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# Optimal sizing of a run-of-river small hydropower plant

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#### Abstract

The sizing of a small hydropower plant of the run-of-river type is very critical for the cost effectiveness of the investment. In the present work, a numerical method is used for the optimal sizing of such a plant that comprises two hydraulic turbines operating in parallel, which can be of different type and size in order to improve its efficiency. The study and analysis of the plant performance is conducted using a newly developed evaluation algorithm that simulates in detail the plant operation during the year and computes its production results and economic indices. A parametric study is performed first in order to quantify the impact of some important construction and operation factors. Next, a stochastic evolutionary algorithm is implemented for the optimization process. The examined optimization problem uses data of a specific site and is solved in the single and two-objective modes, considering, together with economic, some additional objectives, as maximization of the produced energy and the best exploitation of the water stream potential. Analyzing the results of various optimizations runs, it becomes possible to identify the most advantageous design alternatives to realize the project. It was found that the use of two turbines of different size can enhance sufficiently both the energy production of the plant and the economic results of the investment. Finally, the sensitivity of the plant performance to other external parameters can be easily studied with the present method, and some indicative results are given for different financial or hydrologic conditions. © 2007 Elsevier Ltd. All rights reserved.

Keywords: Optimal sizing; Small hydropower plant; Run-of-river; Plant simulation; Cost benefit analysis

## 1. Introduction

Small scale hydropower constitutes a cost effective technology for rural regions in developing countries and, on the other hand, is a still growing sector in Europe where the installed capacity was 8 GW by 2002 and the further potential is estimated at 18 GW [1], while the European Commission aims to achieve a 50% increase in capacity by the year 2010.

Most of the small hydropower (SHP) plants are of the run-of-river type, which is much different in design, appearance and impact from conventional large hydroelectric projects. There is no water storage reservoir except the small head pond capacity and all diverted water returns to the stream below the power house, whereas the environmental impact is minor. The problem of optimum design of a SHP plant is very critical for the cost effectiveness of the investment. The difficulty in sizing the components of the plant and mainly in determining its installed capacity arises from the non-uniformity and seasonal variation of the natural flow rate combined with the lack of an upstream reservoir of important volume.

The optimization of small run-of-river hydropower plants design has been examined in various studies. The first known approach was performed by Gingold [2], who concluded that the optimum size of the turbines is not well defined. This is mainly due to the fact that the efficiency of the turbines is taken to be constant for all the operating points. According to the approach of Fahlbush [3], the optimum size of the turbine is derived by maximisation of the net benefit. He gives an analytical solution after a stochastic formulation of the problem. Da Deppo et al.

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[4] use a mathematical model of cost effectiveness of run-ofriver hydro installations suitable for microcomputer application. The efficiency of the turbine varies depending on the operation point. Bleinc [5] presents the results of a computer software developed to examine the influence of hydrologic data on energy production and on the design and optimum sizing of the installation. A case study is also presented. Papantonis and Andriotis [6] optimized the size and the number of identical turbines in parallel operation for a specific hydraulic site.

More recently, Voros et al. [7] developed an empirical short-cut design method for selection of the nominal flow rate of hydraulic turbines. Their study includes the effect of several site characteristics. Montanari [8] has developed a procedure for the best sizing of the Michell-Banki turbine in order to achieve the best exploitation of hydraulic energy in low head sites. A general method for economic evaluation of a SHP scheme is presented by Liu et al. [9], and some new quantitative criteria are introduced to describe the performance and the efficiency of the turbine unit. Analytical simulation of the energy production and comprehensive cost benefit sensitivity analysis of SHP plants in Greece has been performed by Kaldellis et al. [10]. The capacity factor of the installation, along with the market electricity price and the first installation cost are found that mostly affect the viability of the investment. Hosseini et al. [11] underlined that the most important design parameter of a run-of-river plant is the installed capacity of the turbine, and they optimized this value for a particular case study, performing detailed evaluation of the economic, technical and reliability indices of the plant. A global numerical model for optimization of a SHP system configuration was developed by Lopes de Almeida et al. [12] using non-linear programming. In addition to the detailed technoecoanalysis. the software accounts for nomic the investment risk related to hydrologic conditions and to market variability.

In all the above methods and studies, the SHP unit consists of one or more identical hydraulic turbines. However, the installation of two hydraulic turbines operating in parallel that may differ in size and even in type could increase energy production and improve the economic results of the investment. The numerical study of this configuration is the aim of the present work. Moreover, in the methods used so far, the optimum sizing of the turbine for a specific hydraulic site is performed keeping constant some other parameters of the problem, such as the penstock diameter. In many cases, the latter has a strong impact on the technical (net head) and economic (purchase and installation cost, energy losses, etc.) feasibility of the plant. For this reason, a more cost effective design is examined in which the penstock is constructed in three consecutive sections of different diameter and/or wall thickness.

#### 2. Simulation of the SHP plant operation

The main components of the SHP scheme examined here are shown in Fig. 1, along with the design parameters. The gross head h and the total penstock length L are known for a specific site, therefore, there are nine design variables for optimization: the type and the size (nominal power or flow rate  $Q_1$  and  $Q_2$ ) of the two turbines, the nominal diameters,  $D_1$ ,  $D_2$  and  $D_3$  of the three pipe sections and the length of two of them,  $L_1$  and  $L_2$ .

The net head H of the unit is equal to the gross head h minus the head losses of the penstock that correspond to the flow rate  $Q_T$  conducted to the water turbines. Since the penstock is composed of three sections of different length and diameter, the net head H is computed as:

$$H = h - (\delta h_1 + \delta h_2 + \delta h_3) \tag{1}$$

where  $\delta h_i$  are the head losses of each section (i = 1, 2, 3) calculated by application of the Darcy–Weisbach formulation ( $\lambda$  and  $\zeta$  are the linear and minor loss coefficients, respectively):



Fig. 1. Schematic plant layout and parameters for optimization.

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Fig. 2. Standardised small hydraulic turbines data: (a) selection nomograph; (b) normalized efficiency curves.

$$\delta h_i = \left(\lambda_i \cdot \frac{L_i}{D_i} + \zeta_i\right) \cdot \frac{8Q_T^2}{\pi^2 g D_i^4} \tag{2}$$

The coefficients of linear hydraulic losses can be computed by the well known White–Colebrook formula (Moody diagram) for a specific flow Reynolds number and a given pipe wall roughness. The wall thickness of each section is selected to withstand the maximum expected static pressure in the particular pipe. Only nominal values of pipe diameter and wall thickness are considered using tabulated data that correspond to welded steel pipes. For every combination of the three sections, the total purchase and installation cost of the penstock is calculated applying proper cost correlations.

The most suitable turbine type for a specific set of net head H and nominal flow rate  $Q_r$  can be selected from the nomograph of Fig. 2a, which summarises the production program of the manufacturers of standardized hydraulic turbines with output between 50 and 10000 kW. For each turbine type of the nomograph, a typical efficiency curve is introduced in dimensionless form,  $\eta/\eta_r$ , as a function of the load of the turbine,  $Q/Q_r$ , where the subscript r holds for the nominal (best efficiency) operating point of the turbine. Such typical curves for the small turbines considered here are illustrated in Fig. 2b. The maximum efficiency  $\eta_r$  of a turbine depends on the type and size of the unit, and its value is about 88-91% for all types, except cross flow where it is lower, about 84%. A similar procedure is adopted for selection of the nominal size and estimation of the efficiency of the electrical generator.

The operating range of a selected type is introduced according to the specifications of the turbine's manufacturers. The turbine can operate when the water flow rate is between a minimum  $Q_{min}$  and a maximum  $Q_{max}$  value. The lower limit is about 50% of the nominal flow rate for the Francis type, but can be down to almost 15% for the Pelton turbine. In the simplest plant layout, where only one turbine is installed, the operating schedule is as follows: for  $Q_S < Q_{min}$  the turbine is shut down (mainly due to unsteady operation), while for  $Q_S > Q_{max}$  the flow rate through the turbine is equal to  $Q_{max}$  and hence the difference  $(Q_S - Q_{max})$  cannot be exploited and overflows to the natural bed of the river.

However, in the case of two parallel turbines, which are, in general, of different type and/or size, the operating schedule becomes more complex. When the water stream flow rate falls below the minimum limit of the smaller turbine the turbines are stopped, while when it exceeds the cumulative capacity of the turbines, they operate at their maximum limit. In the mid region, the plant may operate with only one or with both turbines in order to completely exploit the available water flow with the best possible efficiency. With the aid of the example sketched in Fig. 3, the following operation modes can be encountered (assuming that  $Q_{2,min}$  is less than  $Q_{1,min}$ ):

- (1)  $Q_S < Q_{2,min}$ : Both turbines shut down. No energy production.
- (2)  $Q_{1,min} \leq Q_S \leq Q_{1,max}$ : One turbine (T2) in operation.
- (3)  $Q_{2,min} < Q_S < Q_{2,min}$ : One turbine (T2 or T1) in operation, whichever achieves the best efficiency.
- (4)  $Q_{2,max} < Q_S < Q_{1,max}$ : One turbine (T1) in operation.
- (5)  $Q_{1,max} < Q_S < Q_{1,max} + Q_{2,min}$ : Both turbines in operation, optimum  $Q_T$  distribution.
- (6)  $Q_{1,max} + Q_{2,min} < Q_S < Q_{1,max} + Q_{2,max}$ : Both turbines in operation, optimum  $Q_T$  distribution.
- (7)  $Q_{1,max} + Q_{2,max} < Q_S$ : Both turbines in operation at maximum flow rate  $Q_{T,max} < Q_S$ .

The detailed schedule is programmed in the numerical simulation algorithm. The optimum distribution of the total flow rate to the two turbines in modes 5 and 6, as well as the selection of the most efficient turbine in mode 3, is automatically computed.



Fig. 3. Modes of operation of two installed turbines.

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The generated electric power of each operating unit j (water turbine and electrical generator) is obtained from:

$$P_j = \gamma \cdot H \cdot Q_j \cdot \eta_j \quad j = 1,2 \tag{3}$$

where *H* is the net head corresponding to the exploited flow rate *Q*,  $\gamma$  is the specific weight of the water and  $\eta$  is the product of the efficiencies of the hydraulic turbine, the electrical generator and the transformer. The cumulative nominal power of the two turbines must not exceed 10 MW, and this constraint is included in the evaluation algorithm along with the minimum power limit of 50 kW.

The first step for optimization of the SHP plant is the detailed simulation of its operation during a one year period. In order to calculate the annually produced electric energy, a year is divided into 100 equal time periods  $\delta t$ , and the mean flow rate during each such period is taken from the flow duration curve of the incoming stream. The solid line in Fig. 4 represents the flow duration curve extracted from the hydrologic data of a specific water stream (Krathis River, Greece), which is used as a standard case in the present study. The energy generated in period *i* is calculated from the relation:

$$\delta E_i = \gamma \cdot H \cdot (Q_{1,i} \cdot \eta_{1,i} + Q_{2,i} \cdot \eta_{2,i}) \cdot \delta t \tag{4}$$

where  $Q_1$  and  $Q_2$  are the water flow rates through the corresponding turbines, one or both of which may be zero depending on the available stream flow rate. The sum of  $\delta E_i$  for the 100 time periods gives the total energy produced during a year. A sensitivity study showed that the use of 100 divisions gives practically independent results in all cases.

The annual income of the power plant can be computed from the generated energy and the selling tariffs for electric energy. The capital cost of the plant is calculated as the sum of the cost of the various parts and components (civil works, electrical and hydro-mechanical equipment purchase and installation, project design and supervision), which are introduced in parametric form using empirical cost estimation relations [13]. For the complete economic evaluation of the plant, additional financial and fiscal parameters are introduced, such as the annual operation



Fig. 4. Flow duration curve of the water stream.

and maintenance cost, the construction period (of the order of 2 years), subsidization, taxation and electricity price escalation rate, financing cost (interest rate), financing period, taken equal to 20 years, etc. The economic analysis is based on the dynamic evaluation method of Net Present Value (NPV). Additional criteria are also used, such as the internal rate of return (IRR), the benefit to cost ratio (BCR) etc.

The optimization software is based on the evolutionary algorithms approach, and it has been developed and brought to market by the Laboratory of Thermal Turbomachinery, NTUA [14]. The use of stochastic optimization is advantageous for multi-parametric problems like the one examined here, which are described by non-linear and discontinuous cost functions. Moreover, all the free design variables can be handled at the same time and the optimal solution is obtained automatically, whereas most of the optimization methodologies in the literature require performance of parametric or sensitivity studies. The optimizer can be applied in a single or multi-objective mode and has been successfully used by the authors in a recent study [15].

## 3. Results and discussion

#### 3.1. Parametric study

In order to obtain a preliminary picture of the plant performance and economic prospects, the evaluation algorithm is used to study the influence of the absolute and relative size (nominal power) of the two hydraulic turbines, assuming a standard penstock configuration. Both turbines are taken as Francis type, which is a reasonable selection, since for the examined hydraulic site, the gross head is 200 m, and the mean stream flow rate is about 2.1 m<sup>3</sup>/s (see Fig. 2a).

The results of the parametric study are plotted in Fig. 5 as a function of the total nominal flow rate of the two turbines,  $Q_{r,tot} = Q_{1,r} + Q_{2,r}$ , and include three different turbine size combinations, as well as the case when only one turbine is installed. Fig. 5a demonstrates the impact on the energy production index  $E_{f}$ , which is computed as the sum of the generated energy divided by the energy potential of the natural stream at the gross head h during a year's period. The produced energy increases with the turbines size because a larger machine exploits better the higher stream flow rate periods. However, all curves exhibit a maximum above which production decreases, since large turbines cannot operate during a low stream flow period when the flow is below their safe operation limit. The best performance is obtained for  $Q_{r,tot}$  between 3 and 5 m<sup>3</sup>/s and for a turbine size ratio between 0.25 and 0.5. The single turbine unit (dotted line) exhibits much lower and size dependent efficiency because the operating range, determined by the turbine flow rate limits, is more restricted. The maximum attainable energy production with the combination of two hydraulic turbines appears for the present case to be about 25% higher than that with a single turbine.

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Fig. 5. Impact of turbines size on: (a) energy production index; (b) water exploitation index; (c) Net present value; (d) capital cost per installed KW.

Similar is the picture of Fig. 5b for the water exploitation index,  $W_f$ , which is the fraction of the stream flow that passes through the operating turbines. The maximum value approaches 95% and is clearly obtained by a size ratio of 0.5 and at about the same flow rate region as previously. On the other hand, the corresponding index of the single turbine plant does not exceed the value of 75% that constitutes the lowest approval limit for a SHP investment in Greece.

Concerning the economic indices, the *NPV* curves also show a maximum but now at smaller  $Q_{r,tot}$ , about 3 m<sup>3</sup>/sec for all three size ratios (Fig. 5c). Also, the unit cost per installed kW,  $C_N$ , decreases as expected with the size of the turbines, and it does not seem to depend on the specific size ratio (Fig. 5d). The single turbine exhibits lower corresponding  $C_N$  values, but this cannot compensate for its poor performance, which is quantified by its considerably lower *NPV* (Fig. 5c).

#### 3.2. Optimization

In the first, single-objective optimization study, the target is to find the proper plant configuration that minimizes or maximizes the value of some operation or economic parameters. A number of 5000 evaluations (simulations of 1 year plant operation) were found adequate for the optimizer to converge to the minimum cost function value. The results are summarized in Table 1 and include the maximization of the previously introduced efficiency indices  $E_f$ and  $W_f$ , the economic indices NPV and BCR and an additional load coefficient  $L_f$  that gives the ratio of the mean (annually) produced power to the installed nominal power, similar to the capacity factor used in Ref. [10]. The maximum value of the latter can be more than unity, but this happens only when the turbines are so small that they always operate close to their maximum limit, thus giving a non-feasible solution, as shown in the first row of Table 1.

The objectives of maximum  $E_f$  or  $W_f$  result in a similar plant design, namely two Francis turbines with size ratio 0.4 and total nominal flow rate and power of the order of 5 m<sup>3</sup>/s and 8.5 MW, respectively. Similar also are the corresponding results for the two economic indices, *NPV* and *BCR*. This configuration achieves the maximum energy production, about 80% of the water stream potential (or 28,300 MWh). The remaining 20% is lost due to hydraulic and other losses and in the 5% of water that bypasses the machines when the stream flow rate is out of their operation range ( $W_f = 95\%$ ). However, as the low load coefficient value indicates, the installed power is too high, and this increases the investment capital cost, resulting in a longer pay back period of almost 9 years.

This problem is solved when the objective is maximization of the *NPV*. The resulting system in the 4th row of Table 1 comprises again two Francis turbines but at the much reduced size of 5 MW in total. The nominal penstock diameter decreases from DN 1200 in the first pipe section to DN 1000 in the third section in order to use smaller wall thickness. This configuration reduces considerably the capital cost and gives the highest *NPV* of almost 8 M€, while

Table 1	
Results of the single-objective optimization	study

	$L_{f}$	$E_{f}$	$W_{f}$	NPV (M€)	BCR	Turbine types	Size ratio	$Q_{r,tot}$ (m <sup>3</sup> /s)	$P_{r,tot}$ (kW)	
$L_f$	1.16	0.24	0.29	-1.04	0.82	P–P	0.47	0.5	860	
$\vec{E_f}$	0.39	0.80	0.95	5.81	1.59	F–F	0.405	4.8	8460	
$W_f$	0.37	0.78	0.95	5.56	1.58	F–F	0.4	5.4	8720	
NPV	0.61	0.74	0.91	7.94	2.20	F–F	0.504	3.0	5040	
BCR	0.73	0.68	0.84	7.48	2.31	F–F	0.78	2.3	3830	

the pay back period turns down to 5.7 years. However, this improvement is achieved at the expense of the produced energy, which is now reduced to about 26,900 MWh ( $E_f = 0.74$ ). The water exploitation index also decreases 4% units but remains well above 75%.

The last row of Table 1 contains the results after maximizing the BCR value. The latter becomes slightly higher compared to the previous row (max. NPV), however, the installed capacity becomes even lower, reducing further the amount of the produced energy  $(E_f)$  and also the NPV of the investment. These results are almost identical with the ones obtained by maximizing the IRR index. The BCR value depends on the size of the unit and cannot be a credible economic index. However, the BCR variation curve as a function of plant size and cost can provide a reliable estimation of the most efficient investment: the most profitable size is in the region where the slope of this curve becomes 45° [13]. The BCR curve is calculated for the present case, and it was found that the installed power must be in the range of 4600-5500 kW, which includes the optimal value of 5040 kW found previously for the maximum NPV.

The two-objective optimization mode is also used to study the interdependence of some of the objectives. This mode implies two different and usually competitive objectives, and hence, the algorithm does not lead to a single optimal solution but produces an array of points that correspond to optimal combinations of the objective values, forming a Pareto Front [14]. The first example combines an economic objective, the maximum NPV, with an operation target, maximization of the load coefficient  $L_f$ . As shown in Fig. 6a, the points of the computed Pareto Front represent the best combination of the  $NPV-L_f$  values or the maximum possible corresponding NPV for each  $L_f$  value. As expected, the two objectives are competitive: an increase of the NPV implies an increase of the installed power and, hence, a smaller load coefficient value, and vice versa.

Information about the corresponding values of all the other design parameters can be easily obtained from such diagrams. For example, in Fig. 6a, the dotted line shows the corresponding variation of the  $W_f$  index along the points of the Pareto Front. It can be observed that the water exploitation limit of 75% divides the diagram into two parts and clearly separates the feasible region (right part). Also, Fig. 6b depicts the regions of the diagram where different turbine types combinations should be used. It can be seen that two Francis turbines (F–F) produce the best NPV values, whereas two Pelton wheels can give the highest  $L_f$  values but without fulfilling the  $W_f$  constraint.

In a second two-objective optimization the NPV is combined with the principal operation objective, the energy production index  $E_f$ , the maximization of which can also be seen as an environmental target. The Pareto Front in Fig. 7 shows that the range of  $E_f$  is rather small compared to the NPV range, indicating that the latter must be the most decisive criterion for plant sizing. However, the asymptotic form of the Pareto Front reveals a very small increase of NPVnear its right edge, whereas the corresponding reduction of the installed power and of both the  $E_f$  and  $W_f$  indices is significant (Fig. 7a and b). Consequently, instead of the maximum NPV, an alternative selection could be a point



Fig. 6. Two-objective (max. NPV-max. Lf) optimization. Computed Pareto Front, and: (a) Water exploitation index; (b) Turbine types combination.

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Fig. 7. Two-objective (max. NPV-max. E<sub>f</sub>) optimization. Computed Pareto Front, and: (a) Total installed power; (b) water exploitation index.

near the maximum NPV that can give better operation results, as the points marked in Fig. 7a and b.

#### 3.3. Sensitivity study

The sensitivity of the plant operation and economic prospects to some critical parameters is examined. Concerning, at first, the purely economic parameters and the plant configuration found previously that exhibits the maximum NPV of the investment (Table 1), it was found that a 10% increase in the annual discount rate causes about the same reduction in the NPV. Similarly, when the construction cost becomes 10% higher, the NPV decreases by about 8%. More important appears to be the electricity selling price that determines the plant benefits. When this price reduces by 10%, the NPV drop becomes almost double, about 19%.

The hydrologic conditions, on the other hand, can have a significant impact on both the operation and the economic behavior of the plant. In order to study such effects, it was assumed that either the cumulative stream flow during a year is different or the duration curve profile becomes more or less uniform, as shown in Fig. 4. In the first case, the flow is varied between 50% and 200% of the standard case, assuming a similar duration profile. The performance of the plant having the maximum *NPV* (Table 1), as well as



Fig. 8. Effects of mean stream flow variation.

of an alternative design discussed in the previous chapter 3.2 (Fig. 7a), are compared in Fig. 8. The alternative plant has higher nominal power and flow rate, 6300 kW and  $3.65 \text{ m}^3$ /s, respectively. In Fig. 8, it is clear that this design becomes superior in both generated energy and *NPV* when a hydrologic year is richer than the standard, whereas the differences would be small during dryer hydrologic years. However, the construction cost is about 14% greater (about 7.2 M€ compared to 6.3 M€ of the maximum *NPV* scheme).

Finally, the impact of potential changes in the flow duration curve pattern is examined using a more uniform or a sharper profile than the standard one (Fig. 4) that represents a more or less distributed stream flow rate during the financing period. As expected, the smoother profile results in an increase of the annually produced energy E, while the opposite is valid for the sharper profile. The calculations showed that for the max. *NPV* plant, the corresponding variations in the produced energy are +8% and -19% of the standard case. The alternative design is again more flexible, and the plant production more stable: the energy variations are +5% and -14% for the smooth and for the sharp profile, respectively.

# 4. Conclusions

The use of two water turbines of different size and/or type in a small hydropower plant is numerically investigated by simulating in detail the plant operation and performance, and with the use of a powerful optimization software. It was found that the net present value, along with the annual energy production constitute the two principal objectives, the maximization of which can lead to the most advantageous design alternatives for this scheme.

The optimal sizing in terms of economic benefits of the investment does not coincide with the one that maximizes exploitation of the hydraulic potential. However, based on the results of a two-objective optimization study, the main design parameters can be selected in order to produce a compromise solution that combines a high *NPV* with enhanced energy production. This unit has somewhat higher

construction cost but appears to be less sensitive to hydrologic conditions, thus reducing the associated project risks.

The size ratio of the turbines that produces the best results is always in the range of 0.4–0.5, verifying that the use of different size turbines improves the capability of the plant to respond more efficiently to seasonal flow rate variations.

## References

- Paish O. Small hydro power: technology and current status. Renew Sust Energy Rev 2002;6:537–56.
- [2] Gingold PR. The optimum size of small run-of-river plants. Water Power Dam Construct 1981;33(11):35–9.
- [3] Fahlbush F. Optimum capacity of a run-of-river plant. Water Power Dam Construct 1983;35(3):45–8.
- [4] Da Deppo L, Datei C, Fioretto V, Rinaldo A. Capacity and type of units for small run-of-river plants. Water Power Dam Construct 1984;36(10):33–8.
- [5] Bleinc C. Utilisation de micro-ordinateur pour le dimensionnement des mini-centrales hydroelectriques. Societe Hydrotechnique de France, XXe Journees de l' Hydraulique, IV.3.1., Lyon 1989.
- [6] Papantonis DE, Andriotis G. Optimization of the size and number of turbines for a small hydropower plant. In: Proceedings, Hidroenergia 93, vol. III. Munchen, 1993. p. 59–68.

- [7] Voros NG, Kiranoudis CT, Maroulis ZB. Short-cut design of small hydroelectric plants. Renew Energy 2000;19:545–63.
- [8] Montanari R. Criteria for the economic planning of a low power hydroelectric plant. Renew Energy 2003;28:2129–45.
- [9] Liu Y, Ye L, Benoit I, Liu X, Cheng Y, Morel G, et al. Economic performance evaluation method for hydroelectric generating units. Energy Convers Manage 2003;44:797–808.
- [10] Kaldellis JK, Vlachou DS, Korbakis G. Techno-economic evaluation of small hydro power plants in Greece: a complete sensitivity analysis. Energy Policy 2005;33:1969–85.
- [11] Hosseini SMH, Forouzbakhsh F, Rahimpoor M. Determination of the optimal installation capacity of small hydro-power plants through the use of technical, economic and reliability indices. Energy Policy 2005;33:1948–56.
- [12] Lopes de Almeida JPPG, Lejeune AGH, Sa Marques JAA, Cunha MC. OPAH a model for optimal design of multi-purpose small hydropower plants. Adv Eng Softw 2006;37:236–47.
- [13] Papantonis D. Small hydro power stations. 2nd ed. Athens: Simeon; 2002.
- [14] Giannakoglou KC. Design of optimal aerodynamic shapes using stochastic optimization methods and computational intelligence. Prog Aerosp Sci 2002;38:43–76.
- [15] Anagnostopoulos J, Papantonis DE. Optimum sizing of a pumpedstorage plant for the recovery of power rejected by wind farms. In: ERCOFTAC design optimization international conference, N.T.U.A., Athens, 2004.