat{K}_{Da}(n)\Delta\hat{e}(n) = \hat{K}_{Pa}(n)\hat{e}(n) - \hat{K}_{Pa}(n-1)\hat{e}(n-1). \tag{64}$$

Considering small sampling intervals, the above can be described in a continuous form by the following nonlinear differential equation:

$$\hat{K}_{Pa} + \frac{d\hat{K}_{Pa}}{d\hat{e}}\,\hat{e} - \hat{K}_{Da} = 0. \tag{64}$$

The gain dependency has the disadvantage of obtaining optimum nonlinear tuning of individual ANG terms. As an example, in the type V controller, both integral and derivative gains follow the nonlinear proportional action in terms of nonlinear tuning. In case of optimum nonlinear tuning, this requires a compromise for achieving best performance. The conventional type III controller shows a highly complex gain dependency. The independent nonlinear gain control in types II and VI controllers allows the design to achieve the best independent nonlinear tuning in terms of ANG values.

B. Low-Level Tuning

This tuning level is described by the apparent linear PID gains (ALG). The nonlinear tuning has a direct effect on the normalized controller surface or curve, whereas the ALG terms adjust the overall magnifications, similar to a linear PID controller. From a practical point of view, the behavior of the linear gains is expected to be similar to the three gains of a linear PID controller. In coupled structures, these gains provide magnifications for all dimensions in the error state space. Therefore, any increase of a single ALG term also indirectly affects the overall magnification of the other two ANG values as well. In the common two-input coupled structure, the complex nature of the linear proportional gain can be compared with other linear gain terms. Practically, the overall gains are controlled by the tuning variables provided in Table VI. The coupled nature of tuning variables in the linear proportional gain of type III controller makes it difficult for designers to adapt linear PID tuning heuristics. Even the low level tuning heuristics developed in [44] are applicable only for the PI version of a type III controller. The design of low-level gain terms of two-input fuzzy controllers using a sliding mode approach [29] is limited only to PD type controllers. In [35] and [36], this proportional action complexity was avoided by having a separate one-input fuzzy integral controller connected to a two-input coupled fuzzy PD controller. Again, decoupled rule structures or one-input fuzzy PID controllers provide better individual overall tuning, enabling the control engineer to use accumulated PID tuning knowledge to obtain optimum overall tuning of the fuzzy PID controllers.

VII. Summary and Conclusions

This paper describes research to provide control engineers with fundamental information about the design aspects of fuzzy PID controllers and a selection procedure by evaluating the functional behaviors of structures. This systematic analysis has facilitated the identification of different fuzzy PID controller structures, particularly decoupled and one-input type controllers, which have not been commonly used in previous applications. It is known that the curse of dimensionality is a major problem in fuzzy controller design today [45]. In controller designs, the identification of fuzzy controller parameters relating the plant dynamics or performance is particularly challenging. In most cases extensive computer simulations or exhaustive numerical search techniques are used for solving the multidimensional problem. In our work, this high dimensional design was identified as a two-level tuning problem. The choice of any fuzzy PID structure should be done based on the efficiency of these tuning levels while seeking superior performance. Our study also has shown the explicit representation of high-level tuning by ANG terms. For optimal design one has to choose the nonlinear tuning parameters for varying the ANG terms.

The type V controller is the simplest, with the nonlinear tuning accomplished through the fuzzy proportional action. However, the gain dependency in this controller avoids independent tuning of integral and derivative nonlinear gains. The rule decoupled structures and one-input fuzzy structures have the advantage of identifying individual PID actions in terms of their nonlinear tuning parameters. Types II and VI structures offer independent gain control for both of the tuning levels. The type VI controller is more analogous to a linear PID controller, where each control action is nonlinearly related to the error. The system can be made exactly like a linear PID controller by selecting nonlinear tuning parameters to produce a linear function for the proportional signal. As an example, the proper selection of s_1 and s_2 in the one-input nonlinear like fuzzy controller element allows the fuzzy output to be almost linear (curve C in Fig. 14). Therefore proper selection of nonlinear tuning parameters can produce the linear controller as a special case of the fuzzy PID controller. This particular feature makes the fuzzy controller always perform either better than or equal to a linear PID controller and avoids the poorer performance of the fuzzy controllers as experienced in [7]. The scaling factors for the error can be readily computed by knowing its maximum deviation, which is usually available with the response data. With proper choice of nonlinear tuning, the type II controller also can be made with a perfect incremental (velocity) type PID controller. Due to the derivative error inputs, this structure is sensitive to noise [48]. However the error derivatives provide additional information and enhance the generalized damping of the control system [47]. Thus the type II structure may make the controller more robust than the type VI controller.

In this study we have proposed an equivalent linear controller analysis to identify second level or overall tuning terms. The ALG terms derived from the ELC analysis have the same effect as the three PID gains of a linear PID controller. Also we have shown that the final overall tuning task can be simplified to a three term tuning problem. Therefore one can find suitable tuning heuristics for the ALG tuning terms by correlating existing linear PID tuning methods.

All coupled structures have the disadvantage of using a large number of rules compared to decoupled structures. Since the nonlinearity tuning parameters are associated with the rules, the parameter growth also increases with the rule growth. Therefore, rule decoupled structures are quite advantageous in terms of using the least number of nonlinearity tuning parameters, thus enabling one to perform efficient and easy high level tuning for attaining optimum performance.

The design of a fuzzy controller requires the building a knowledge based system with the specific nonlinearity to generate a specific performance of the process response. The variation of tuning parameters is always related to the performance. Therefore, development of a suitable tuning scheme for fuzzy PID controllers requires consideration of the two tuning levels, where one level matches the plant dynamics and the nonlinear behavior and the second level provides the necessary magnifications to PID control actions.

From this study it can be concluded that the Mamdani-type conventional two-input fuzzy PID structure produces an inferior performance in terms of functional behaviors. These drawbacks can be summarized as follows.

1) The coupled rules produce an associated PID action and therefore identifying nonlinear tuning parameters for the nonlinearity (or high-level) tuning is difficult.
2) The complex and coupled nature of both linear and nonlinear gains makes the tuning of fuzzy PID controllers an extremely a difficult task, and therefore its applications are limited to either the PI or PD versions.
3) With linear rules, [see (17)] the nonlinearity obtained by changing membership functions of the consequent fuzzy variables is limited [33]. Therefore any nonlinearity tuning for better control performance requires an exhaustive search of large numbers of rules for obtaining an optimum control surface.

In this paper, we have also described a new analytical solution procedure for the output of a general three-input LLFLC system. The input transformation procedure reduces the number of nonlinear expressions required to represent multi-phase solutions for any LLFLC structure. The LLFLC structure can be used as the basic controller structure to compare the dynamic characteristics of different fuzzy controller structures.

Appendix
Derivation of the Nonlinear Term β_3

For this derivation, assume the incremental input values from the reference modal positions given in Step 3 of the solution procedure shown in Section IV-B satisfy the following condition:

$$\frac{\delta x_{3,k}}{a_3} \le \frac{\delta x_{2,j}}{a_2} \le \frac{\delta x_{1,i}}{a_1} \le 0.5$$

or

$$\frac{\delta x_{3,k}}{a_3} \le \frac{\delta x_{2,j}}{a_2} \le 1 - \frac{\delta x_{1,i}}{a_1} \le 0.5.$$

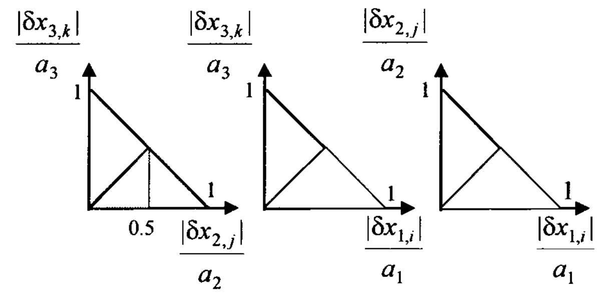

Fig. 15. Relative positions of inputs.

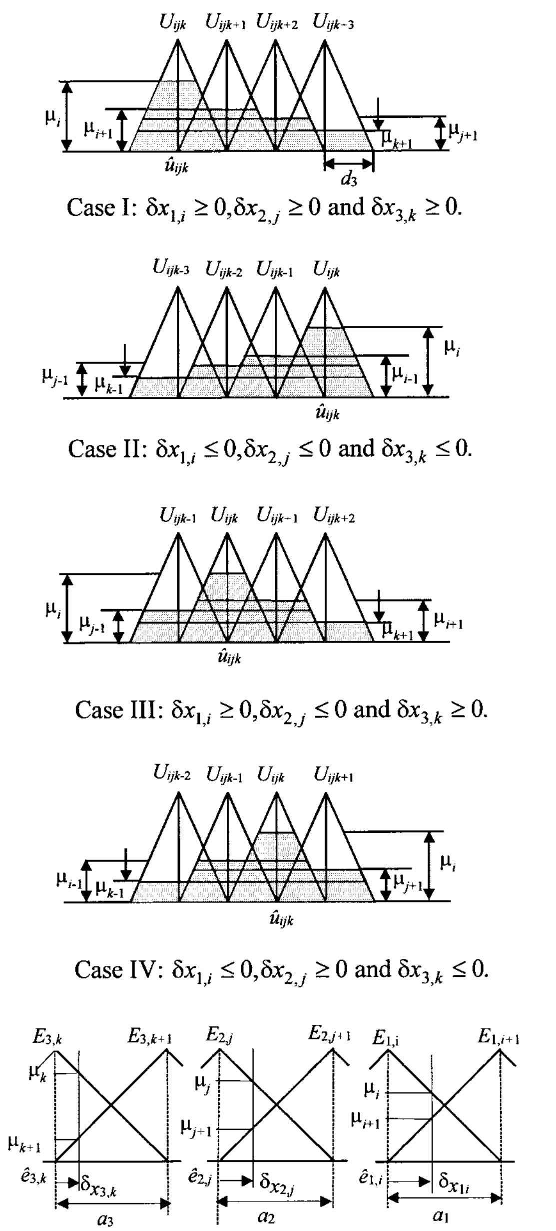

Fig. 16. Fuzzy output shapes corresponding to different input conditions. The incremental inputs are measured from the modal positions. The subscript $ijk \equiv i + j + k$.

The shaded areas in Fig. 15 show these relative input conditions. This particular region is selected to give a simple and concise expression for the nonlinear term β_3. Any other region

TABLE VII
RULE IMPLICATION AND FUZZY OUTPUTS FOR CASE I

Rule	$\hat{e}_1$	$\hat{e}_2$	$\hat{e}_3$	$\hat{u}$	h	$h_{\max}$
R1	$E_{1,i}$	$E_{2,j}$	$E_{3,k}$	U_{ijk}	μ_i	μ_i
R2	$E_{1,i+1}$	$E_{2,j}$	$E_{3,k}$	U_{ijk+1}	μ_{i+1}	
R3	$E_{1,i}$	$E_{2,j+1}$	$E_{3,k}$	U_{ijk+1}	μ_{j+1}	μ_{i+1}
R4	$E_{1,i}$	$E_{2,j}$	$E_{3,k+1}$	U_{ijk+1}	μ_{k+1}	
R5	$E_{1,i+1}$	$E_{2,j+1}$	$E_{3,k}$	U_{ijk+2}	μ_{j+1}	
R6	$E_{1,i+1}$	$E_{2,j}$	$E_{3,k+1}$	U_{ijk+2}	μ_{k+1}	μ_{j+1}
R7	$E_{1,i}$	$E_{2,j+1}$	$E_{3,k+1}$	U_{ijk+2}	μ_{k+1}	
R8	$E_{1,i+1}$	$E_{2,j+1}$	$E_{3,k+1}$	U_{ijk+3}	μ_{k+1}	μ_{k+1}

TABLE VIII
RULE IMPLICATION AND FUZZY OUTPUTS FOR CASE II

Rule	$\hat{e}_1$	$\hat{e}_2$	$\hat{e}_3$	$\hat{u}$	h	$h_{\max}$
R1	$E_{1,i}$	$E_{2,j}$	$E_{3,k}$	U_{ijk}	μ_i	μ_i
R2	$E_{1,i-1}$	$E_{2,j}$	$E_{3,k}$	U_{ijk-1}	μ_{i-1}	
R3	$E_{1,i}$	$E_{2,j-1}$	$E_{3,k}$	U_{ijk-1}	μ_{j-1}	μ_{i-1}
R4	$E_{1,i}$	$E_{2,j}$	$E_{3,k-1}$	U_{ijk-1}	μ_{k-1}	
R5	$E_{1,i-1}$	$E_{2,j-1}$	$E_{3,k}$	U_{ijk-2}	μ_{j-1}	
R6	$E_{1,i-1}$	$E_{2,j}$	$E_{3,k-1}$	U_{ijk-2}	μ_{k-1}	μ_{j-1}
R7	$E_{1,i}$	$E_{2,j-1}$	$E_{3,k-1}$	U_{ijk-2}	μ_{k-1}	
R8	$E_{1,i-1}$	$E_{2,j-1}$	$E_{3,k-1}$	U_{ijk-3}	μ_{k-1}	μ_{k-1}

in the incremental input space is then transformed to this space by the input transformation shown in Step 4 of the solution procedure in Section IV-B. Consider the linear rule base given for the general three-input case by (17) in Section IV-A. For given crisp inputs $\{\hat{e}_1^*, \hat{e}_2^*, \hat{e}_3^*\}^T$ the final control decision U' is determined by applying the Z–M min–max reasoning (compositional rule of inference) as described in [11]. It is given by

$$\mu_{U'}(\hat{u}) = \max_{i,\,j,\,k} \min\left[\mu_{E_{1,\,i}}(\hat{e}_1^*),\, \mu_{E_{2,\,j}}(\hat{e}_2^*),\, \mu_{E_{3,\,k}}(\hat{e}_3^*)\right].$$

The fuzzy inference will fire a maximum of eight fuzzy rules to produce eight nonzero fuzzy outputs (clipped outputs) against any arbitrary three fuzzy singleton input values. The clipped fuzzy output produced by a single rule inference is a trapezoid. After the union of all clipped outputs, the final fuzzy set, U' can have four different shapes with respect to the reference crisp output position $\hat{u}_{i+j+k}$. The reference output is when all crisp inputs are at membership modal positions. The input conditions and the resultant fuzzy outputs corresponding to each case are shown in Fig. 16. For convenience the subscript $i + j + k$ is represented by ijk. The membership functions used for each rule fired and the heights (h) of the trapezoids produced for each rule are shown in Table VII–X. As an example, the rule R1 shown in Table VIII reads as "**If** ($\hat{e}_1$ is $E_{1,\,i}$ and $\hat{e}_2$ is $E_{2,\,j}$ and $\hat{e}_3$ is $E_{3,\,k}$) **then** $\hat{u}$ is U_{i+j+k}." The rules having the same output fuzzy labels are combined by the "max" operation. Thus the maximum height of the trapezoids having the same support sets is described by ($h_{\max}$).

TABLE IX
RULE IMPLICATION AND FUZZY OUTPUTS FOR CASE III

Rule	$\hat{e}_1$	$\hat{e}_2$	$\hat{e}_3$	$\hat{u}$	h	$h_{\max}$
R1	$E_{1,i}$	$E_{2,j-1}$	$E_{3,k}$	U_{ijk-1}	μ_{j-1}	μ_{j-1}
R2	$E_{1,i}$	$E_{2,j}$	$E_{3,k}$	U_{ijk}	μ_i	
R3	$E_{1,i+1}$	$E_{2,j-1}$	$E_{3,k}$	U_{ijk}	μ_{j-1}	μ_i
R4	$E_{1,i}$	$E_{2,j-1}$	$E_{3,k+1}$	U_{ijk}	μ_{k+1}	
R5	$E_{1,i+1}$	$E_{2,j-1}$	$E_{3,k+1}$	U_{ijk+1}	μ_{k+1}	
R6	$E_{1,i}$	$E_{2,j}$	$E_{3,k+1}$	U_{ijk+1}	μ_{k+1}	μ_{i+1}
R7	$E_{1,i+1}$	$E_{2,j}$	$E_{3,k}$	U_{ijk+1}	μ_{i+1}	
R8	$E_{1,i+1}$	$E_{2,j}$	$E_{3,k+1}$	U_{ijk+2}	μ_{k+1}	μ_{k+1}

TABLE X
RULE IMPLICATION AND FUZZY OUTPUTS FOR CASE IV

Rule	$\hat{e}_1$	$\hat{e}_2$	$\hat{e}_3$	$\hat{u}$	h	$h_{\max}$
R1	$E_{1,i-1}$	$E_{2,j}$	$E_{3,k-1}$	U_{ijk-2}	μ_{k-1}	μ_{k-1}
R2	$E_{1,i-1}$	$E_{2,j}$	$E_{3,k}$	U_{ijk-1}	μ_{i-1}	
R3	$E_{1,i-1}$	$E_{2,j+1}$	$E_{3,k-1}$	U_{ijk-1}	μ_{k-1}	μ_{i-1}
R4	$E_{1,i}$	$E_{2,j}$	$E_{3,k-1}$	U_{ijk-1}	μ_{k-1}	
R5	$E_{1,i}$	$E_{2,j}$	$E_{3,k}$	U_{ijk}	μ_i	
R6	$E_{1,i-1}$	$E_{2,j+1}$	$E_{3,k}$	U_{ijk}	μ_{j+1}	μ_i
R7	$E_{1,i}$	$E_{2,j+1}$	$E_{3,k-1}$	U_{ijk}	μ_{k-1}	
R8	$E_{1,i}$	$E_{2,j+1}$	$E_{3,k}$	U_{ijk+1}	μ_{j+1}	μ_{j+1}

TABLE XI
THE NONLINEAR OUTPUT TERM

Case	I	II	III	IV
β_3	α_1	$-\alpha_1$	α_2	$-\alpha_2$

Defuzzification: The COA based defuzzified value can be expressed as [11]

$$\hat{u} = \frac{\displaystyle\int_{u \in U} u\mu_u(u)\, du}{\displaystyle\int_{u \in U} \mu_u(u)\, du}$$

where the membership function U' with its support set is given by $U = \{u|\mu_u(u) > 0\}$. This refers to the center of the shaded areas shown in Fig. 16. From these diagrams the membership heights shown in the $h_{\max}$ columns of Table VI can be expressed as follows:

$$\mu_{r+1} = \mu_{r-1} = \frac{|\delta x_{w,r}|}{a_w}, \qquad \mu_r = 1 - \frac{|\delta x_{w,r}|}{a_w}$$
$$\text{where } r = i, j, k \text{ and } w = 1, 2, 3.$$

Using the membership equations above and applying the COA defuzzification method for each case, we can obtain the nonlinear term β_3 as shown in the Table XI. The values α_1 and α_2 are given by (27) with

$$|m_1| = \frac{|\delta x_{1,i}|}{a_1}$$
$$|m_2| = \frac{|\delta x_{2,j}|}{a_2}$$
$$|m_3| = \frac{|\delta x_{3,k}|}{a_3}.$$

Table II given in Section IV-B shows 8 cases with respect to the sign of the incremental inputs. The extra four cases shown have been transformed to the region shown in Fig. 15 by modifying the modal position and thus changing the direction of the maximum incremental input. This procedure would eliminate the use of an excessive number of formulae for representing different input conditions.

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