Stabilization of Nonlinear Systems Under Variable Sampling: A Fuzzy Control Approach

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***Abstract*—This paper investigates the problem of stabilization for a Takagi–Sugeno (T–S) fuzzy system with nonuniform uncertain sampling. The sampling is not required to be periodic, and the only assumption is that the distance between any two consecutive sampling instants is less than a given bound. By using the input delay approach, the T–S fuzzy system with variable uncertain sampling is transformed into a continuous-time T–S fuzzy system with a delay in the state. Though the resulting closed-loop state-delayed T–S fuzzy system takes a standard form, the existing results on delay T–S fuzzy systems cannot be used for our purpose due to their restrictive assumptions on the derivative of state delay. A new condition guaranteeing asymptotic stability of the closed-loop sampled-data system is derived by a Lyapunov approach plus the free weighting matrix technique. Based on this stability condition, two procedures for designing state-feedback control laws are given: one casts the controller design into a convex optimization by introducing some overdesign and the other utilizes the cone complementarity linearization idea to cast the controller design into a sequential minimization problem subject to linear matrix inequality constraints, which can be readily solved using standard numerical software. An illustrative example is provided to show the applicability and effectiveness of the proposed controller design methodology.**

***Index Terms*—Input delay, linear matrix inequality, nonlinear systems, sampled-data control, Takagi–Sugeno fuzzy systems.**

I. INTRODUCTION

CONTROL of nonlinear systems is a difficult problem because no systematic mathematical tools are available to help find necessary and sufficient conditions for guaranteeing their stability and performance. By using a Takagi–Sugeno (T–S) fuzzy plant model, we can express a nonlinear system as a weighted sum of some simple linear subsystems [5], [9], [14], [23], [27], [35]–[39]. This model provides a fixed structure to some nonlinear systems and greatly facilitates the analysis and synthesis of the systems under consideration. Therefore, the last decade witnessed a rapidly growing interest in T–S fuzzy systems, and many important results have been reported. Among these references, to mention a few, stability analyses are investigated in [23], [38], and [39], stabilizing and H_∞ control strategies are proposed in [5], [6], [9], [25], [27], [46], and [47], H_∞ filter designs are reported in [44], and reliable control strategies are presented in [42] and [43]. These results are concerned with many classes of T–S fuzzy systems, including T–S fuzzy systems with parameter uncertainties [25], T–S fuzzy systems with state delays [8], [45], T–S fuzzy systems with actuator saturation [7], T–S fuzzy systems with singular perturbations [28], [29], T–S stochastic fuzzy systems [40], T–S sampled-data fuzzy systems [22], [34], and so on.

On the other hand, in practical and modern control systems, computers are usually used as digital controllers to control continuous-time systems [11]. In such a system, a digital computer is used to sample and quantize a continuous-time measurement signal to produce a discrete-time signal, and then produce a discrete-time control input signal, which is further converted back into a continuous-time control input signal using a zero-order hold. Such control systems involve both continuous-time and discrete-time signals in the continuous-time framework and are referred to as sampled-data systems. Analysis and synthesis of sampled-data systems have been investigated in a number of papers (see, for instance, [2], [10], [12], [15], [24], [32], [33], [41] and the references therein). Among these references, two main approaches have been used. The first one is based on the lifting technique, in which the system under consideration is transformed into an equivalent finite-dimensional discrete system [2]. Recently, Lall and Dullerud gave a linear matrix inequality (LMI) solution to sampled-data output-feedback H_∞ control by using the lifting technique when the sampling and the hold operators are periodic and their rates are commeasurable [24]. Lifting-based solutions are usually computationally complicated, as they include the evaluation of the matrices of lifted systems. The second one is more direct, and is based on the representation of the system in the form of hybrid discrete/continuous models. The solution is obtained in terms of differential Riccati equations with jumps. Recently, Hu *et al.* applied the hybrid system approach to robust sampled-data H_2 control for the case of uniform sampling [21]. To overcome difficulties of solving differential inequalities with jumps, a piecewise linear in time Lyapunov function has been suggested, and LMI solutions have been obtained that do not depend on the sampling interval, and thus are quite conservative. Besides these two main approaches, the continuous-time systems with digital control also can be modelled as continuous-time systems with delayed control inputs, which were introduced in [1] and [30]. In this approach, the digital control law is represented as delayed control between two sampling instants. This approach has been further developed to robust H_∞ sampled-data control of linear

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systems. The most significant advantage of this input delay approach over the other two main approaches is that it does not require the sampling distances to be constant. In other words, this approach can be applied to systems with nonuniform uncertain sampling.

In this paper, we aim at solving the problem of sampled-data stabilization for T–S fuzzy systems with nonuniform uncertain sampling. Sampled-data control of T–S fuzzy systems has been investigated in a few papers, mainly by the aforementioned hybrid discrete/continuous approach (see, for instance, [22] and [31]). It is worth noting that these pieces of work are based on the assumption that sampling is made periodic, and the existing results are generally difficult to be extended to systems with variable sampling. The uncertain sampling may happen when the sampler contains uncertainties or the mathematical model we use is not ideally consistent with the sampling equipment. To the best of the authors' knowledge, so far no attempt has been made towards solving the problem of stabilization for T–S fuzzy systems with nonuniform uncertain sampling. This problem still remains challenging, which motivates the present study.

In this paper, the input delay approach is adopted to solve the problem of state-feedback stabilization for T–S fuzzy systems with nonuniform uncertain sampling. More specifically, we do not require the sampling to be periodic; the only assumption is that the distance between any two consecutive sampling instants is less than a given bound. By using the input delay approach, the T–S fuzzy system with variable uncertain sampling is transformed into a continuous-time T–S fuzzy system with a delay in the state. Though the resulting closed-loop state-delayed T–S fuzzy system takes a standard form, the existing results on delay T–S fuzzy systems cannot be used for our purpose due to their restrictive assumptions on the derivative of state delay. A new condition guaranteeing asymptotic stability of the closed-loop sampled-data system is derived by the Lyapunov approach plus the free weighting matrix technique recently developed by He *et al.* [19], [20]. Based on this stability condition, two procedures for designing state-feedback control laws are given: one casts the controller design into a convex optimization by introducing some overdesign and the other utilizes the cone complementarity linearization (CCL) idea [13] to cast the controller design into a sequential minimization problem subject to LMI constraints, which can be readily solved using standard numerical software [16].

The remainder of this paper is organized as follows. The problem to be solved is formulated mathematically in Section II. Main results, including stability analysis and controller design, are presented in Section III. Section IV gives an illustrative example. We conclude this paper in Section V.

The notation used throughout this paper is fairly standard. The superscript "T" stands for matrix transposition; $\mathbb{R}^n$ denotes the n -dimensional Euclidean space; and the notation $P > 0$ means that P is real symmetric and positive definite. In symmetric block matrices or long matrix expressions, we use an asterisk $(*)$ to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

II. Problem Formulation

Consider the nonlinear system

$$\dot{x}(t) = f(x(t), u(t)) \tag{1}$$

where $f(\cdot)$ is a nonlinear function, $x(t) \in \mathbb{R}^n$ is the state vector, and $u(t) \in \mathbb{R}^m$ is the control input. For state-feedback sampled-data stabilization, only discrete measurements of $x(t)$ can be used for control purpose, that is, we only have measurements $x(t_k)$ at the sampling instant t_k with

$$0 = t_0 < t_1 < \cdots < t_k < \cdots < \lim_{k\to\infty} t_k = \infty. \tag{2}$$

For nonlinear systems, it is now well known that a good approximation is provided by the so-called T–S fuzzy modeling. This model is based on the suitable choice of a set of linear subsystems, according to rules associated with some physical knowledge and some linguistic characterization of the properties of the system. These linear subsystems properly describe, at least locally, the behavior of the nonlinear system for a predefined region of the state space. The T–S model for the nonlinear system in (1) is given by the following [37].

♦ *Plant Rule i:* IF $\theta_1(t)$ is μ_{i1} and $\theta_2(t)$ is μ_{i2} and $\cdots$ and $\theta_p(t)$ is μ_{ip}, THEN

$$\dot{x}(t) = A_i x(t) + B_i u(t), \quad i = 1, \ldots, r \tag{3}$$

where $\mu_{i1}, \ldots, \mu_{ip}$ are fuzzy sets, A_i, B_i are constant matrices of compatible dimensions, r is the number of IF-THEN rules, and $\theta(t) = [\theta_1(t),\ \theta_2(t),\ \ldots,\ \theta_p(t)]$ is the premise variable vector. Throughout this paper, it is assumed that the premise variables do not depend on the input variable $u(t)$ explicitly. Given a pair of $(x(t), u(t))$, the final output of the fuzzy system is inferred as

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\theta(t))[A_i x(t) + B_i u(t)] \tag{4}$$

where

$$h_i(\theta(t)) = \omega_i(\theta(t)) \Big/ \sum_{i=1}^{r} \omega_i(\theta(t))$$
$$\omega_i(\theta(t)) = \prod_{j=1}^{p} \mu_{ij}(\theta_j(t))$$

with $\mu_{ij}(\theta_j(t))$ representing the grade of membership of $\theta_j(t)$ in μ_{ij}. Then, it can be seen that

$$\omega_i(\theta(t)) \ge 0, \quad i = 1, 2, \ldots, r$$
$$\sum_{i=1}^{r} \omega_i(\theta(k)) > 0$$

for all t. Therefore, for all t, we have

$$h_i(\theta(t)) \ge 0, \quad i = 1, 2, \ldots, r$$
$$\sum_{i=1}^{r} h_i(\theta(t)) = 1.$$

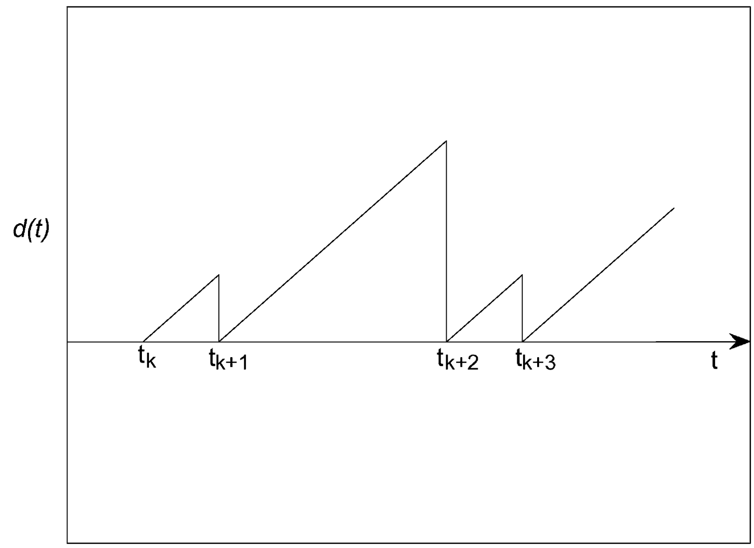

Fig. 1. Illustration of time delay $d(t)$.

The purpose of this paper is to design a controller, based on the parallel distributed compensation (PDC) technique, such that the resultant closed-loop system is asymptotically stable under the nonuniform sampling (2). For the fuzzy model represented by (3) or (4), the fuzzy PDC controller shares the same IF parts with the following structure.

♦ *Controller Form:* **Rule** i: IF $\theta_1(t)$ is μ_{i1} and $\theta_2(t)$ is μ_{i2} and $\cdots$ and $\theta_p(t)$ is μ_{ip}, THEN

$$u(t) = K_i x(t_k), \quad t_k \leq t < t_{k+1}, \quad i = 1, \ldots, r. \tag{5}$$

Thus, the controller in (5) can be represented by the following input–output form:

$$u(t) = \sum_{i=1}^{r} h_i(\theta(t)) K_i x(t_k), \quad t_k \leq t < t_{k+1}. \tag{6}$$

For the above controller, we have assumed that the premise variables can be continuously measured online, and a zero-order hold is placed after each subcontroller. Throughout this paper, we make the following assumption.

Assumption 1: It is assumed that the distance between any two sampling instants is bounded by c $(c > 0)$. That is

$$t_{k+1} - t_k \leq c, \quad \forall k \geq 0. \tag{7}$$

Remark 1: The sampled-data control for the nonlinear system in (1) through a fuzzy system approach has been investigated in [22] and [34]. However, all these papers consider the case with a constant sampling distance g, that is, for any sampling instant t_k, they assume $t_{k+1} - t_k = g$. However, the problem of sampled-data control with nonuniform sampling, which has many applications such as networked control systems, is more challenging, especially for nonlinear systems. It is worth noting that the existing results obtained for periodic sampling case (such as those obtained in [22], [34]) cannot be directly generalized to variable sampling case, which constitutes the main motivation of this paper.

III. Main Results

A. Basic Idea

In this section, we will present our main results for the sampled-data stabilization problem described above through an input delay approach. The key idea behind this approach is that we represent the sampling instant t_k as

$$t_k = t - (t - t_k) = t - d(t) \tag{8}$$

where $d(t) = t - t_k$. By connecting the system in (4) and the controller in (6), we have the closed-loop system as follows.

♦ *Closed-Loop System:*

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t)) h_j(\theta(t)) [A_i x(t) + B_i K_j x(t_k)], \quad t_k \leq t < t_{k+1}. \tag{9}$$

By noticing (8), the closed-loop system in (9) can be rewritten as

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t)) h_j(\theta(t)) [A_i x(t) + B_i K_j x(t - d(t))]. \tag{10}$$

Now, we have transformed the sampled-data closed-loop system in (9) into a continuous-time system with a time-varying delay $d(t)$ in the state. In the following, we will investigate how to design a stabilizing sampled-data controller based on the transformed closed-loop system in (10).

Remark 2: From (7) and (8), it is easy to see that $0 \leq d(t) < c$ and $\dot{d}(t) = 1$, $\forall t_k < t < t_{k+1}$, for all $k > 0$. An illustration of the time delay $d(t)$ is given in Fig. 1. Therefore, the sampled-data fuzzy system in (9) can be seen as a particular class of the state-delayed fuzzy system in (10). The asymptotic stability of system (9) will be guaranteed if (10) is asymptotically stable.

Remark 3: Though the fuzzy sampled-data closed-loop system has been transformed into a continuous-time fuzzy system with a time-varying delay in the state, it is worth pointing out that the existing results for fuzzy time-delay systems cannot be used for our purpose. To the best of the authors' knowledge, the results obtained for fuzzy delay systems can be generally divided into two categories: delay-independent and delay-dependent results. Easily understandable, the delay-independent results cannot be used for system (10) due to their ignorance of the delay size. Moreover, the existing delay-dependent stability results also cannot be applied to system (10) because most of them are based on the assumption that $\dot{d}(t) \leq \tau < 1$ or assume the delay to be constant. Note here in our problem the derivative of time delay $d(t)$ at the sampling instant does not exist (please refer to Fig. 1). In what follows, we will develop a new procedure for stability analysis and controller synthesis.

B. Stability Analysis

In this section, we are concerned with the stability analysis of the closed-loop system. More specifically, assuming that all the subcontroller gains K_i, $i = 1, \ldots, r$, are known, we shall study the conditions under which the closed-loop system in (10) is asymptotically stable. The following theorem shows that asymptotic stability of the closed-loop sampled-data system can be guaranteed if there exist some matrices satisfying certain LMIs. This theorem will play an instrumental role in the controller design.

Theorem 1: Consider the fuzzy system in (4) with Assumption 1, and suppose the gain matrices K_i, $i = 1, \ldots, r$, of the subsystem controllers (6) are given. The closed-loop sampled-data system in (9) is asymptotically stable if there exist matrices $P > 0$, $M > 0$, X_{ij}, and Y_{ij} satisfying

$$\begin{bmatrix} \Psi_1 & \Psi_2 & -X_{ij} - X_{ji} \\ * & \Psi_3 & -Y_{ij} - Y_{ji} \\ * & * & -2c^{-1}M \end{bmatrix} < 0, \quad 1 \leq i \leq j \leq r \tag{11}$$

where

$$\begin{aligned} \Psi_1 \triangleq\; & PA_i + A_i^T P + cA_i^T \\ & \times MA_i + X_{ij} + X_{ij}^T + PA_j \\ & + A_j^T P + cA_j^T MA_j + X_{ji} + X_{ji}^T \\ \Psi_2 \triangleq\; & PB_iK_j + cA_i^T \\ & \times MB_iK_j - X_{ij} + Y_{ij}^T + PB_jK_i \\ & + cA_j^T MB_jK_i - X_{ji} + Y_{ji}^T, \\ \Psi_3 \triangleq\; & cK_j^T B_i^T MB_iK_j - Y_{ij}^T - Y_{ij} \\ & + cK_i^T B_j^T MB_jK_i - Y_{ji}^T - Y_{ji}. \end{aligned}$$

Proof: Define the following Lyapunov–Krasovskii functional:

$$V(x_t) = x^T(t)Px(t) + \int_{-c}^{0}\int_{\beta}^{0} \dot{x}^T(t+\alpha)M\dot{x}(t+\alpha)d\alpha d\beta \tag{12}$$

where

$$x_t = x(t+\theta), \quad -2c \leq \theta \leq 0$$

and $P > 0$, $M > 0$ are matrices to be determined. Then, along the solution of system (10), the time derivative of $V(x_t)$ is given by (13), where $\Xi(t,\alpha)$ is given in (14)

$$\begin{aligned} \dot{V}(x_t) =\; & \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) \\ & + c\dot{x}^T(t)M\dot{x}(t) - \int_{t-c}^{t} \dot{x}^T(\alpha)M\dot{x}(\alpha)\,d\alpha \\ \leq\; & 2x^T(t)P\left\{\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t))[A_ix(t) \right. \\ & \left. + B_iK_jx(t-d(t))]\right\} + c\dot{x}^T(t)M\dot{x}(t) \\ & - \int_{t-d(t)}^{t} \dot{x}^T(\alpha)M\dot{x}(\alpha)\,d\alpha \\ =\; & \frac{1}{d(t)}\int_{t-d(t)}^{t} \Xi(t,\alpha)\, d\alpha, \end{aligned} \tag{13}$$

$$\begin{aligned} \Xi(t,\alpha) \triangleq\; & 2x^T(t)P\left\{\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t))[A_ix(t) \right. \\ & \left. + B_iK_jx(t-d(t))]\right\} \\ & + c\dot{x}^T(t)M\dot{x}(t) - d(t)\dot{x}^T(\alpha)M\dot{x}(\alpha). \end{aligned} \tag{14}$$

In addition, we have (15) as shown at the bottom of the next page. By the Newton–Leibniz formula, we have

$$\int_{t-d(t)}^{t} \dot{x}(\alpha)d\alpha = x(t) - x(t-d(t)). \tag{16}$$

Then, for any matrices X_{ij} and Y_{ij}, we have (17) and thus (18)

$$\begin{aligned} \Lambda \triangleq\; & \frac{1}{d(t)}\int_{t-d(t)}^{t} [x^T(t)\;\; x^T(t-d(t))] \\ & \times \begin{bmatrix} X_{ij} \\ Y_{ij} \end{bmatrix} [x(t) - x(t-d(t)) - d(t)\dot{x}(\alpha)]d\alpha = 0 \end{aligned} \tag{17}$$

$$\begin{aligned} \bar{\Lambda} \triangleq\; & \frac{1}{d(t)}\int_{t-d(t)}^{t} \left\{\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t)) \right. \\ & \times [x^T(t) \quad x^T(t-d(t))] \\ & \left. \times \begin{bmatrix} X_{ij} \\ Y_{ij} \end{bmatrix} [x(t) - x(t-d(t)) - d(t)\dot{x}(\alpha)]\right\} d\alpha = 0. \end{aligned} \tag{18}$$

Adding $2\bar{\Lambda}$ in (18) to (13) yields (19) as shown at the bottom of the page. On the other hand, by noticing $d(t) < c$, from (11) it is not difficult to get

$$\begin{bmatrix} \Psi_1 & \Psi_2 & -d(t)X_{ij} - d(t)X_{ji} \\ * & \Psi_3 & -d(t)Y_{ij} - d(t)Y_{ji} \\ * & * & -2d(t)M \end{bmatrix} < 0, \quad 1 \le i \le j \le r \tag{20}$$

which implies

$$\Phi_{ij} + \Phi_{ji} < 0, \quad 1 \le i \le j \le r \tag{21}$$

and thus

$$\Phi_{ii} < 0, \quad i = 1, 2, \ldots, r \tag{22}$$

$$\Phi_{ij} + \Phi_{ij} < 0, \quad 1 \le i < j \le r. \tag{23}$$

Then, for any i, it follows from (22) that there exists a scalar ρ such that

$$\Phi_{ii} + \mathrm{diag}\{\rho I, 0, 0\} < 0. \tag{24}$$

Then, from (19)–(24), we can obtain

$$\dot{V}(x_t) \le -\rho |x(t)|^2.$$

This implies the asymptotic stability of the closed-loop system in (9) and (10), and the proof is completed. □

Remark 4: From the above proof, we can see that no model transformation is performed in order to obtain the delay-dependent stability condition. It should be noted that in deriving delay-dependent stability and performance conditions, a common approach is to transform the original

$$\begin{aligned} c\dot{x}^T(t)M\dot{x}(t) &= c\left\{\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t))[A_i x(t) + B_i K_j x(t-d(t))]\right\}^T M \\ &\quad \times \left\{\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t))[A_i x(t) + B_i K_j x(t-d(t))]\right\} \\ &= c\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r} h_i(\theta(t))h_j(\theta(t))h_m(\theta(t))h_n(\theta(t))[A_i x(t) \\ &\quad + B_i K_j x(t-d(t))]^T M[A_m x(t) + B_m K_n x(t-d(t))] \\ &= c\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r} h_i(\theta(t))h_j(\theta(t))h_m(\theta(t))h_n(\theta(t)) \\ &\quad \times \frac{1}{2}\left\{\begin{array}{c} [A_i x(t) + B_i K_j x(t-d(t))]^T M^{1/2} M^{1/2}[A_m x(t) + B_m K_n x(t-d(t))] + \\ [A_i x(t) + B_i K_j x(t-d(t))]^T M^{1/2} M^{1/2}[A_m x(t) + B_m K_n x(t-d(t))] \end{array}\right\} \\ &\le c\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t))[A_i x(t) + B_i K_j x(t-d(t))]^T M[A_i x(t) + B_i K_j x(t-d(t))] \end{aligned} \tag{15}$$

$$\begin{aligned} \dot{V}(x_t) &\le \frac{1}{d(t)}\int_{t-d(t)}^{t}\left[\eta^T(t,\alpha)\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t))\Phi_{ij}\eta(t,\alpha)\right]d\alpha \\ &= \frac{1}{d(t)}\int_{t-d(t)}^{t}\left\{\eta^T(t,\alpha)\left[\sum_{i=1}^{r} h_i(\theta(t))\Phi_{ii} + \sum_{i=1}^{r-1}\sum_{j=i+1}^{r} h_i(\theta(t))h_j(\theta(t))\left(\Phi_{ij} + \Phi_{ji}\right)\right]\eta(t,\alpha)\right\}d\alpha, \\ \eta^T(t,\alpha) &\triangleq [x^T(t) \quad x^T(t-d(t))\dot{x}^T(\alpha)], \\ \Phi_{ij} &\triangleq \begin{bmatrix} PA_i + A_i^T P + cA_i^T M A_i + X_{ij} + X_{ij}^T & PB_i K_j + cA_i^T M B_i K_j - X_{ij} + Y_{ij}^T & -d(t)X_{ij} \\ * & cK_j^T B_i^T M B_i K_j - Y_{ij}^T - Y_{ij} & -d(t)Y_{ij} \\ * & * & -d(t)M \end{bmatrix} \end{aligned} \tag{19}$$

system into another one by using the Newton–Leibniz formula. In this framework, usually one has to employ some bounding techniques to find upper bounds for the inner product between two vectors. These bounding techniques involve some matrix inequalities, such as the well-known inequality $-2x^Ty \leq x^TRx + y^TR^{-1}y$. Employing these inequalities will inevitably introduce some overdesign into the derived conditions. However, it is worth emphasizing that in our derivation, no system transformation has been performed to the original system and thus no inequality is needed for seeking upper bounds of the inner product between two vectors. This feature has the potential to yield less conservative results.

Remark 5: It can be seen from the above development that the input delay approach, though effective for the sampled-data systems, has introduced some overdesign when treating the sampling as a delayed input. The conservativeness comes from the fact that the delay $d(t)$ in the transformed system in (10) is only one particular type of all nondifferentiable delays, while the condition presented in Theorem 1 is suitable for all nondifferentiable delays. Therefore, how to develop a stability condition that removes this overdesign for the delay shown in Fig. 1 is an interesting topic that is worthy of further investigation.

C. Controller Design

Theorem 1 presents an LMI condition by which the closed-loop sampled-data system is asymptotically stable. It is worth noting that for given subsystem matrices (A_i, B_i) and distributed controller matrices K_i, (11) is a set of strict LMIs with respect to matrix variables P, M, X_{ij}, Y_{ij}, and thus can be efficiently solved by available numerical software. However, when the subsystem controller gains K_i are not known, (11) is a set of nonlinear matrix inequalities. By observation, it is not easy to transform this set of nonlinear matrix inequalities to an equivalent set of LMIs. In what follows, we will present two procedures for controller design, that is, to find the subsystem controller gains K_i.

Theorem 2: Consider the fuzzy system in (4) with Assumption 1. A stabilizing controller in the form of (6) exists, such that the closed-loop sampled-data system in (9) is asymptotically stable, if there exist matrices $L > 0$, $R > 0$, $\bar{X}_{ij}$, $\bar{Y}_{ij}$, and $\bar{K}_i$ satisfying

$$\begin{bmatrix} \Xi_1 & \Xi_2 & -\bar{X}_{ij} - \bar{X}_{ji} & LA_i^T & LA_j^T \\ * & \Xi_3 & -\bar{Y}_{ij} - \bar{Y}_{ji} & \bar{K}_j^T B_i^T & \bar{K}_i^T B_j^T \\ * & * & 2c^{-1}(R - 2L) & 0 & 0 \\ * & * & * & -c^{-1}R & 0 \\ * & * & * & * & -c^{-1}R \end{bmatrix} < 0, \quad 1 \leq i \leq j \leq r \tag{25}$$

where

$$\begin{aligned} \Xi_1 &\triangleq (A_i + A_j)L + L(A_i + A_j)^T + \bar{X}_{ij} \\ &\quad + \bar{X}_{ij}^T + \bar{X}_{ji} + \bar{X}_{ji}^T, \\ \Xi_2 &\triangleq B_i\bar{K}_j + B_j\bar{K}_i - \bar{X}_{ij} + \bar{Y}_{ij}^T - \bar{X}_{ji} + \bar{Y}_{ji}^T \\ \Xi_3 &\triangleq -\bar{Y}_{ij}^T - \bar{Y}_{ij} - \bar{Y}_{ji}^T - \bar{Y}_{ji}. \end{aligned} \tag{26}$$

Moreover, if the above condition has a feasible solution, the gains K_i of the subsystem controllers in (5) are given by

$$K_i = \bar{K}_i L^{-1}. \tag{27}$$

Proof: Suppose there exist matrices $L > 0$, $R > 0$, $\bar{X}_{ij}$, $\bar{Y}_{ij}$, and $\bar{K}_i$ satisfying (25); we will prove that there must exist matrices $P > 0$, $M > 0$, X_{ij}, Y_{ij}, and K_i satisfying (11).

First, since $R > 0$, we have $(R - L)R^{-1}(R - L) \geq 0$, which is equivalent to

$$-LR^{-1}L \leq R - 2L. \tag{28}$$

Thus, from (25) and (28), we have

$$\begin{bmatrix} \Xi_1 & \Xi_2 & -\bar{X}_{ij} - \bar{X}_{ji} & LA_i^T & LA_j^T \\ * & \Xi_3 & -\bar{Y}_{ij} - \bar{Y}_{ji} & \bar{K}_j^T B_i^T & \bar{K}_i^T B_j^T \\ * & * & -2c^{-1}LR^{-1}L & 0 & 0 \\ * & * & * & -c^{-1}R & 0 \\ * & * & * & * & -c^{-1}R \end{bmatrix} < 0, \quad 1 \leq i \leq j \leq r. \tag{29}$$

Performing a congruence transformation to (29) by $\mathrm{diag}\{L^{-1}, L^{-1}, L^{-1}, I, I\}$, we have (30) as shown at the bottom of the next page, where

$$\begin{aligned} \bar{\Xi}_1 &\triangleq L^{-1}(A_i + A_j)L + (A_i + A_j)^T L^{-1} \\ &\quad + L^{-1}(\bar{X}_{ij} + \bar{X}_{ij}^T + \bar{X}_{ji} + \bar{X}_{ji}^T)L^{-1} \\ \bar{\Xi}_2 &\triangleq L^{-1}\left(B_i\bar{K}_j + B_j\bar{K}_i\right)L^{-1} \\ &\quad + L^{-1}\left(-\bar{X}_{ij} + \bar{Y}_{ij}^T - \bar{X}_{ji} + \bar{Y}_{ji}^T\right)L^{-1} \\ \bar{\Xi}_3 &\triangleq -L^{-1}\left(\bar{Y}_{ij}^T + \bar{Y}_{ij} + \bar{Y}_{ji}^T + \bar{Y}_{ji}\right)L^{-1} \end{aligned}$$

which, by Schur complement, is equivalent to

$$\begin{bmatrix} \bar{\Xi}_4 & \bar{\Xi}_5 & -L^{-1}\left(\bar{X}_{ij} + \bar{X}_{ji}\right)L^{-1} \\ * & \bar{\Xi}_6 & -L^{-1}\left(\bar{Y}_{ij} + \bar{Y}_{ji}\right)L^{-1} \\ * & * & -2c^{-1}R^{-1} \end{bmatrix} < 0, \quad 1 \leq i \leq j \leq r \tag{31}$$

where

$$\begin{aligned} \bar{\Xi}_4 &\triangleq \bar{\Xi}_1 + cL^{-1}A_i^TR^{-1}A_iL^{-1} \\ &\quad + cL^{-1}A_j^TR^{-1}A_jL^{-1} \\ \bar{\Xi}_5 &\triangleq \bar{\Xi}_2 + cL^{-1}A_i^TR^{-1}B_i\bar{K}_jL^{-1} \\ &\quad + cL^{-1}A_j^TR^{-1}B_j\bar{K}_iL^{-1} \\ \bar{\Xi}_6 &\triangleq \bar{\Xi}_3 + cL^{-1}\bar{K}_j^TB_i^TR^{-1}B_i\bar{K}_jL^{-1} \\ &\quad + cL^{-1}\bar{K}_i^TB_j^TR^{-1}B_j\bar{K}_iL^{-1}. \end{aligned}$$

Now, define the following matrix variables:

$$P \triangleq L^{-1}, \quad M \triangleq R^{-1}, \quad K_i \triangleq \bar{K}_iL^{-1}$$
$$X_{ij} \triangleq L^{-1}\bar{X}_{ij}L^{-1}, \quad Y_{ij} \triangleq L_{ij}^{-1}\bar{Y}L^{-1}.$$

By substituting the above matrix variables into (31), we readily obtain (11), which means that there exist matrices $P > 0$, $M > 0$, X_{ij}, Y_{ij}, and K_i satisfying (11), and thus the controller gains defined in (27) render the closed-loop sampled-data system in (9) to be asymptotically stable. □

Remark 6: Theorem 2 presents a sufficient condition for determining the gains of subsystem controllers. Condition (25) is a set of LMIs, and thus can be efficiently solved by using standard numerical software. It is worth pointing out that the condition in Theorem 2 is not equivalent to that in Theorem 1. In other words, the condition in Theorem 2 is more stringent than that in Theorem 1. In the following, we will present another controller design procedure.

Theorem 3: Consider the fuzzy system in (4) with Assumption 1. A stabilizing controller in the form of (6) exists, such that the closed-loop sampled-data system in (9) is asymptotically stable, if there exist matrices $L > 0$, $R > 0$, $\bar{X}_{ij}$, $\bar{Y}_{ij}$, and $\bar{K}_i$ satisfying

$$\begin{bmatrix} \Xi_1 & \Xi_2 & -\bar{X}_{ij} - \bar{X}_{ji} & LA_i^T & LA_j^T \\ * & \Xi_3 & -\bar{Y}_{ij} - \bar{Y}_{ji} & \bar{K}_j^T B_i^T & \bar{K}_i^T B_j^T \\ * & * & -2c^{-1}LR^{-1}L & 0 & 0 \\ * & * & * & -c^{-1}R & 0 \\ * & * & * & * & -c^{-1}R \end{bmatrix} < 0, \quad 1 \le i \le j \le r, \tag{32}$$

where Ξ_1, Ξ_2, and Ξ_3 are given in (26). Moreover, if the above condition has a feasible solution, the gains K_i of the subsystem controllers in (5) are given by (27).

Proof: The proof follows similar lines as those in the proof of Theorem 2, and is thus omitted. □

Remark 7: Theorem 3 presents a sufficient condition for the existence of desired controllers, which is equivalent to the condition in Theorem 1. However, (32) is still a set of nonlinear matrix inequalities, which cannot be directly solved using standard numerical software. In the following, we will present an iterative procedure for solving the nonconvex conditions in Theorem 3.

First, we define a new variable $W > 0$ such that $LR^{-1}L \ge W$ and replace (32) with

$$\Gamma \triangleq \begin{bmatrix} \Xi_1 & \Xi_2 & -\bar{X}_{ij} - \bar{X}_{ji} & LA_i^T & LA_j^T \\ * & \Xi_3 & -\bar{Y}_{ij} - \bar{Y}_{ji} & \bar{K}_j^T B_i^T & \bar{K}_i^T B_j^T \\ * & * & -2c^{-1}W & 0 & 0 \\ * & * & * & -c^{-1}R & 0 \\ * & * & * & * & -c^{-1}R \end{bmatrix} < 0, \quad 1 \le i \le j \le r \tag{33}$$

$$LR^{-1}L \ge W \tag{34}$$

where Ξ_1, Ξ_2, and Ξ_3 are given in (26). Then, (34) is equivalent to

$$L^{-1}RL^{-1} \le W^{-1}$$

which, by Schur complement, is further equivalent to

$$\begin{bmatrix} -W^{-1} & L^{-1} \\ * & -R^{-1} \end{bmatrix} \le 0.$$

Now, by introducing new matrix variables $\bar{W} > 0$, $\bar{L} > 0$, and $\bar{R} > 0$, the original condition (32) can be represented as (33) and

$$\begin{bmatrix} -\bar{W} & \bar{L} \\ * & -\bar{R} \end{bmatrix} \le 0 \tag{35}$$

$$L\bar{L} = I, \quad R\bar{R} = I, \quad W\bar{W} = I. \tag{36}$$

Denote the following set:

$$\Omega \triangleq \left\{ \begin{pmatrix} L > 0, \bar{L} > 0, R > 0, \bar{R} > 0 \\ W > 0, \bar{W} > 0, \bar{X}_{ij}, \bar{Y}_{ij}, \bar{K}_i \end{pmatrix} : \begin{matrix} \\ (33),\ (35),\ \text{and}\ (36)\ \text{are satisfied} \end{matrix} \right\}$$

as the solution of Theorem 3. It is noted that Ω is not a convex set due to the matrix equality constraints in (36). Several approaches have been proposed to solve such nonconvex feasibility problems, among which the CCL method [13] is the most commonly used one (for instance, the CCL algorithm has been used for solving the controller design problems as well as model reduction problems [17], [18]). The basic idea in CCL algorithm is that if the LMI $\left[\begin{smallmatrix} \mathcal{P} & I \\ I & \mathcal{L} \end{smallmatrix}\right] \ge 0$ is feasible in the $n \times n$ matrix variables $\mathcal{L} > 0$ and $\mathcal{P} > 0$, then $\mathrm{tr}(\mathcal{PL}) \ge n$, and $\mathrm{tr}(\mathcal{PL}) = n$ if and only if $\mathcal{PL} = I$. Very recently, a so-called sequential linear programming matrix method (SLPMM) was proposed for solving such nonconvex feasibility problems, which can be seen as an improved version of the CCL algorithm. As is indicated in [26], the SLPMM algorithm is superior to the CCL algorithm in that it always generates a sequence of iterates with strictly decreasing objective function values and is globally convergent. Here, we will employ the SLPMM algorithm to solve the nonconvex feasibility problem formulated above.

First, for computational purposes, introduce a sufficiently small scalar $\nu > 0$ and replace (33) by

$$\Gamma + \mathrm{diag}\{\nu I,\ 0,\ 0,\ 0,\ 0\} \le 0, \quad 1 \le i \le j \le r. \tag{37}$$

$$\begin{bmatrix} \bar{\Xi}_1 & \bar{\Xi}_2 & -L^{-1}\left(\bar{X}_{ij} + \bar{X}_{ji}\right)L^{-1} & L^{-1}A_i^T & L^{-1}A_j^T \\ * & \bar{\Xi}_3 & -L^{-1}\left(\bar{Y}_{ij} + \bar{Y}_{ji}\right)L^{-1} & L^{-1}\bar{K}_j^T B_i^T & L^{-1}\bar{K}_i^T B_j^T \\ * & * & -2c^{-1}R^{-1} & 0 & 0 \\ * & * & * & -c^{-1}R & 0 \\ * & * & * & * & -c^{-1}R \end{bmatrix} < 0, \quad 1 \le i \le j \le r \tag{30}$$

The idea behind such dealing is to make the condition in (33) closed by introducing the small positive scalar ν. Secondly, to convexify Ω, the equality constraints in (36) can be weakened to the following semidefinite programming relaxations:

$$\begin{bmatrix} L & I \\ I & \bar{L} \end{bmatrix} \geq 0, \quad \begin{bmatrix} R & I \\ I & \bar{R} \end{bmatrix} \geq 0, \quad \begin{bmatrix} W & I \\ I & \bar{W} \end{bmatrix} \geq 0. \tag{38}$$

Note that the equality constraints in (36) correspond to the boundaries of the convex sets in (38). Now, define

$$\Omega_\nu \triangleq \left\{ \begin{pmatrix} L>0, \bar{L}>0, R>0, \bar{R}>0 \\ W>0, \bar{W}>0, \bar{X}_{ij}, \bar{Y}_{ij}, \bar{K}_i \end{pmatrix} : \atop \text{(35), (37), and (38) are satisfied} \right\}.$$

Then, Ω_ν is a closed and convex set and thus the SLPMM algorithm can be applied to find feasible solutions of Theorem 3 via solving the following problem.

Problem SDS (Sampled-Data Stabilization): See (39) at the bottom of the page.

If the solution of the above minimization problem is $3n$, that is, $\min \operatorname{tr}(L\bar{L} + R\bar{R} + W\bar{W}) = 3n$, then the conditions in Theorem 3 are solvable. A detailed SLPMM algorithm adapted to our problem is presented as follows.

Algorithm SDS

Step 1) Given the following parameters:

- δ Error bound to control the solution precision
- N Maximum number of iterations
- ν Sufficiently small positive scalar for (37)

Step 2) Find a feasible set $(L^0>0, \bar{L}^0>0, R^0>0, \bar{R}^0>0,$ $W^0>0, \bar{W}^0>0, \bar{X}_{ij}^0, \bar{Y}_{ij}^0, \bar{K}_i^0) \in \Omega_\nu$. Set $p=0$.

Step 3) Solve the following LMI problem in (40) for the matrix variables $L>0, \bar{L}>0, R>0, \bar{R}>0, W>0, \bar{W}>0, \bar{X}_{ij},$ $\bar{Y}_{ij}, \bar{K}_i$. See (40) at the bottom of the page.

Step 4) Substitute the obtained matrix variables $(L>0, \bar{L}>0,$ $R>0, \bar{R}>0, W>0, \bar{W}>0, \bar{X}_{ij}, \bar{Y}_{ij}, \bar{K}_i)$ into (32). If condition (32) is satisfied, with

$$|\operatorname{tr}(L\bar{L}^p + L^p\bar{L} + R\bar{R}^p + R^p\bar{R} + W\bar{W}^p + W^p\bar{W}) - 6n| \leq \delta, \tag{41}$$

then output the feasible solutions $(L>0, \bar{L}>0, R>0,$ $\bar{R}>0, W>0, \bar{W}>0, \bar{X}_{ij}, \bar{Y}_{ij}, \bar{K}_i)$. EXIT.

Step 5) If $p>N$, EXIT.

Step 6) Calculate $\rho^* \in [0,1]$ by solving

$$\min_{\rho \in [0,1]} \Lambda(\rho)$$

where

$$\Lambda(\rho) \triangleq \operatorname{tr} \left\{ \begin{array}{c} (L^p + \rho(L - L^p))(\bar{L}^p + \rho(\bar{L} - \bar{L}^p)) + \\ (R^p + \rho(R - R^p))(\bar{R}^p + \rho(\bar{R} - \bar{R}^p)) + \\ (W^p + \rho(W - W^p))(\bar{W}^p + \rho(\bar{W} - \bar{W}^p)) \end{array} \right\}.$$

Set

$$\begin{aligned} L^{p+1} &= L^p + \rho^*(L - L^p), \quad \bar{L}^{p+1} = \bar{L}^p + \rho^*(\bar{L} - \bar{L}^p), \\ R^{p+1} &= R^p + \rho^*(R - R^p), \quad \bar{R}^{p+1} = \bar{R}^p + \rho^*(\bar{R} - \bar{R}^p), \\ W^{p+1} &= W^p + \rho^*(W - W^p), \\ \bar{W}^{p+1} &= \bar{W}^p + \rho^*(\bar{W} - \bar{W}^p) \end{aligned}$$

and $p = p+1$, go to Step 3).

Remark 8: In Algorithm SDS, we use (32) and (41) as the stopping criterion since it can be numerically difficult in practice to obtain an optimal solution such that its corresponding minimum value in Step 3) is exactly equal to $3n$.

Remark 9: This algorithm is similar to that developed in [26] for the design of static output-feedback $\mathcal{H}_2/\mathcal{H}_\infty$ controllers. It is noted that the SLPMM-based Algorithm SDS will recover the CCL algorithm [13] by setting $\rho^* \equiv 1$. As indicated in [26],

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$$\min_{(L>0, \bar{L}>0, R>0, \bar{R}>0, W>0, \bar{W}>0, \bar{X}_{ij}, \bar{Y}_{ij}, \bar{K}_i) \in \Omega_\nu} \operatorname{tr}(L\bar{L} + R\bar{R} + W\bar{W}) \tag{39}$$

$$\min_{(L>0, \bar{L}>0, R>0, \bar{R}>0, W>0, \bar{W}>0, \bar{X}_{ij}, \bar{Y}_{ij}, \bar{K}_i) \in \Omega_\nu} \operatorname{tr}(L\bar{L}^p + L^p\bar{L} + R\bar{R}^p + R^p\bar{R} + W\bar{W}^p + W^p\bar{W}) \tag{40}$$

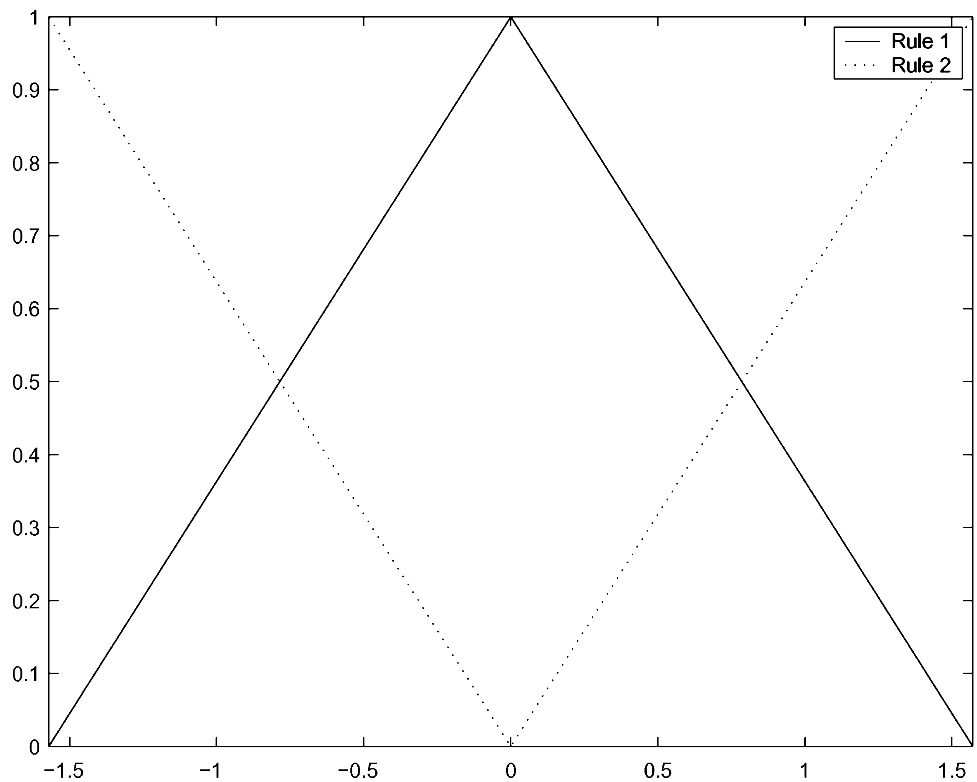

Fig. 2. Membership functions of the two-rule model.

Algorithm SDS always generates a strictly decreasing sequence of the objective function $\mathrm{tr}(L\bar{L} + R\bar{R} + W\bar{W})$ in (39). That is

$$3n \le \mathrm{tr}(\Delta^{q+1}) \le \mathrm{tr}(\Delta^q)$$

where

$$\Delta^q \triangleq L^q\bar{L}^q + R^q\bar{R}^q + W^q\bar{W}^q.$$

Thus, $\mathrm{tr}(\Delta^q)$ always converges to some $\mathrm{tr}(\Delta^*) \ge 3n$. If $\mathrm{tr}(\Delta^*) = 3n$, then an optimal solution $(L^* > 0,\ \bar{L}^* > 0,\ R^* > 0,\ \bar{R}^* > 0,\ W^* > 0,\ \bar{W}^* > 0,\ \bar{X}^*_{ij},\ \bar{Y}^*_{ij},\ \bar{K}^*_i) \in \Omega$. Upon the obtained feasible solution, desired gains K_i of the subsystem controllers in (5) can be obtained by (27).

IV. Illustrative Example

In this section, we will use an example to illustrate the applicability of the sampled-data controller design procedure proposed in this paper. Consider the problem of balancing and swing-up of an inverted pendulum on a cart. The equations of the pendulum motion are given by [4]

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = \frac{\begin{bmatrix} g\sin(x_1(t)) - amlx_2^2(t)\sin(2x_1(t))/2 \\ -a\cos(x_1(t))u(t) \end{bmatrix}}{4l/3 - aml\cos^2(x_1(t))} \tag{42}$$

where $x_1(t)$ is the angle (in radians) of the pendulum from the vertical, $x_2(t)$ is the angular velocity, and $u(t)$ is the force applied to the cart (in newtons). $g = 9.8$ m/s^2 is the gravity constant, m is the mass of the pendulum, M is the mass of the cart, $2l$ is the length of the pendulum, and $a = 1/(m + M)$. Here we choose $m = 2.0$ kg, $M = 8.0$ kg, and $2l = 1.0$ m in simulations [3].

The control objective here is to balance the inverted pendulum for the approximate range $x_1(t) \in (-(\pi/2), (\pi/2))$ through a sampled-data control approach. First, we represent the system in (42) by a two-rule Takagi–Sugeno fuzzy model [39]

$$\begin{aligned} \text{Model Rule 1}: \quad & \text{IF } x_1(t) \text{ is about } 0 \\ & \text{THEN } \dot{x}(t) = A_1x(t) + B_1u(t) \\ \text{Model Rule 2}: \quad & \text{IF } x_1(t) \text{is about } \pm\frac{\pi}{2}\left(|x_1(t)| < \frac{\pi}{2}\right) \\ & \text{THEN } \dot{x}(t) = A_1x(t) + B_1u(t) \end{aligned} \tag{43}$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ \frac{g}{4l/3 - aml} & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ -\frac{a}{4l/3 - aml} \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 0 & 1 \\ \frac{2g}{\pi(4l/3 - aml\beta^2)} & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ -\frac{a\beta}{4l/3 - aml\beta^2} \end{bmatrix}$$

and $\beta = \cos(88^\circ)$ (notice that when $x_1(t) = \pm\frac{\pi}{2}$, the system is uncontrollable). Membership functions for Rules 1 and 2 are shown in Fig. 2.

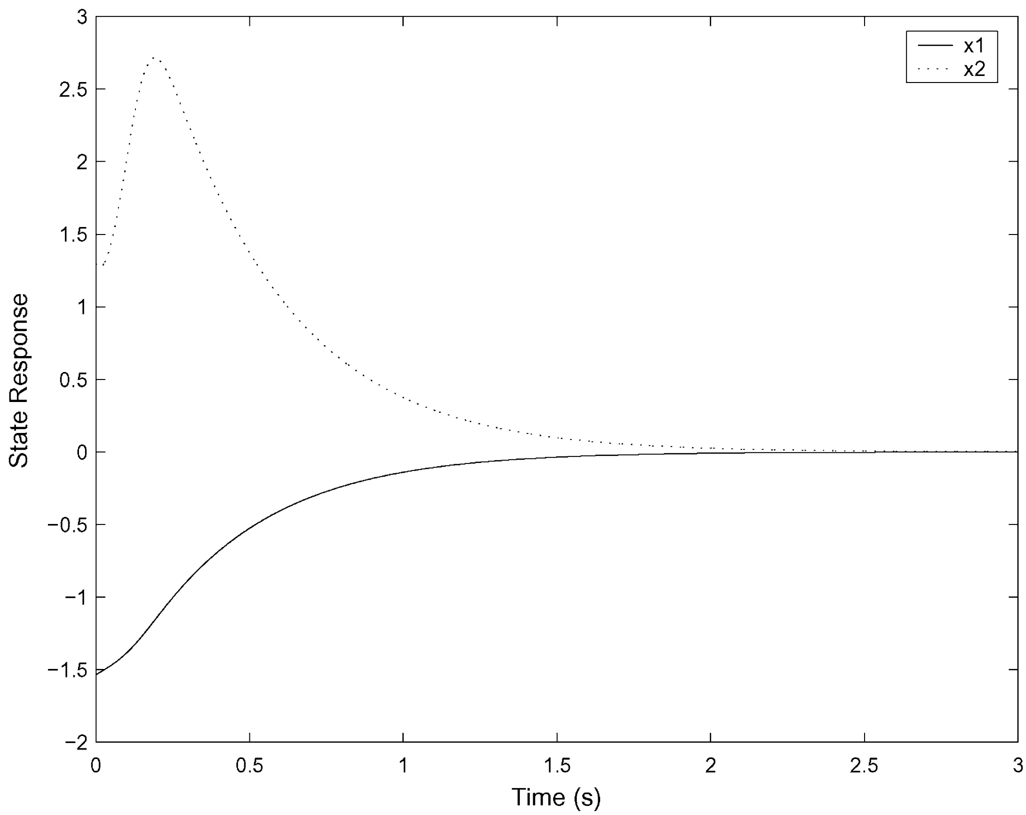

Fig. 3. State response of the closed-loop sampled-data system.

Now assume that the state variable $x(t)$ is measured at the sampling instant t_k, where t_k satisfies Assumption 1 with $c =$ 15 ms (that is, the sampling distances are allowed to change with time but do not exceed 15 ms). By using the sampled-data controller design method (Theorem 1), we obtain the following PDC control law:

$$\begin{aligned} \text{Model Rule 1 :}\quad & \text{IF } x_1(t) \text{ is about } 0 \\ & \text{THEN } u(t) = K_1 x(t_k), \quad t_k \le t < t_{k+1} \\ \text{Model Rule 2 :}\quad & \text{IF } x_1(t) \text{ is about } \pm\frac{\pi}{2}\left(|x_1(t)| < \frac{\pi}{2}\right) \\ & \text{THEN } u(t) = K_2 x(t_k), \quad t_k \le t < t_{k+1} \end{aligned}$$

where

$$K_1 = [533.4344 \quad 174.9688]$$
$$K_2 = [1962.9537 \quad 734.6736].$$

To illustrate the asymptotic stability of the closed-loop system, we assume that the initial condition of the state is $x(0) = [-1.53\ 1.3]^T$ and the sampling instant t_k is generated randomly with the constraint that $t_{k+1} - t_k \le 15$ ms for all t_k. The state response of the closed-loop sampled-data system is depicted in Fig. 3, from which we can see that the closed-loop sampled-data system is asymptotically stable, showing the effectiveness of the proposed controller design procedure. Finally, by maximizing c in Theorem 1, we obtain that the maximum value of c is 18 ms, such that the condition in Theorem 1 has feasible solutions, and the controller gain matrices are given by

$$K_1 = [621.4791 \quad 186.5781]$$
$$K_2 = [1931.7639 \quad 626.9138].$$

V. Concluding Remarks

In this paper, we have investigated the problem of state-feedback stabilization for T–S fuzzy systems with nonuniform uncertain sampling. This problem was solved through an input delay approach, which represented the hybrid system with both continuous and discrete signals as a continuous-time system with a delay in the state. The sampling is not required to be periodic, and the only assumption is that the distance between any two consecutive sampling instants is less than a given bound. An LMI-based stability condition has been obtained for the closed-loop sampled-data fuzzy system, upon which two procedures have been proposed for designing desired state-feedback controllers. An illustrative example has been used to show the effectiveness of the proposed controller design procedures. The results reported here can be further extended to T–S fuzzy systems with uncertain parameters and to $\mathcal{H}_\infty$ control of T–S fuzzy systems.

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