GNSS Receiver Autonomous Integrity Monitoring (RAIM) for Multiple Outliers

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Abstract
Receiver Autonomous Integrity Monitoring (RAIM) procedures for Global Navigation Satellite Systems (GNSS) are required for safety and liability critical applications. While existing RAIM techniques, generally based on a single outlier assumption model, are adequate today they will not be justifiable for the next generation of GNSS. In this paper, a scheme for outlier identification, which uses the w-test and the associated correlation information of the test statistics to make 'smart' decisions for identification of multiple outliers, is presented as an alternative to the conventional w-test. It is also proposed that, as a minimum, the conventional w-test should also implicitly include the determination of the corresponding correlation coefficients with a warning signaled to the user regarding any adjustments comprising highly correlated statistics. Detailed simulations and analyses have been performed to assess the performance of the new 'smart' scheme and improved conventional procedure. Results show that the scheme is capable of detecting and isolating single and multiple outliers to comparable levels with the conventional w-test procedure with a significant reduction in the computational load.

1. Introduction.

Receiver Autonomous Integrity Monitoring (RAIM) procedures for Global Navigation Satellite Systems (GNSS) are required, by the International Civil Aviation Organisation, to satisfy the following definition of integrity: *a measure of the trust which can be placed in the correctness of the information supplied by the total system. Integrity includes the ability of a system to provide timely warnings to the user when the system should not be used for the intended operation* (Ober, 2003). Meeting this definition requires that, if outliers contaminate the data set and cannot be removed or appropriately dealt with in a timely way, then the user should be alerted of the failure. Due to modern computer processing power the treatment of outliers, and if necessary the time-to-alert responses, can be achieved via some very intensive RAIM procedures, such as recursive or iterative adjustment approaches. However, time is still a major factor concerning RAIM procedures as it limits the sampling rate of the receiver. Additionally, the design of the algorithms should be relatively simple in order to permit certification (Ober, 2003) and easy implementation. Further discussion on receiver software complexity as a significant threat to GNSS integrity can be found in Goodman (2003).

The statistical redundancy based w-test procedure for outlier identification was first introduced in Baarda (1968) for use in geodetic networks. Since then the procedure has been adopted and used extensively in quality control schemes for GPS positioning. Cross et al. (1994) presents a quality control scheme for differential GPS positioning using the w-test. In the event of multiple failures occurring it is suggested to reject only the largest failure and repeat adjustment computation until no further outliers are identified. Miller et al. (1997) identifies the correlations of the w-test statistics as a 'widening' problem for outlier identification. Miller et al. (1997) also recommends removing only the largest test failure as a single outlier. It is also shown and discussed that the procedure can be unreliable when the redundancy is low. The correlation problem relating to misidentification is also discussed in Tiberius (1998). Again, only the results of low redundancy differential GPS positioning situations are presented and the conventional approach of removing only the largest failing w-statistics as the outlier is maintained. Wang & Chen (1999) present generalised outlier detection and reliability theory capable of treating multiple outliers provided it is known which measurements are contaminated. Recently, Hwang & Brown (2005) propose a new RAIM algorithm capable of managing two simultaneous faults. The procedure is highly complex and requires a considerable increase in the number of hypothesis tests and is still limited by the number of outliers that can be identified. The authors justify the sufficiency of the simultaneous dual fault assumption based on the likelihood of satellite faults. However, this does not include the likelihood of faults due to signal obscuration or receiver malfunction.

In statistical literature, there have been numerous publications addressing the multiple outlier issue over the last 30 years. Generally speaking, the most common and successful methods pertaining to RAIM have been based on statistics derived from the residuals of the navigation adjustment. These redundancy based procedures accommodate two major approaches to the multiple outlier identification problem: inward or outward search procedures in which outliers are either identified and removed, one at a time, from a contaminated set or the remaining inliers are added, one at a time, to an outlier free set, respectively. Atkinson & Riani (1997) show that the outward, or forward search procedures generally out perform the inward methods for the identification of multiple outliers. Clustering techniques, which attempt to sort the data into clusters of like data, have also been shown to be capable of multiple outlier identification but such methods are prone to exhibiting higher than desirable levels of false alarms Wisnowski et al. (2001).

In this paper, it is shown that an extended w-test identification procedure can be used for the simultaneous removal of multiple outliers simply and effectively, provided the navigation adjustment is sufficiently resilient to the contaminating measurements. The procedure is designed to maintain computational efficiency and avoid complexity. Furthermore, the limitations of the conventional w-test procedure and the new extended procedure proposed herein are discussed.

2. The RAIM Scheme.

Statistical testing procedures focused on the reliability of detecting fault measurements or outliers have generally been the basis for current RAIM techniques. With more than five satellites, the contaminating measurement can generally be identified, depending on the correlation of detection statistics. If the statistics are highly correlated, the likelihood of flagging the wrong measurement as the outlier is severe. It should be noted that greater redundancy and geometric strength of the measurement system significantly reduces the correlation of the test statistics and therefore, improves the capability of RAIM procedures for both detecting and identifying the outliers, including multiple outlier scenarios. Thus, the separability of an adjustment should be considered when evaluating GNSS RAIM performance where, the separability measure is used to assess the capability of GNSS receivers to correctly identify the outlier from the measurements processed. For a detailed geometrical analysis with respect to reliability and separability readers are referred to Hewitson and Wang (2005)

2.1. Outlier Identification.

Herein, the *w*-test is used to identify the outliers because of its simplicity and relatively effective performance (Baarda 1968, Cross et al. 1994, Teunissen 1998). The test statistic is the normalised residual for an observation and therefore belongs to the standard normal distribution when no outlier is present in the adjustment and a non-central normal distribution in the presence of an outlier. The *w*-test statistic is (Baarda 1968, Cross et al. 1994, Teunissen 1998):

$$w_i = \frac{\nabla \hat{S}_i}{\sqrt{Q_{\nabla \hat{S}_i}}} = \frac{e_i^T P \hat{v}}{\sqrt{e_i^T P Q_{\hat{v}} P e_i}} \tag{1}$$

and the critical value to test $|w_i|$ against is $N_{1-\alpha/2}(0,1)$ where α is the significance level of the test. $\nabla \hat{S}_i$ is the least squares estimation of the magnitude of the i^{th} outlier, $Q_{\nabla \hat{S}}$ is the variance covariance (VCV) matrix of $\nabla \hat{S}$, P is the weight matrix of the measurements, $Q_{\hat{v}}$ is the VCV matrix of the estimated residuals and e_i is a unit vector in which the i^{th} element has a value equal to one and dictates the measurement to be tested.

Teunissen (1991) states that the w-test statistics are uniformly-most-powerful-invariant test statistics and have the highest probability of correctly detecting and identifying model errors of any test statistic provided they are formulated with identical assumptions. A w-test statistic corresponding to each and every observation is incrementally evaluated and where the test statistic exceeds the critical value for the desired significance level, the corresponding measurement is flagged as a possible outlier. However, as the test was originally derived for the identification of a single outlier, only the largest absolute failure is assumed to correspond to a real outlier. For this reason, only one outlier is identified per least squares adjustment. In order to determine whether or not any more outliers exist in the observations, the adjustment must be recomputed with the previously identified outliers removed until no more outliers are detected or the remaining redundancy is insufficient.

2.2. Separability.

Highly correlated statistics increase the levels of false alarms and outlier masking. If there is a strong correlation between two statistics and the measurement corresponding to one is a real detectable outlier, then the other statistic is likely to exceed the critical value as well, thus creating difficulty to distinguish the real outlier. In some cases the non-outlying correlated measurement can have a larger test statistic (see Section 3). The ability to accurately identify an outlier, referred to as separability, is therefore dependent upon the outlier magnitude and the correlation of the test statistics. The degree of correlation of two test statistics is determined through derivation of the correlation coefficient (Förstner 1983, Tiberius 1998):

$$\rho_{ij} = \frac{e_i^T P Q_{\hat{v}} P e_j}{\sqrt{e_i^T P Q_{\hat{v}} P e_i} \cdot \sqrt{e_j^T P Q_{\hat{v}} P e_j}} \qquad (2)$$

where e_j is a unit vector in which the i^{th} element has a value equal to one and dictates the measurement to be tested.

The correlation coefficient can be interpreted as the cosine of the angle between the vectors e_i and e_j with respect to the metric $PQ_{\hat{v}}P$ (Förstner 1983).

3. Limitations of the *W*-Test.

It is reasonable to argue that missed detection is a far more serious problem than a false alarm as the removal of a good measurement is not adversely influential. The only serious concern with false alarms is the reduction of the adjustment redundancy upon their removal. On the other hand, an undetected or masked outlier can have significant impact on a solution, particularly if it is large. In such cases the fit of the adjustment is biased, which not only results in an erroneous solution but also is likely to cause several false alarms due to the correlations of the test statistics and may mask other smaller outliers. As a large outlier can cause many test failures for outlier identification by biasing the adjustment centre we need to be able to separate the false alarms from the true outliers. Sufficient redundancy and geometric strength are required to ensure correct identification. Misidentification can occur quite frequently when the redundancy is low and the test statistics are highly correlated.

In addition, as the w-test was developed to identify only a single fault in an adjustment, its ability to identify multiple outliers has various shortcomings. The major shortcoming, which needs addressing, also relates to the correlation problem regarding the test statistics. These correlations, together with the outlying measurements, result in a complex situation of influence and interaction amongst all w-statistics. This complexity increases with the number of outliers and increasing correlation of the test statistics. Therefore, part of the solution to the multiple outlier problem is to decrease the w-statistic correlations through strengthening the geometry and redundancy of the adjustment. A second major shortcoming is the efficiency of the w-test when identifying multiple faults. The iterative nature of the test requires a re-computation of the adjustment for every outlier identified. Such a procedure can have serious implementation issues, especially in real-time systems. Finally, another limitation is its inability to detect small outliers. This shortcoming is far less significant than those previously mentioned as small outliers have little effect on solution, relatively. Improved geometry and redundancy also improves the *w*-test capability in this regard (Hewitson 2003, Hewitson et al. 2004).

4. Extended *W*-Test for Multiple Outliers.

When dealing with multiple outliers there can be a severe impact upon the adjustment. In some cases, in particular with many outlying measurements or say 3 large ones, the centre of the adjustment (ie. the solution) can shift and cause good measurements to appear as the outliers. Upon removal of these false alarms the adjustment can then be found to appear statistically sound. That is, the measurements fit the model well according to the variance factor and no outliers are detected. The real outliers however, remain in the adjustment. Investigations are required to determine the extent of such an effect on the final solution but are not within the scope of this paper.

In order to improve the integrity of the current procedure it is proposed that the conventional test should also implicitly include the determination of the corresponding correlation coefficients with a warning signaled to the user regarding any adjustments comprising highly correlated statistics. Note it is not reasonable to simply remove highly correlated measurements, as doing so will decrease the redundancy and geometric strength, thereby worsening the problem.

Such a procedure can then be further extended to identify multiple outliers without having to readjust the entire solution after each identification. The major advantages of this are computational efficiency gain and the preservation of the redundancy and geometry of the initial adjustment. The extension requires iterative re-evaluations of the statistics after each outlier has been identified using correlation coefficients of the test statistics. The iterations are performed as part of the RAIM procedure on the initial adjustment until no more outliers are identified. However, as for any outlier identification procedure, the adjustment itself must be sufficiently resilient to the influence of the contaminating measurements. In other words, the ability to identify the multiple outliers is limited by the redundancy and geometric strength of the adjustment. The process of identification for the first outlier is identical to the conventional approach, where the largest test statistic exceeding the critical value of the test is identified as an outlier. However, instead of readjusting without the previously identified outlier and performing the test again in search for other outliers, the extended approach removes the influence, due to the correlation, of the identified outlier's test statistic upon the remaining statistics and a reduced sub set of test statistics is obtained. This procedure is repeated until no more outliers are identified. That is to say, other outliers can be iteratively unmasked by removing the effect of larger biases on the remaining statistics due to their spatial correlations. The following algorithm is used to reduce the *w*-test statistics:

$$w_{r_s,i} = w_{r_{s-1},i} - w_{r_{s-1},\max} \times \rho_{i,\max} \qquad (3)$$

where $w_{rs,i}$ is the reduced *w*-test statistic of the current iteration (s) corresponding to the i^{th} measurement, $w_{rs\_1,i}$ is the test statistic corresponding to the i^{th} of the previous iteration (s_1), $w_{rs\_1,max}$ is the test statistic corresponding to the maximum failure of the previous iteration and $\rho_{i,\max}$ is the correlation coefficient of the i^{th} measurement and the measurement corresponding to the maximum failure in the previous epoch. As the largest test failure generally corresponds to the outlying measurement with the greatest impact on the adjustment solution, the extended procedure iteratively removes the influence of the largest outlier on the other remaining statistics and thereby unmasks any further outliers. The influence is estimated by multiplying the largest *w*-statistic with the correlation coefficients corresponding to it and the other statistics. This influence is then simply subtracted from the current statistics. The extended *w*-test procedure can be outlined as follows:

1. Execute initial adjustment and compute *w*-test statistics.
2. Identify the largest *w*-test failure and flag the corresponding measurement as an outlier.
3. Estimate the influence of the identified outlier on the other statistics using $w_{\max} \times \rho_{i,\max}$ for *i=1:n,* where n is the current number of statistics.
4. Reduce the *w*-test statistics by removing the influence of the identified outlier on the other statistics by subtracting the estimated influence from the current statistics.
5. Remove the identified outlier from the current and future reduced sets, *n=n-1.*
6. Go to step 2 and repeat until no more outliers are identified.

This extended w-test maintains algorithmic simplicity while markedly improving the computational efficiency of the conventional test in the case of multiple outliers. For single or zero fault scenarios the conventional and extended procedures are theoretically identical.

5. Simulated Performance Studies.

To assess the performances of both the conventional and extended w-test procedures, several Monte Carlo simulations have been run. The results shown here only include single and quadruple outlier scenarios per epoch over 24 hrs for GPS-only, combined GPS/GLONASS and combined GPS/GLONASS/Galileo systems. Performances were measured in terms of correct detections, false alarm rates and computation time. It should be noted here that the correlations of the statistics are computed for both procedures. Even though the conventional w-test does not strictly require them, they are essential to ensure that the geometry is strong enough to isolate outliers and avoid the problems discussed in Section 3 based on the reasoning given in Section 4. Positioning results are also included to highlight this concern.

The sampling interval used was 100 seconds and the nominal constellation designs of each were implemented for measurement generation. Here, the nominal constellations for GPS and GLONASS were implemented as described in Hoffman-Wellenhoff *et al.* (2003), Coordinational Scientific Information Center, Russian Federation Ministry of Defence (2002) respectively and the Galileo constellation was compiled from information in Dinwiddy et al. (2004) and European Commission and European Space Agency (2002). Simulated measurements are based on single frequency point positioning with a standard deviation of 3m and a masking angle of 5° was implemented. The critical value for the w-test was 3.29, corresponding to 99.9% confidence and the outliers were randomly induced with a magnitude between 0m – 80m. We have simulated the cases of single, double, triple and quadruple outlier scenarios. Due to the limited space, we only discuss the two of the cases below.

5.1. Single Outlier Scenario.

Here, the performances of the two procedures are analysed with respect to single outliers for the GPS, GPS/GLONASS and GPS/GLONASS/Galileo systems. Table 1 shows the Percentages of correct detections and false alarm rates as well as the total time taken to perform the computations, the number of acceptable solutions and the position error means and standard deviations for all 865 epochs. The criterion for an acceptable position is that the final solution passes the variance factor test with 99% confidence. The conventional and extended procedures are identical for the correct detection rates for all 3 system configurations. The false alarm rates were also identical with the exception of the GPS/GLONASS case, where the extended procedure delivered slightly lower rates than the conventional method. The extended procedure also exhibited slight improvements in computation time for all scenarios. Furthermore, the number of acceptable solutions, postion error means and standard deviations are all roughly equivalent for both procedures.

Table 1. 24-hour Monte Carlo results for single outlier scenario.

	Conventional			Extended		
	GPS	GPS/GLONASS	GPS/GLONASS/ Galileo	GPS	GPS/GLONASS	GPS/GLONASS/ Galileo
Total Outliers	865	865	865	865	865	865
Total Detections	866	881	883	866	878	883
Correct Detections (%)	97.46	100.00	100.00	97.46	100.00	100.00
False Alarms (%)	2.66	1.82	2.04	2.66	1.48	2.04
Acceptable Solutions (%)	99.42	99.77	99.54	99.42	99.42	99.54
Total Time (s) w/ Correlation Information	18.56	33.05	75.49	17.33	31.55	73.49
Hor Pos error	0.658	0.028	0.018	0.658	0.038	0.018
Vert Pos Error	-0.031	-0.002	-0.056	-0.031	0.003	-0.056
Hor Std	8.306	2.216	1.639	8.306	2.179	1.639
Ver Std	10.505	3.226	2.370	10.505	3.206	2.370

5.1. Quadruple Outlier Scenario.

The results for the simulation including 4 outliers in every epoch are given in Table 2. With respect to detection rates, the extended procedure now outperforms the conventional procedure for both the GPS-only and the GPS/GLONASS configurations and delivers nearly identical performance for the GPS/GLONASS/Galileo case. This is the first time detection rates of less than 100% have been given by either procedure for the GPS/GLONASS/Galileo case. The false alarm rates of the extended procedure are greater for all system configurations than those of the conventional procedure. Differences in the false alarm rates of the extended approach with respect to the conventional method are +0.14%, +5.82% and +1.31% for the GPS-only, GPS/GLONASS and GPS/GLONASS/Galileo configurations, respectively. The positioning accuracy is notably better for the extended procedure with respect to the GPS-only and GPS/GLONASS configurations and marginally worse for the GPS/GLONASS/Galileo scenario. The number of acceptable solutions are 13.76%, 4.4% and 0.03% lower for the extended procedure in comparison with the conventional method with respect to the GPS, GPS/GLONASS and GPS/GONASS/Galileo configurations. Computation times for the extended procedure are 29.1%, 48.2% and 54.1% more efficient than the conventional procedure for the GPS, GPS/GLONASS, GPS/GONASS/Galileo systems, respectively. The relative efficiency of the extended has again increased with respect to the prior simulations.

Table 1. Monte Carlo results for quadruple outlier scenario.

	Conventional			Extended		
	GPS	GPS/GLONASS	GPS/GLONASS/ Galileo	GPS	GPS/GLONASS	GPS/GLONASS/ Galileo
Total Outliers	3460	3460	3460	3460	3460	3460
Total Detections	1944	3818	3494	2084	4139	3540
Correct Detections (%)	30.03	88.90	99.86	32.11	89.42	99.83
False Alarms (%)	46.55	19.43	1.12	46.69	25.25	2.43
Solutions (%)	86.94	97.23	99.42	73.18	92.83	98.84
Total Time (s) w/ Correlation Information	22.30	58.39	152.06	15.80	30.27	69.82
Hor Pos error	1.223	0.863	0.039	1.735	0.202	0.012
Vert Pos Error	-2.335	3.675	-0.085	1.894	3.386	-0.043
Hor Std	59.156	17.772	1.784	51.764	15.556	1.945
Ver Std	89.871	29.082	2.573	75.643	25.697	2.870

5.3 Separability Warning Results.

The results above compare the two procedures for different scenarios without utilising the computed separability information; the following results are used to investigate the influence of separability warnings on accepting position solutions. The separability warnings used herein are warnings that accompany a solution when any of the computed correlation coefficients for that solution are greater than a specified level. The results include position solutions obtained without quality control, with the conventional procedure and with the extended procedure. The results for the conventional and extended procedures shown in black correspond to solutions comprising correlation coefficients greater than a specified level. It should be noted that, the specified level used herein is chosen empirically and requires further investigation to determine a theoretical value depending on redundancy, significance levels and other specific requirements. The conventional procedure performs the separability test as part of every adjustment, while the extended procedure performs it for only the first and last adjustments. Even though only the information of the first adjustment is used for outlier identification the final test is included as an additional integrtity check. All plotted solutions, except for the 'without RAIM' solutions, have passed a final variance factor test with a confidence level of 99%.

As an example, separability warning results are given for the GPS/GLONASS/Galileo 4 outlier scenario in Figure 1. In this case the critical value for the separability warning was selected as 0.60. In this case the conventional procedure with separability warning performs very well. The separability warning enabled extended procedure also works well albeit some small outlying solutions. It is possible that a lower critical value for the separability warning is required by the extended procedure. However, some further and extensive investigations will be required to determine this. Here, both procedures with the separability warning flag some inlying solutions with high spatial correlations.

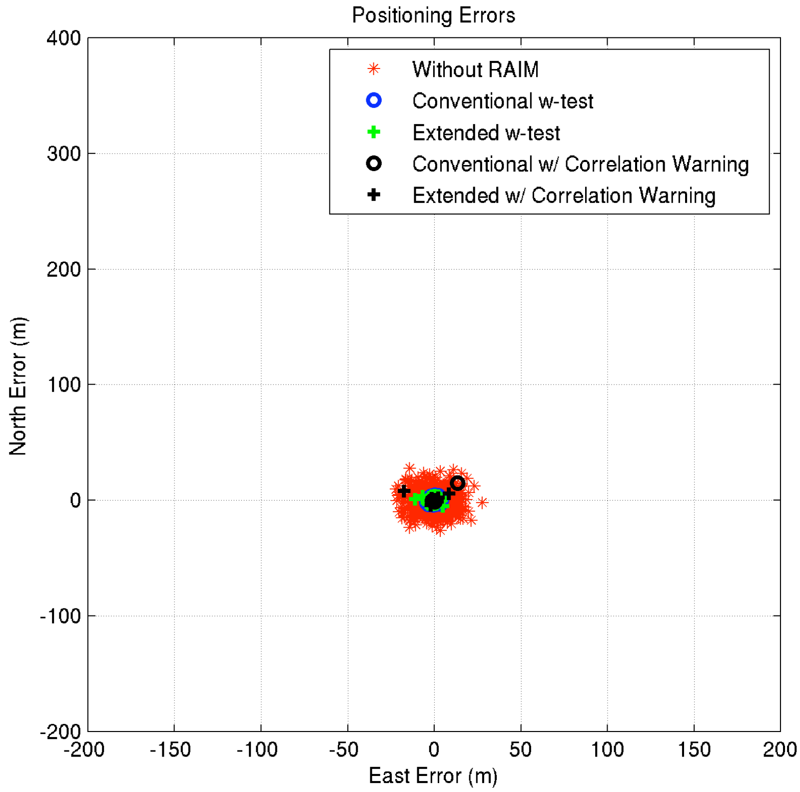

Figure 1. Positioning error results, with and without separability warnings, for the GPS/GLONASS/Galileo configuration and 4 outliers per epoch (warning value, $\rho > 0.60$).

A 5m positioning error threshold was chosen to examine the impact of the separability warning on the GPS/GLONASS/Galileo configuration with 4 outliers. In Figure 2, the results for the same same scenario with the corrleation coefficient lowered to 0.5 are shown. For the results using the 0.6 separability warning value, the conventional procedure flagged 0.23% of the solutions with a positioning error of less than 5m and missed 0% of those with positioning errors greater than 5m. The extended procedure flagged 1.53% of the inlying solutions with a separability warning and missed 60.00% of the outlying solutions. With the separability warning value dropped to 0.5, see Table 10, the conventional procedure flagged 5.83% of the inlying solutions and missed 33.33% of the outlying solutions. The results for extended procedure showed that 8.20% of the inlying solutions were flagged and 0% of the outlying solutions were flagged.

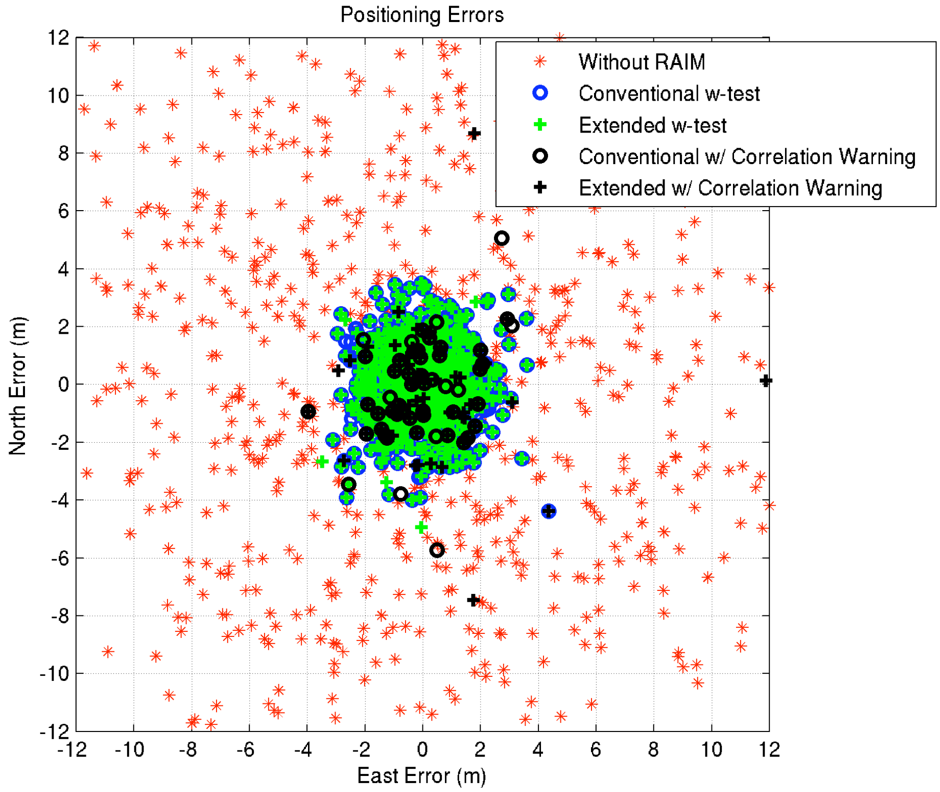

Figure 2. Zoomed in positioning error results, with and without separability warnings, for the GPS/GLONASS/Galileo configuration and 4 outliers per epoch (warning value, $\rho > 0.50$).

These results suggest that the conventional approach with separability warnings may be better than the extended procedure with separability warning for strong geometries such as the GPS/GLONASS/Galileo configuration but worse for weaker geometries such as the GPS/Galileo of GPS-only configurations. It should also be noted that the GPS-only configuration has been shown to be poorly separable and therefore highly unreliable with respect to multiple outliers. The results also show that the implementation of separability warnings is an effective way of reducing the number of contaminated and unacceptable solutions, which pass the other integrirty checks.

6. Conclusions.

Next generation RAIM algorithms need to be developed to match the advances being made in GNSS. The greater levels of visibility, which will be delivered to GNSS users with the advent of Galileo and the restored GLONASS system, will increase the likelihood of multiple fault occurrences. However, even though the *w*-test statistics are uniformly-most-powerful-invariant test statistics it was not originally developed for multiple fault identification. The extended *w*-test can be implemented to improve on the conventional procedure in terms of the computational efficiency, with comparable or even better levels of detection rates as the conventional procedure under certain conditions, albeit at the expense of greater levels of false alarms. However, initial results suggest that the extended procedure may be a better option for weaker geometries. Dealing with multiple outliers is inevitably a difficult task with obvious risks due to their combined influence on the solution. The first priority in dealing with such circumstances should be to identify when these risks are significant. The separability information is ideally suited to such tasks and the concept of a separability warning measure has been introduced in this paper for the first time. However, optimizing the utilisation of this information still requires extensive research.

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